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Mobile Robotics

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Lecture 3

Outline

- Wheeled locomotion
- Coordinate system & math
- Kinematics

Motivation

- Autonomous mobile robots move around in the environment. Therefore **ALL** of them:
  - They need to know **where** they **are**.
  - They need to know **where** their **goal** is.
  - They need to know **how** to get there.

- **Odometry!**
- Robot:
  - I know how fast the wheels turned =>
  - I know how the robot moved =>
  - I know where I am ☺

Odometry

- Robot:
  - I know how fast the wheels turned =>
  - I know how the robot moved =>
  - I know where I am ☺

- Marine Navigation: Dead reckoning (using heading sensor)

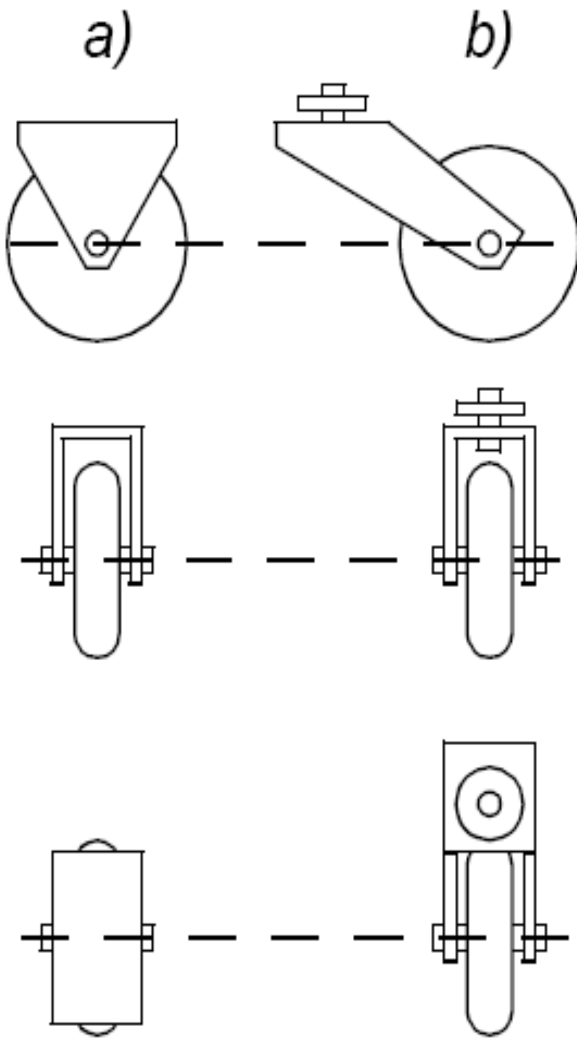
- Sources of error (AMR pages 269 - 270):
  - Wheel slip
    - Uneven floor contact (non-planar surface)
    - Robot kinematic: tracked vehicles, 4 wheel differential drive..
  - Integration from speed to position: Limited resolution (time and measurement)
  - Wheel misalignment
  - Wheel diameter uncertainty
  - Variation in contact point of wheel

Mobile Robots with Wheels

- Wheels are the most appropriate solution for most applications
- Three wheels are sufficient to guarantee stability
- With more than three wheels an appropriate suspension is required
- Selection of wheels depends on the application

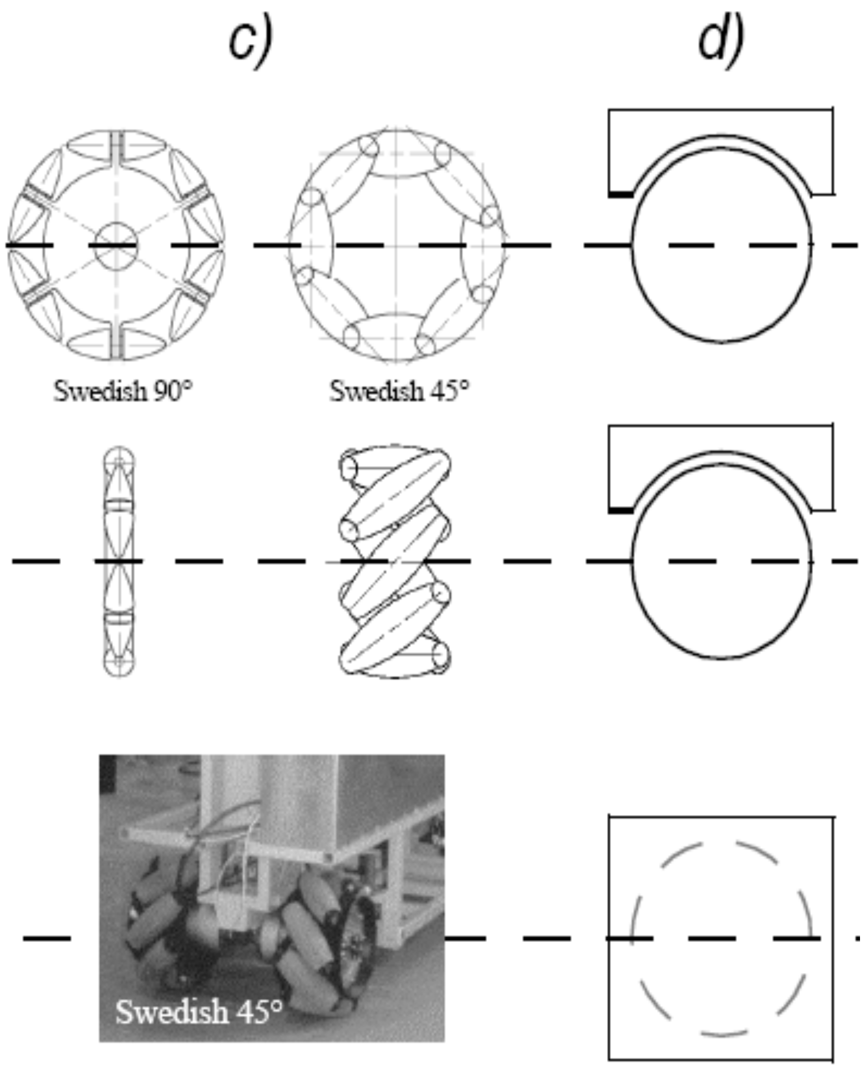
The Four Basic Wheels Types

- a) Standard wheel: Two degrees of freedom; rotation around the (motorized) wheel axle and the contact point
- b) Castor wheel: Three degrees of freedom; rotation around the wheel axle, the contact point and the castor axle



The Four Basic Wheels Types

- c) Swedish wheel: Three degrees of freedom; rotation around the (motorized) wheel axle, around the rollers and around the contact point
- d) Ball or spherical wheel: Suspension technically not solved

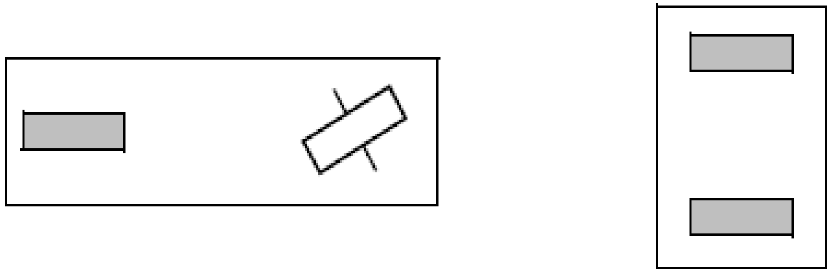


Characteristics of Wheeled Robots and Vehicles

- Stability of a vehicle is be guaranteed with 3 wheels
  - center of gravity is within the triangle with is formed by the ground contact point of the wheels.
- Stability is improved by 4 and more wheel
  - however, this arrangements are hyperstatic and require a flexible suspension system.
- Bigger wheels allow to overcome higher obstacles
  - but they require higher torque or reductions in the gear box.
- Most arrangements are non-holonomic (see chapter 3)
  - require high control effort
- Combining actuation and steering on one wheel makes the design complex and adds additional errors for odometry.

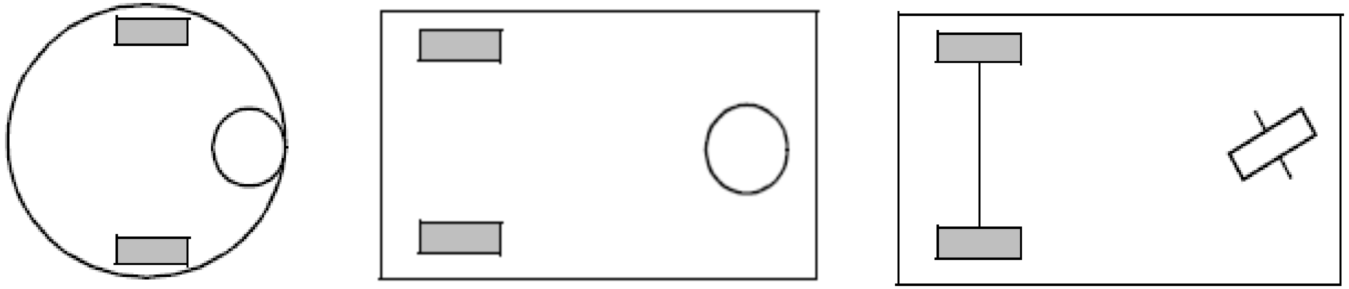
Different Arrangements of Wheels I

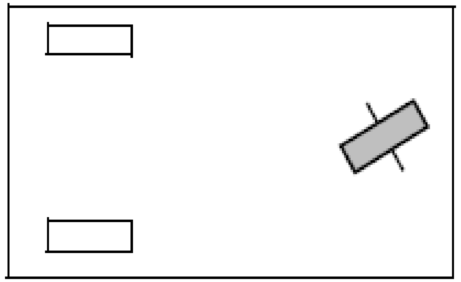
- Two wheels

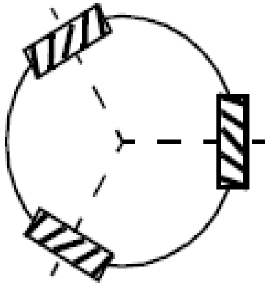


Center of gravity below axle

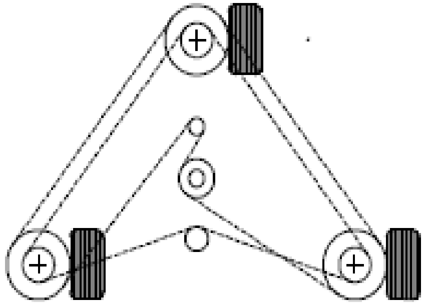
- Three wheels







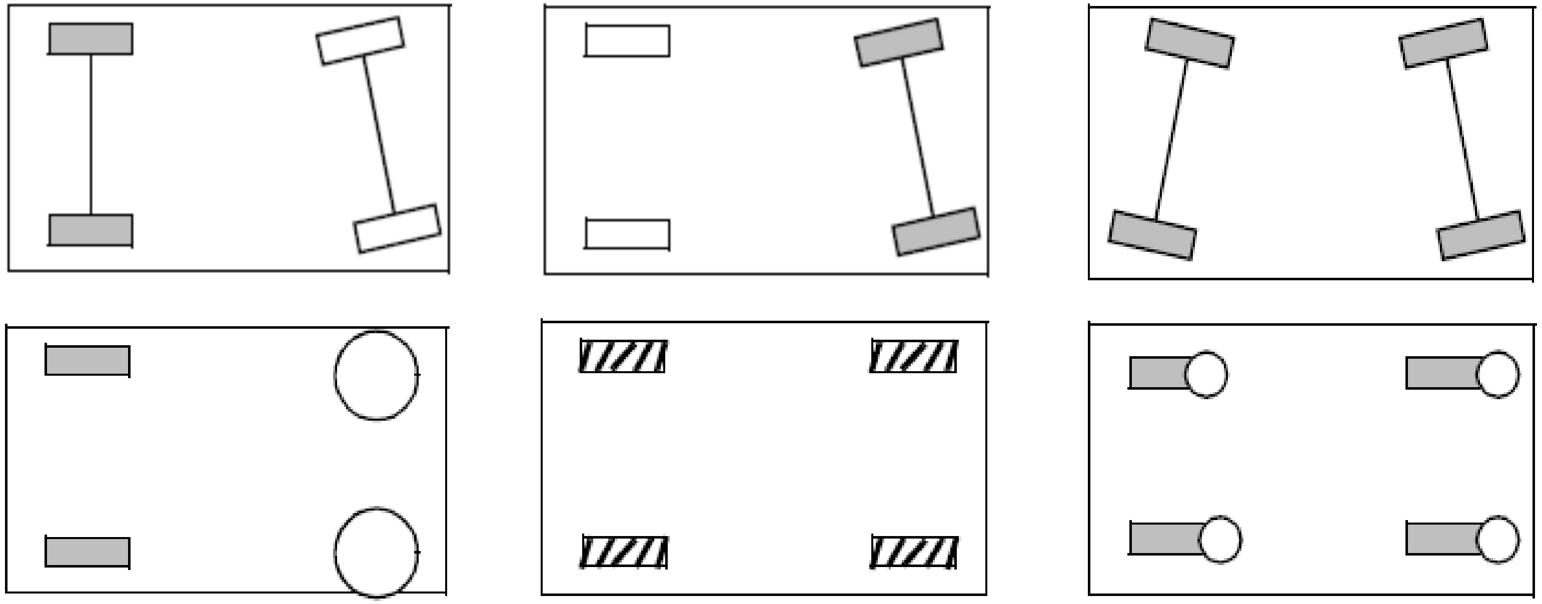
Omnidirectional Drive



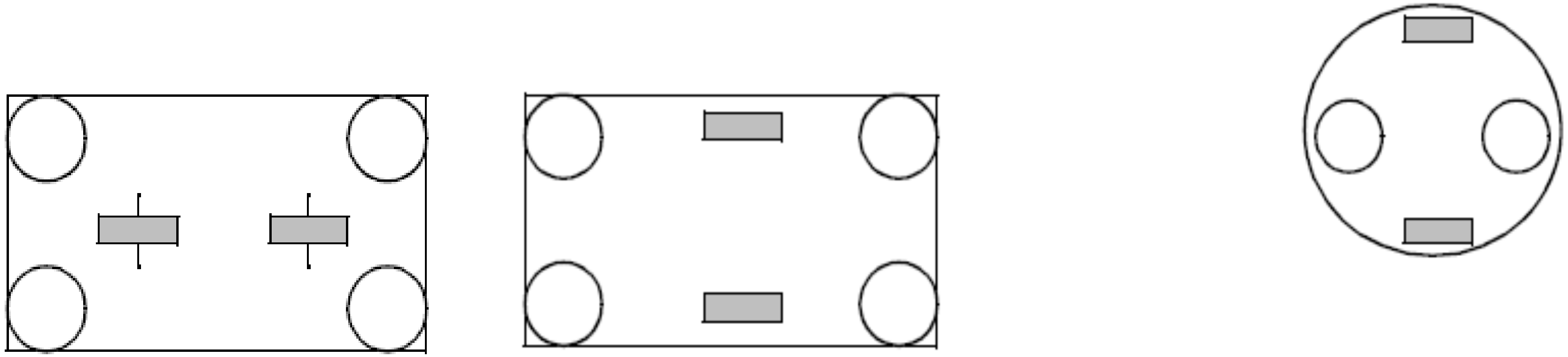
Synchro Drive

Different Arrangements of Wheels II

- Four wheels

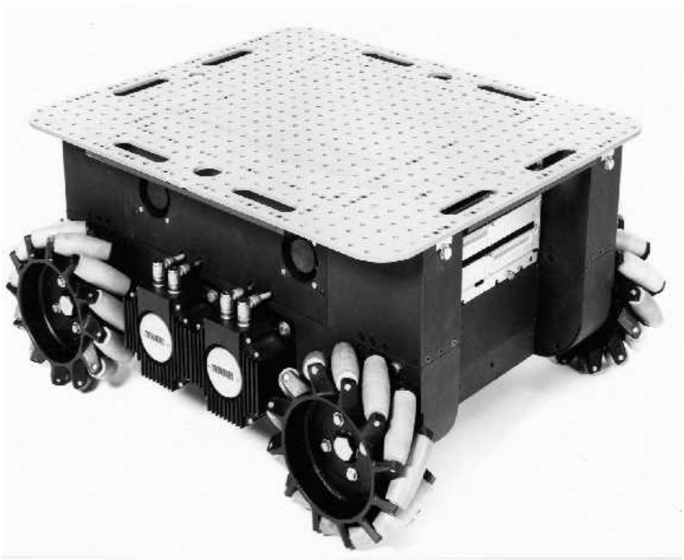


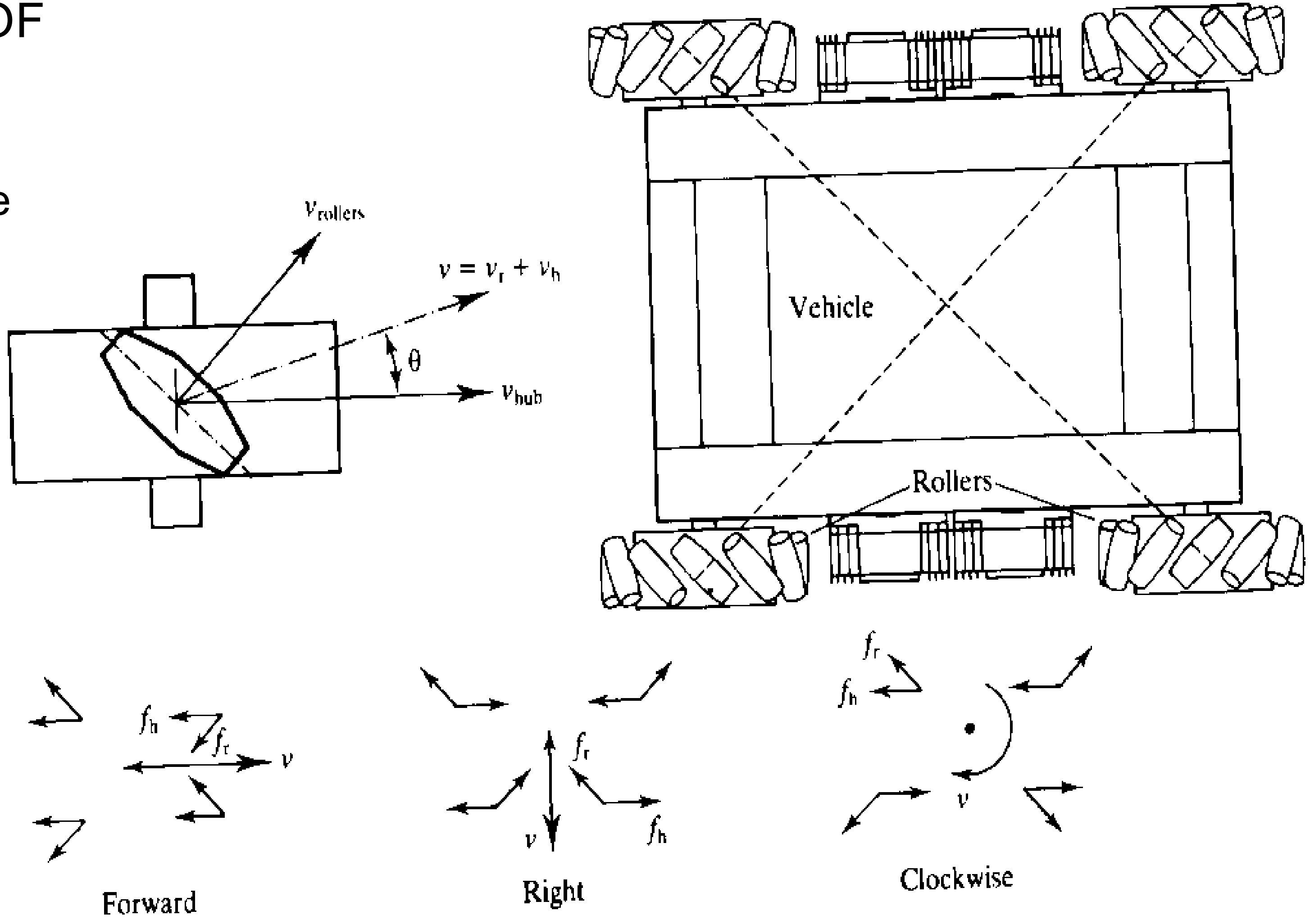
- Six wheels



Uranus, CMU: Omnidirectional Drive with 4 Wheels

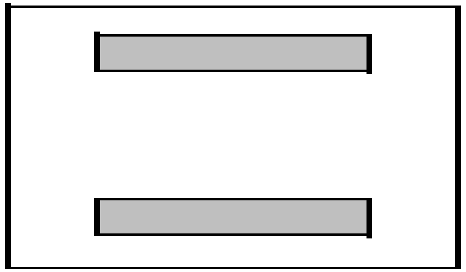
- Movement in the plane has 3 DOF
  - thus only three wheels can be independently controlled
  - It might be better to arrange three swedish wheels in a triangle





Rugbot, Jacobs Robotics: Tracked Differential Drive

- Kinematic Simplification:
  - 2 Wheels, located at the center



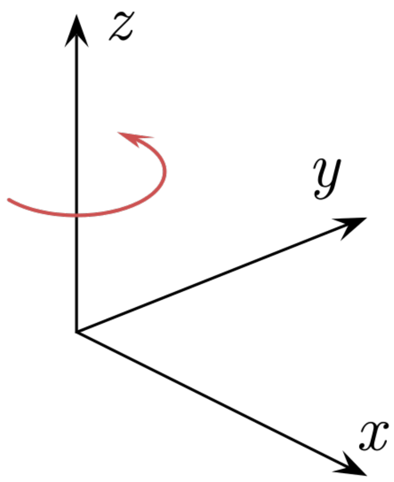
Introduction: Mobile Robot Kinematics

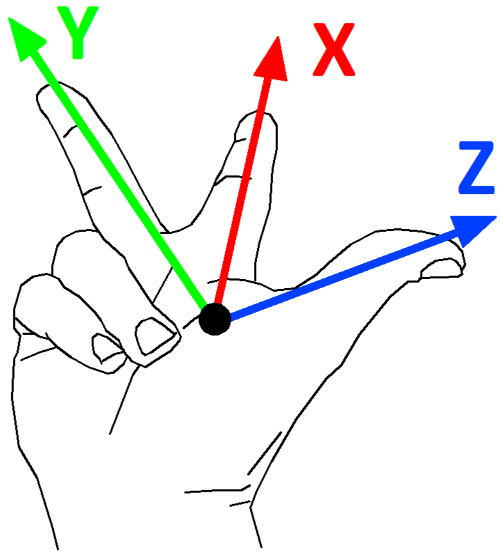
- Aim
  - Description of mechanical behavior of the robot for *design* and *control*
  - Similar to robot manipulator kinematics
  - However, mobile robots can move unbound with respect to its environment
    - there is no direct way to measure the robot's position
    - Position must be integrated over time
    - Leads to inaccuracies of the position (motion) estimate
      -> the number 1 challenge in mobile robotics

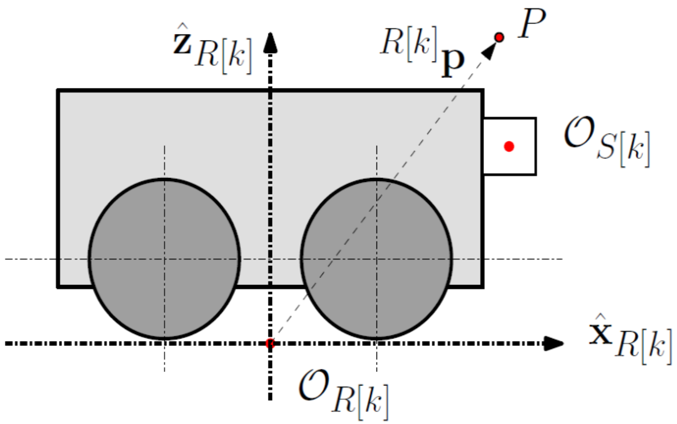
COORDINATE SYSTEM

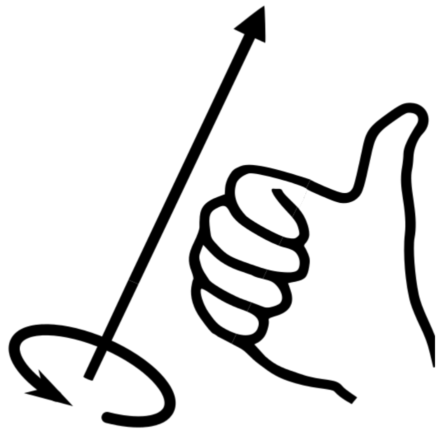
Right Hand Coordinate System

- Standard in Robotics
- Positive rotation around X is anti-clockwise
- Right-hand rule mnemonic:
  - Thumb: z-axis
  - Index finger: x-axis
  - Second finger: y-axis
  - Rotation: Thumb = rotation axis, positive rotation in finger direction
- Robot Coordinate System:
  - X front
  - Z up (Underwater: Z down)
  - Y ???



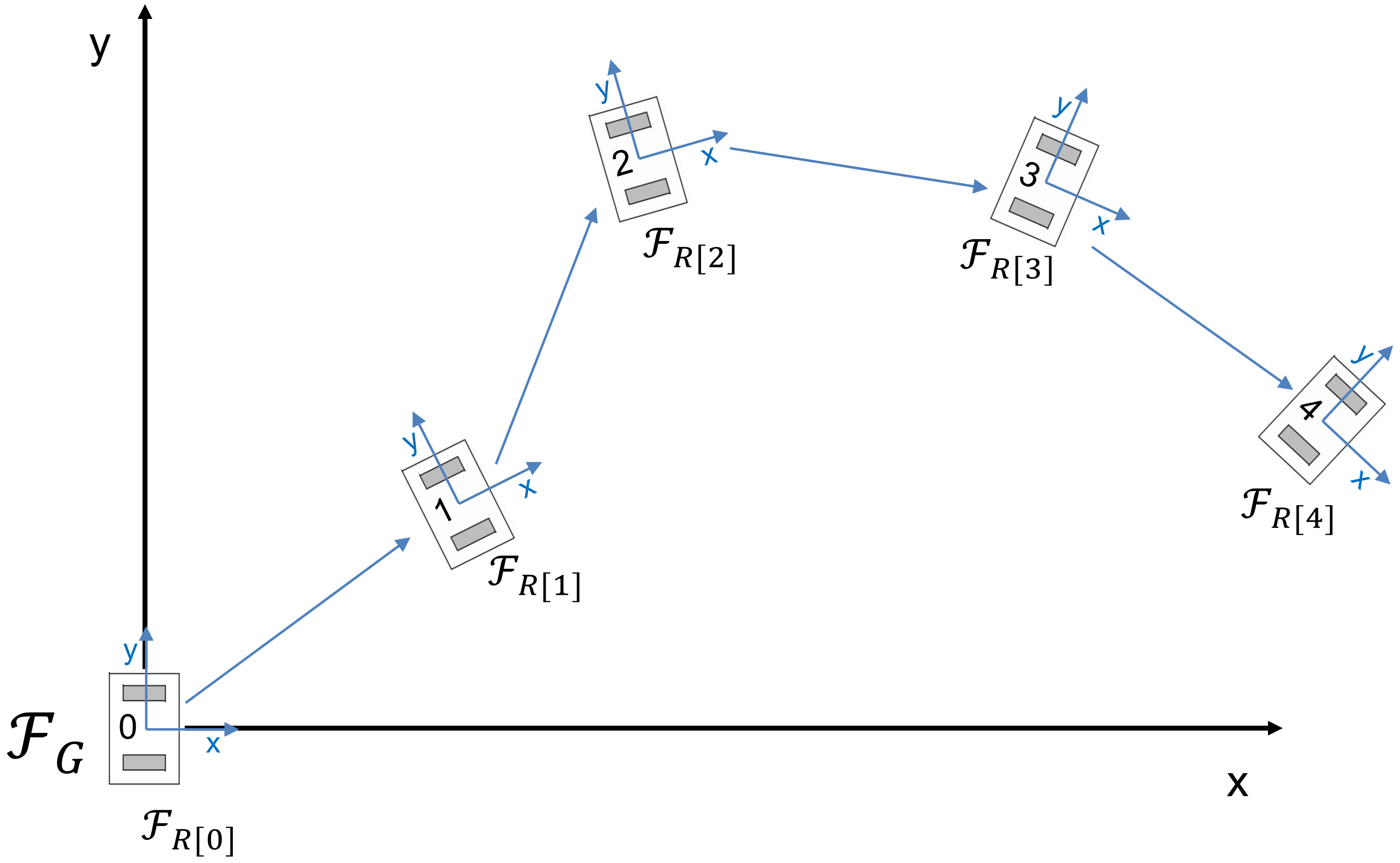






Right Hand Rule http://en.wikipedia.org/wiki/Right-hand_rule

Odometry



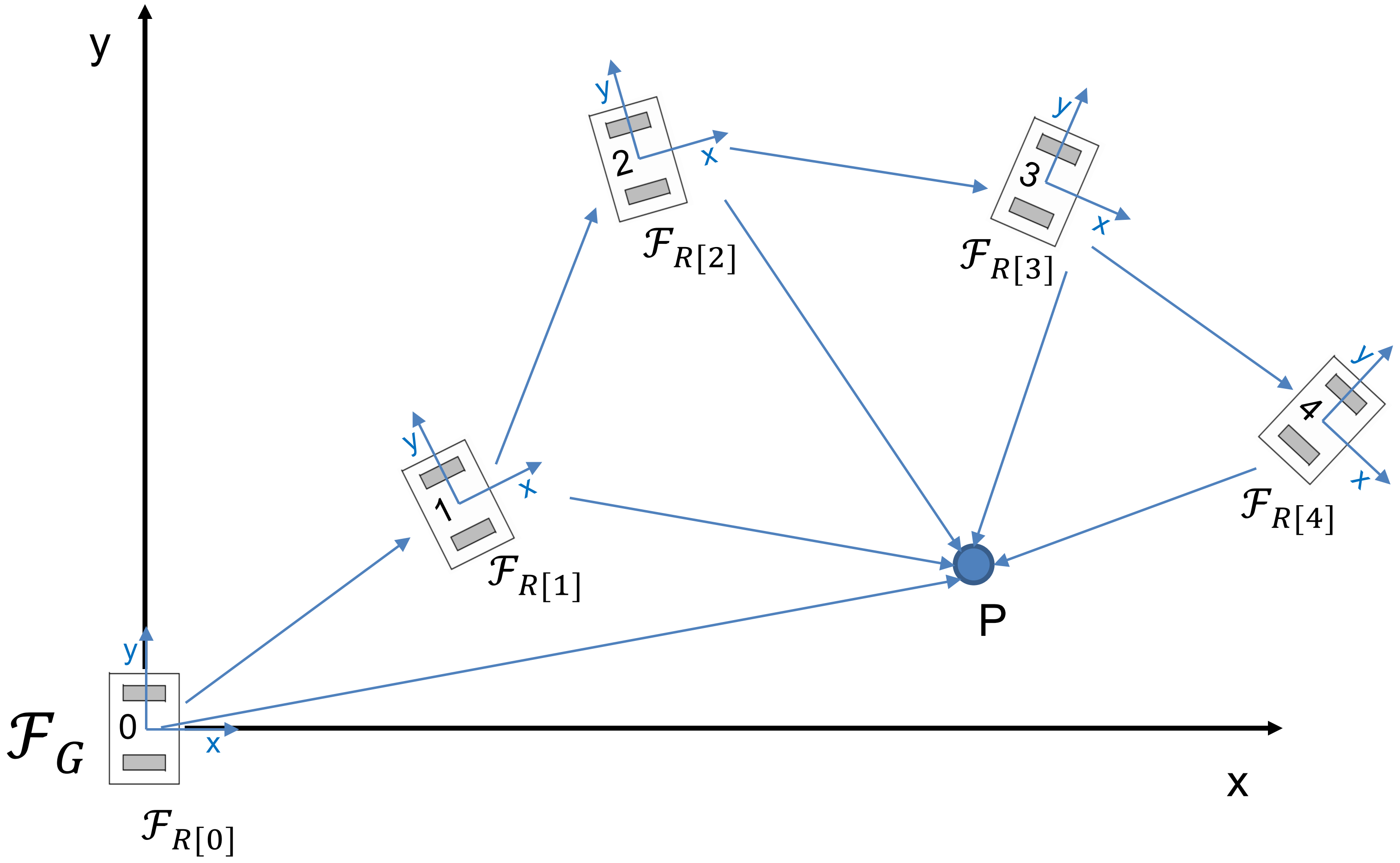
With respect to the robot start pose:
Where is the robot now?

Two approaches – same result:

- Geometry (easy in 2D)
- Transforms (better for 3D)

$\mathcal{F}_{R[X]}$: The ***F***rame of reference (the local coordinate system) of the ***R***obot at the time ***X***

Use of robot frames $\mathcal{F}_{R[X]}$

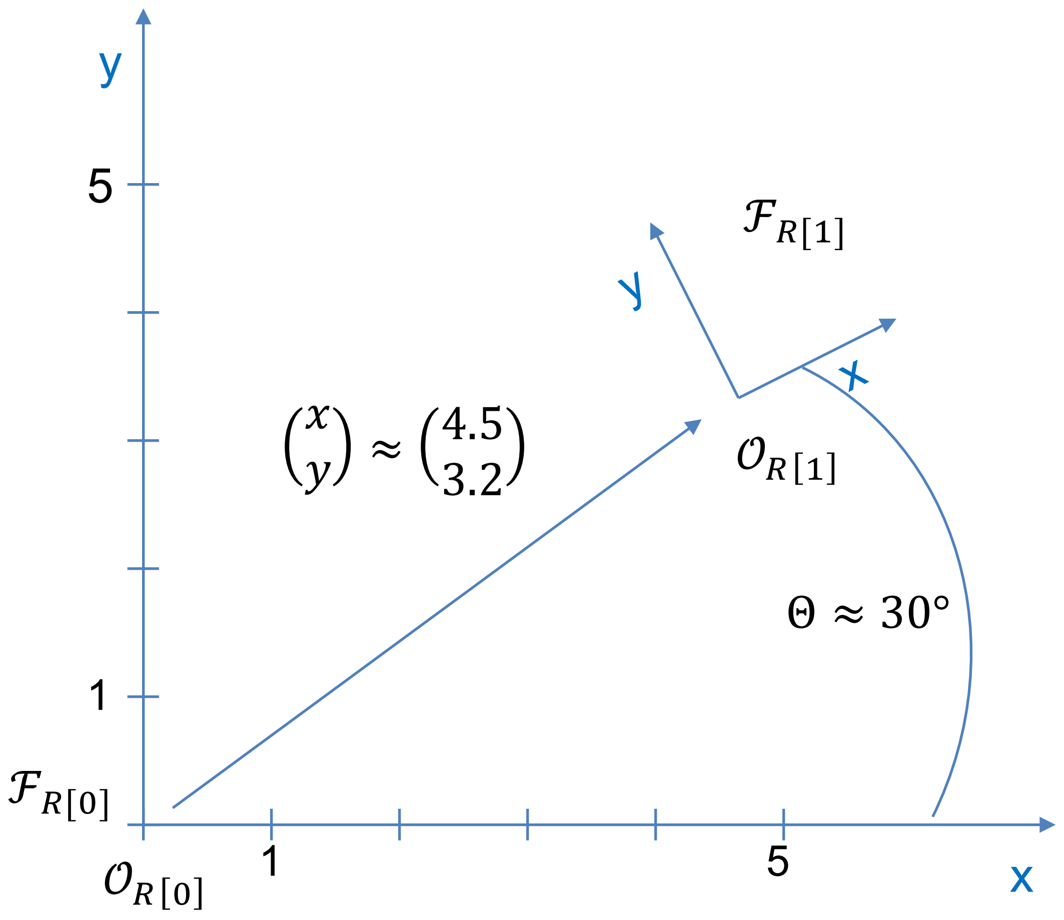


$\mathcal{O}_{R[X]}$: Origin of $\mathcal{F}_{R[X]}$ (coordinates (0, 0)

$\overrightarrow{\mathcal{O}_{R[X]}\mathrm{P}}$: position vector from $\mathcal{O}_{R[X]}$ to point P - $\binom{x}{y}$

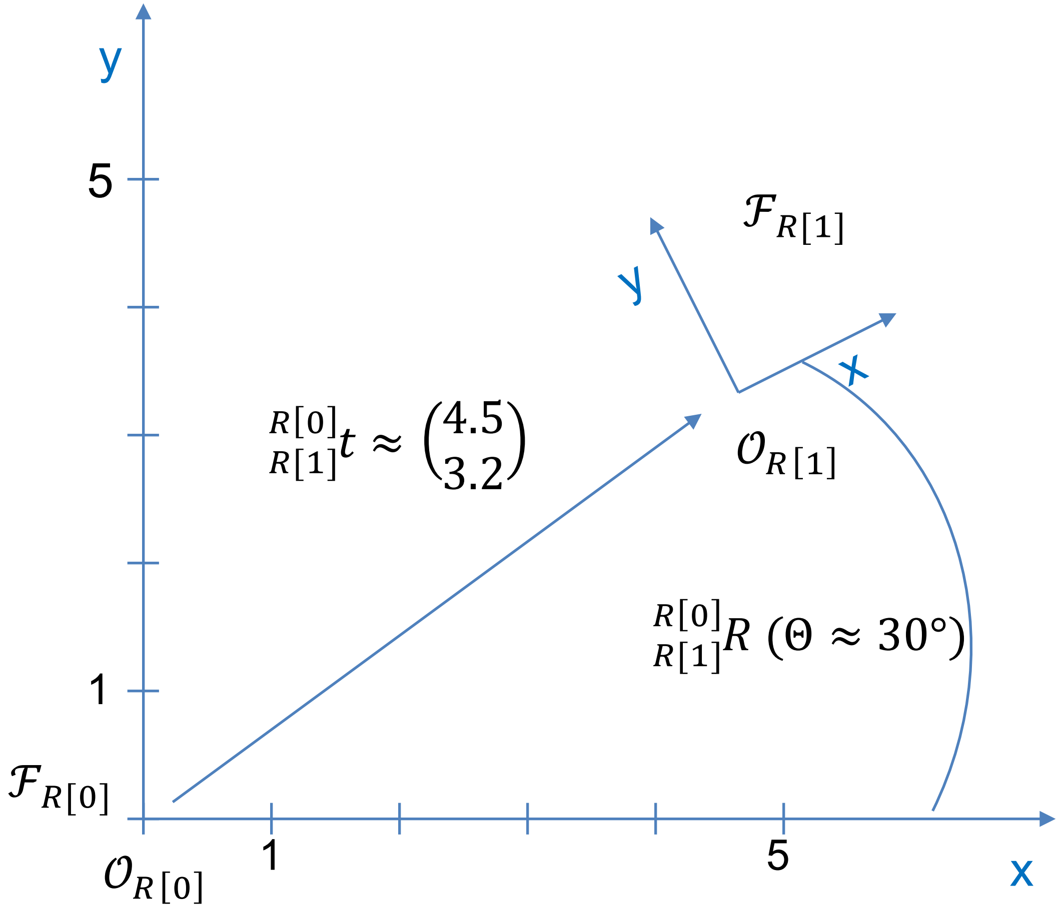
- Object P is observed at times 0 to 4
- Object P is static (does not move)
- The Robot moves (e.g. $\mathcal{F}_{R[0]} \neq \mathcal{F}_{R[1]}$)
- => (x, y) coordinates of P are different in all frames, for example:
  - $\overrightarrow{\mathcal{O}_{R[0]}\mathrm{P}} \neq \overrightarrow{\mathcal{O}_{R[1]}\mathrm{P}}$

Position, Orientation & Pose



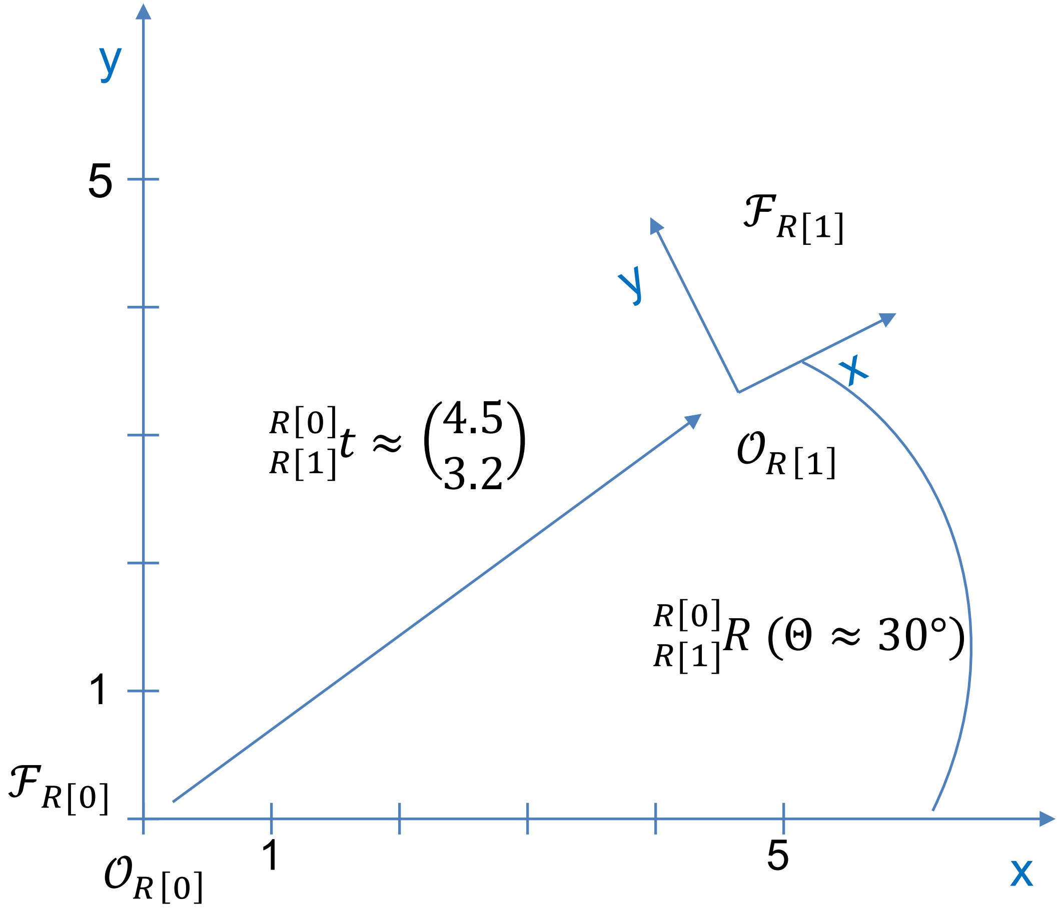
- **Position**:
  - $\begin{pmatrix} x \\ y \end{pmatrix}$ coordinates of any object or point (or another frame)
  - with respect to (wrt.) a specified frame
- **Orientation**:
  - (Θ) angle of any oriented object (or another frame)
  - with respect to (wrt.) a specified frame
- **Pose**:
  - $\begin{pmatrix} x \\ y \\ \Theta \end{pmatrix}$ position and orientation of any oriented object
  - with respect to (wrt.) a specified frame

Translation, Rotation & Transform



- **Translation**:
  - $\begin{pmatrix} x \\ y \end{pmatrix}$ difference, change, motion from one reference frame to another reference frame
- **Rotation**:
  - (Θ) difference in angle, rotation between one reference frame and another reference frame
- **Transform**:
  - $\begin{pmatrix} x \\ y \\ \Theta \end{pmatrix}$ difference, motion between one reference frame and another reference frame

Position & Translation, Orientation & Rotation



- $\mathcal{F}_{R[X]}$: Frame of reference of the robot at time X
- Where is that frame $\mathcal{F}_{R[X]}$?
  - Can only be expressed with respect to (wrt.) another frame (e.g. global Frame $\mathcal{F}_G$) =>
  - Pose of $\mathcal{F}_{R[X]}$ wrt. $\mathcal{F}_G$

- $\mathcal{O}_{R[X]}$: Origin of $\mathcal{F}_{R[X]}$
  - $\overrightarrow{\mathcal{O}_{R[X]}\mathcal{O}_{R[X+1]}}$: **Position** of $\mathcal{F}_{R[X+1]}$ wrt. $\mathcal{F}_{R[X]}$

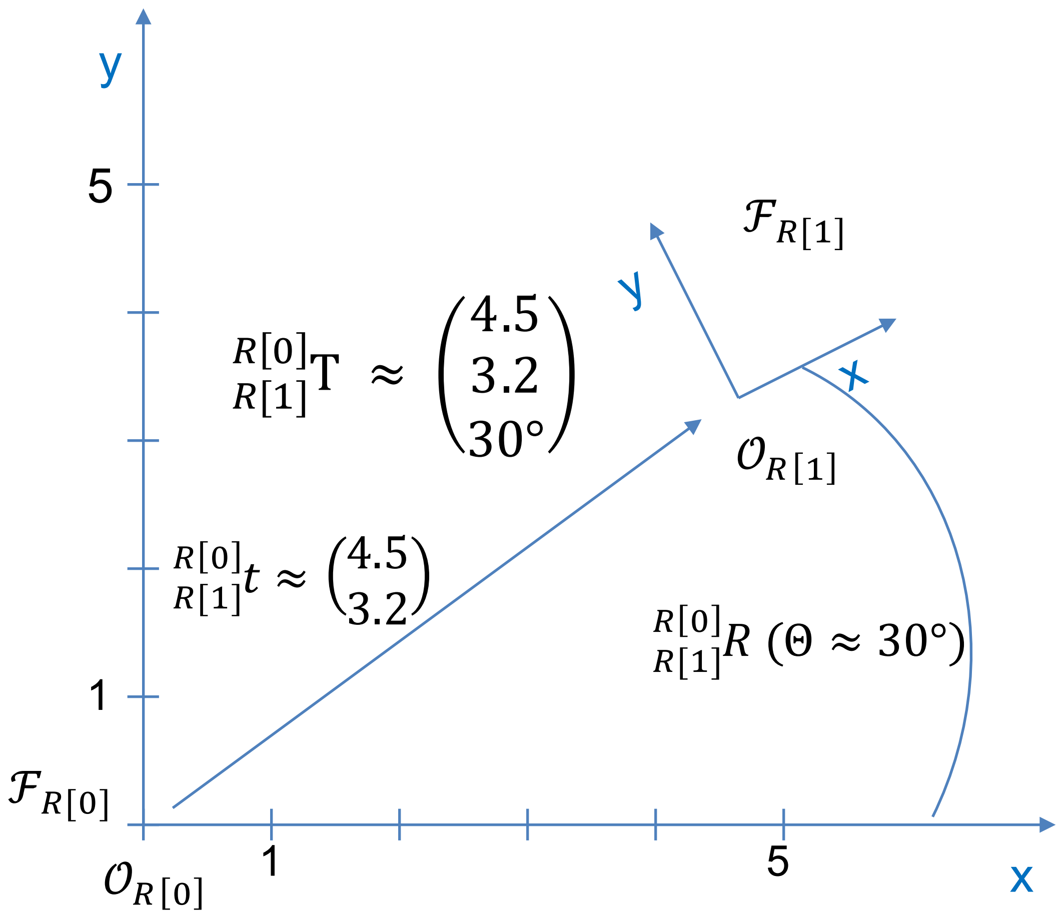
    to $\mathcal{O}_{R[X+1]}$ wrt. $\mathcal{F}_{R[X]}$

    $\triangleq\ ^{R[X]}_{R[X+1]}t$: **Translation**

- The angle Θ between the x-Axes:
  - **Orientation** of $\mathcal{F}_{R[X+1]}$ wrt. $\mathcal{F}_{R[X]}$

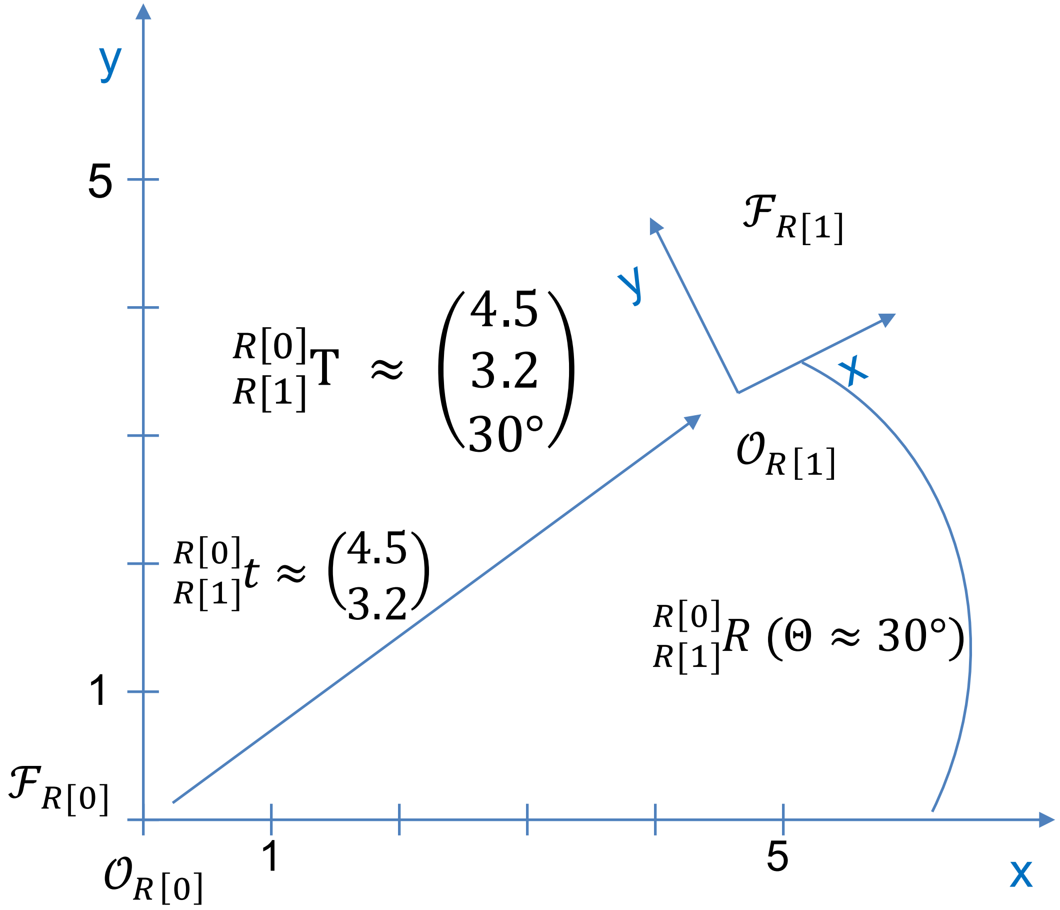
    $\triangleq\ ^{R[X]}_{R[X+1]}R$: **Rotation** of $\mathcal{F}_{R[X+1]}$ wrt. $\mathcal{F}_{R[X]}$

Transform



- ${}^{R[X]}_{R[X+1]}t$: **Translation**
  - Position vector (x, y) of $R[X+1]$ wrt. $R[X]$
- ${}^{R[X]}_{R[X+1]}R$: **Rotation**
  - Angle (Θ) of $R[X+1]$ wrt. $R[X]$
- **Transfrom:** ${}^{R[X]}_{R[X+1]}\mathrm{T} \equiv \begin{Bmatrix} {}^{R[X]}_{R[X+1]}t \\ {}^{R[X]}_{R[X+1]}R \end{Bmatrix}$

Geometry approach to Odometry



We want to know:

- Position of the robot (x, y)
- Orientation of the robot (Θ)
- => together: Pose $\begin{pmatrix} x \\ y \\ \Theta \end{pmatrix}$

With respect to (wrt.) $\mathcal{F}_G$: The global frame; global coordinate system

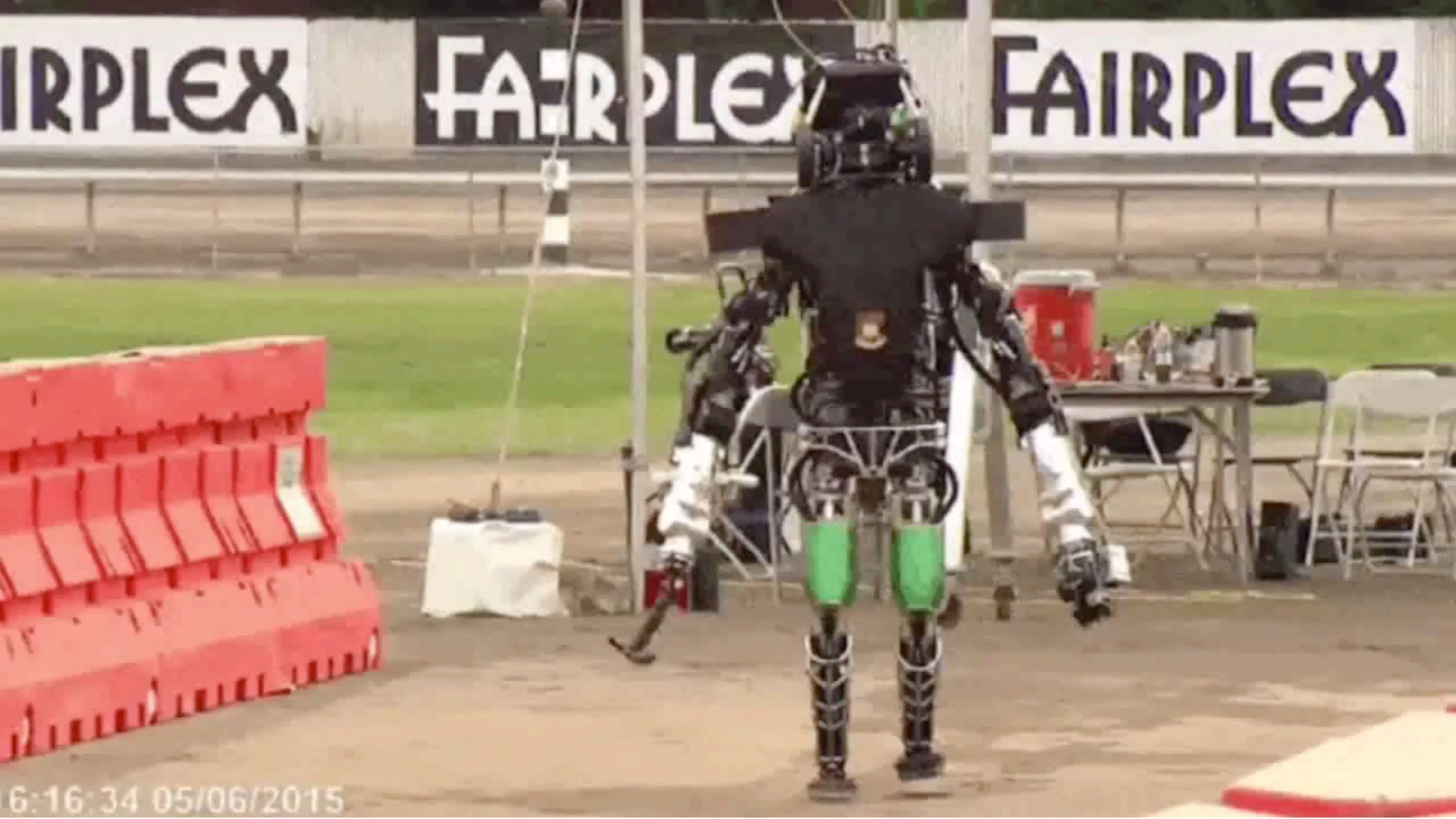
$$\mathcal{F}_{R[0]} = \mathcal{F}_G \Rightarrow \ {}^{G}\mathcal{F}_{R[0]} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$${}^{G}\mathcal{F}_{R[1]} = {}^{R[0]}_{R[1]}\mathrm{T} \approx \begin{pmatrix} 4.5 \\ 3.2 \\ 30° \end{pmatrix}$$

Blackboard: ${}^{R[1]}_{R[2]}\mathrm{T} \approx \begin{pmatrix} 2 \\ 3 \\ 60° \end{pmatrix}$

something else...

- Good hardware for robotics is important
- Good software is also essential!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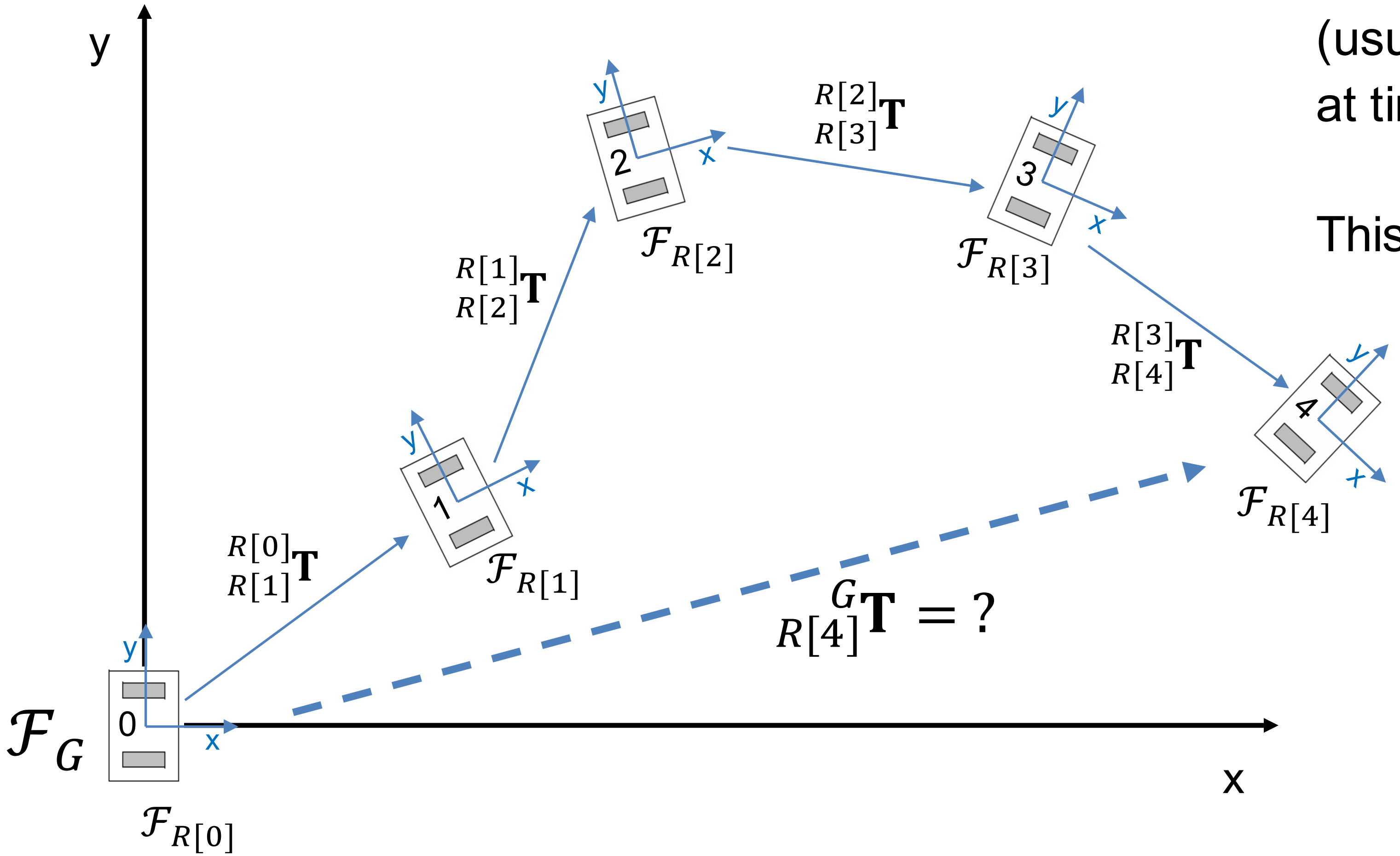
HW1

Mathematical approach: Transforms



Where is the Robot now?

The pose of $\mathcal{F}_{R[X]}$ with respect to $\mathcal{F}_G$ (usually = $\mathcal{F}_{R[0]}$) is the pose of the robot at time X.

This is equivalent to ${}^{G}_{R[X]}\mathbf{T}$

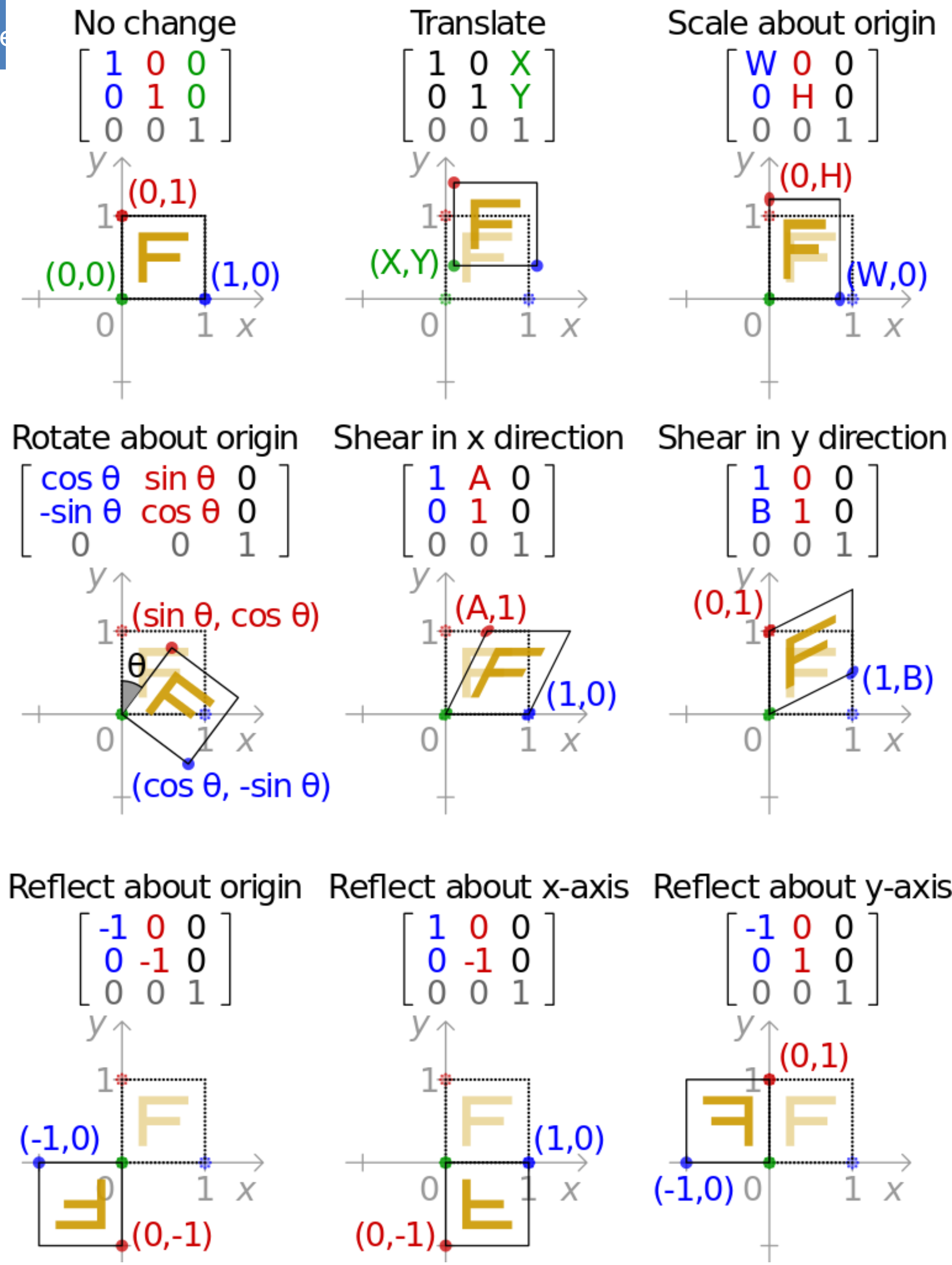
Chaining of Transforms

$$ {}^{G}_{R[X+1]}\mathbf{T} = {}^{G}_{R[X]}\mathbf{T} \; {}^{R[X]}_{R[X+1]}\mathbf{T} $$

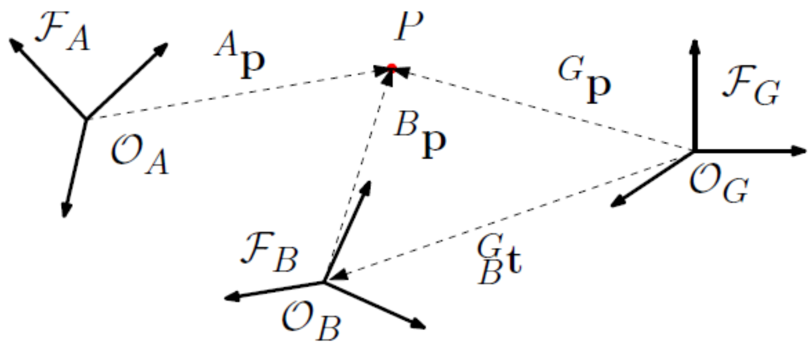
often: $\mathcal{F}_G \equiv \mathcal{F}_{R[0]} \Rightarrow {}^{G}_{R[0]}\mathbf{T} = id$

Affine Transformation

- Function between affine spaces. Preserves:
  - points,
  - straight lines
  - planes
  - sets of parallel lines remain parallel
- Allows:
  - Interesting for Robotics: translation, rotation, (scaling), and chaining of those
  - Not so interesting for Robotics: reflection, shearing, homothetic transforms
- Rotation and Translation: $\begin{bmatrix} \cos\theta & \sin\theta & X \\ -\sin\theta & \cos\theta & Y \\ 0 & 0 & 1 \end{bmatrix}$



Transform



Notation	Meaning
$\mathcal{F}_{\mathrm{R}[k]}$	Coordinate frame attached to object 'R' (usually the robot) at sample time-instant k.
$\mathcal{O}_{\mathrm{R}[k]}$	Origin of $\mathcal{F}_{\mathrm{R}[k]}$.
${}^{\mathrm{R}[k]}\mathbf{p}$	For any general point P, the position vector $\overrightarrow{\mathcal{O}_{\mathrm{R}[k]}P}$ resolved in $\mathcal{F}_{\mathrm{R}[k]}$.
${}^{\mathrm{H}}\hat{\mathbf{x}}_{\mathrm{R}}$	The x-axis direction of $\mathcal{F}_{\mathrm{R}}$ resolved in $\mathcal{F}_{\mathrm{H}}$. Similarly, ${}^{\mathrm{H}}\hat{\mathbf{y}}_{\mathrm{R}}$, ${}^{\mathrm{H}}\hat{\mathbf{z}}_{\mathrm{R}}$ can be defined. Obviously, ${}^{\mathrm{R}}\hat{\mathbf{x}}_{\mathrm{R}} = \hat{\mathrm{e}}_1$. Time indices can be added to the frames, if necessary.
${}^{\mathrm{R}[k]}_{\mathrm{S}[k']}\mathbf{R}$	The rotation-matrix of $\mathcal{F}_{\mathrm{S}[k']}$ with respect to $\mathcal{F}_{\mathrm{R}[k]}$.
${}^{\mathrm{R}}_{\mathrm{S}}\mathbf{t}$	The translation vector $\overrightarrow{\mathcal{O}_{\mathrm{R}}\mathcal{O}_{\mathrm{S}}}$ resolved in $\mathcal{F}_{\mathrm{R}}$.

Transform between two coordinate frames

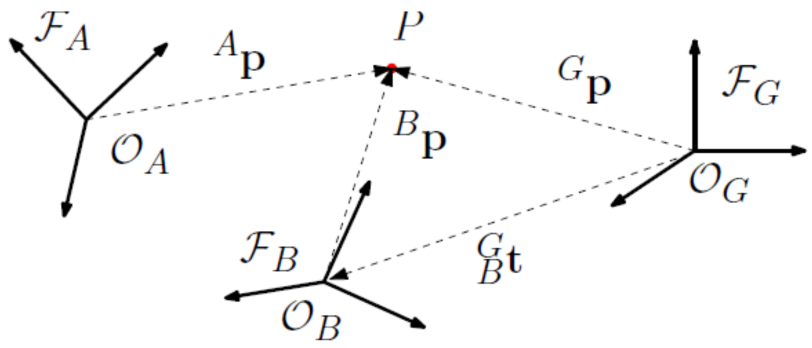
$${}^{\mathrm{G}}_{\mathrm{A}}\mathbf{t} \triangleq \overrightarrow{\mathcal{O}_{\mathrm{G}}\mathcal{O}_{\mathrm{A}}} \text{ resolved in } \mathcal{F}_{\mathrm{G}}$$

$$\begin{pmatrix} {}^{\mathrm{G}}\mathbf{p} \\ 1 \end{pmatrix} \equiv \begin{pmatrix} {}^{\mathrm{G}}_{\mathrm{A}}\mathbf{R} & {}^{\mathrm{G}}_{\mathrm{A}}\mathbf{t} \\ \mathbf{0}_{1\times[2,3]} & 1 \end{pmatrix} \begin{pmatrix} {}^{\mathrm{A}}\mathbf{p} \\ 1 \end{pmatrix} \qquad {}^{\mathrm{G}}_{\mathrm{A}}\mathbf{T} \equiv \left\{ \begin{matrix} {}^{\mathrm{G}}_{\mathrm{A}}\mathbf{t} \\ {}^{\mathrm{G}}_{\mathrm{A}}\mathbf{R} \end{matrix} \right\}$$

$$\begin{aligned} {}^{\mathrm{G}}\mathbf{p} &= {}^{\mathrm{G}}_{\mathrm{A}}\mathbf{R}\ {}^{\mathrm{A}}\mathbf{p} + {}^{\mathrm{G}}_{\mathrm{A}}\mathbf{t} \\ &\triangleq {}^{\mathrm{G}}_{\mathrm{A}}\mathbf{T}(\ {}^{\mathrm{A}}\mathbf{p}). \end{aligned}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta & {}^{\mathrm{G}}_{\mathrm{A}}t_x \\ \sin\theta & \cos\theta & {}^{\mathrm{G}}_{\mathrm{A}}t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Transform: Operations



Transform between two coordinate frames (chaining, compounding):

$$ {}^{G}_{B}\mathbf{T} = {}^{G}_{A}\mathbf{T}\,{}^{A}_{B}\mathbf{T} \equiv \begin{Bmatrix} {}^{G}_{A}\mathbf{R}\,{}^{A}_{B}\mathbf{t} + {}^{G}_{A}\mathbf{t} \\ {}^{G}_{A}\mathbf{R}\,{}^{A}_{B}\mathbf{R} \end{Bmatrix} $$

Inverse of a Transform :

$$ {}^{B}_{A}\mathbf{T} = {}^{A}_{B}\mathbf{T}^{-1} \equiv \begin{Bmatrix} -{}^{A}_{B}\mathbf{R}^{\mathsf{T}}\,{}^{A}_{B}\mathbf{t} \\ {}^{A}_{B}\mathbf{R}^{\mathsf{T}} \end{Bmatrix} $$

Relative (Difference) Transform :

$$ {}^{B}_{A}\mathbf{T} = {}^{G}_{B}\mathbf{T}^{-1}\,{}^{G}_{A}\mathbf{T} $$

See: **Quick Reference to Geometric Transforms in Robotics** by Kaustubh Pathak on the webpage!

Chaining :

$$ {}^{G}_{R[X+1]}\mathbf{T} = {}^{G}_{R[X]}\mathbf{T}\ {}^{R[X]}_{R[X+1]}\mathbf{T} \equiv \begin{Bmatrix} {}^{G}_{R[X]}\mathrm{R}\ {}^{R[X]}_{R[X+1]}t + {}^{G}_{R[X]}t \\ {}^{G}_{R[X]}\mathrm{R}\ {}^{R[X]}_{R[X+1]}\mathrm{R} \end{Bmatrix} = \begin{Bmatrix} {}^{G}_{R[X+1]}t \\ {}^{G}_{R[X+1]}\mathrm{R} \end{Bmatrix} $$

In 2D Translation:

$$ \begin{bmatrix} {}^{G}_{R[X+1]}t_x \\ {}^{G}_{R[X+1]}t_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos {}^{G}_{R[X]}\theta & -\sin {}^{G}_{R[X]}\theta & {}^{G}_{R[X]}\mathrm{t_x} \\ \sin {}^{G}_{R[X]}\theta & \cos {}^{G}_{R[X]}\theta & {}^{G}_{R[X]}\mathrm{t_y} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{R[X]}_{R[X+1]}\mathrm{t_x} \\ {}^{R[X]}_{R[X+1]}t_y \\ 1 \end{bmatrix} $$

In 2D Rotation:

$$ {}^{G}_{R[X+1]}\mathrm{R} = \begin{bmatrix} \cos {}^{G}_{R[X+1]}\theta & -\sin {}^{G}_{R[X+1]}\theta \\ \sin {}^{G}_{R[X+1]}\theta & \cos {}^{G}_{R[X+1]}\theta \end{bmatrix} = \begin{bmatrix} \cos {}^{G}_{R[X]}\theta & -\sin {}^{G}_{R[X]}\theta \\ \sin {}^{G}_{R[X]}\theta & \cos {}^{G}_{R[X]}\theta \end{bmatrix} \begin{bmatrix} \cos {}^{R[X]}_{R[X+1]}\theta & -\sin {}^{R[X]}_{R[X+1]}\theta \\ \sin {}^{R[X]}_{R[X+1]}\theta & \cos {}^{R[X]}_{R[X+1]}\theta \end{bmatrix} $$

In 2D Rotation (simple):

$$ {}^{G}_{R[X+1]}\theta = {}^{G}_{R[X]}\theta + {}^{R[X]}_{R[X+1]}\theta $$

In ROS

- First Message at time 97 : G
- Message at time 103 : X
- Next Message at time 107 : X+1

$$ {}^{G}_{R[X+1]}\mathbf{T} = {}^{G}_{R[X]}\mathbf{T} \; {}^{R[X]}_{R[X+1]}\mathbf{T} $$

$$ {}^{R[X]}_{R[X+1]}t_x $$

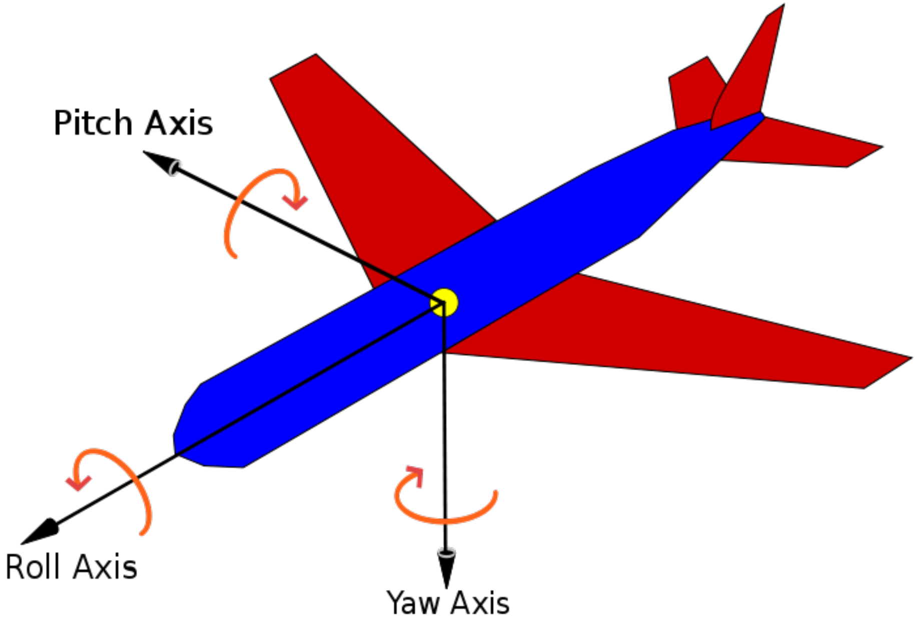
$$ {}^{R[X]}_{R[X+1]}t_y $$

$$ {}^{R[X]}_{R[X+1]}\Theta $$

```
std_msgs/Header header
  uint32 seq
  time stamp
  string frame_id
geometry_msgs/Pose2D pose2D
  float64 x
  float64 y
  float64 theta
```

3D Rotation

- Euler angles: Roll, Pitch, Yaw
  - ☹ Singularities
- Quaternions:
  - Concatenating rotations is computationally faster and numerically more stable
  - Extracting the angle and axis of rotation is simpler
  - Interpolation is more straightforward
  - Unit Quaternion: norm = 1
  - Scalar (real) part: q_0 , sometimes q_w
  - Vector (imaginary) part: $\mathbf{q}$
  - Over determined: 4 variables for 3 DoF



$$\check{\mathbf{p}} \equiv p_0 + p_x\mathbf{i} + p_y\mathbf{j} + p_z\mathbf{k}.$$

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$$

$$\check{\mathbf{q}} = \begin{pmatrix} q_0 & q_x & q_y & q_z \end{pmatrix}^\mathsf{T} \equiv \begin{pmatrix} q_0 \\ \mathbf{q} \end{pmatrix}$$

Transform in 3D

Matrix Euler Quaternion

$$ {}^{G}_{A}\mathbf{T} = \begin{bmatrix} {}^{G}_{A}R & {}^{G}_{A}t \\ 0_{1x3} & 1 \end{bmatrix} = \begin{pmatrix} {}^{G}_{A}t \\ {}^{G}_{A}\Theta \end{pmatrix} = \begin{pmatrix} {}^{G}_{A}t \\ {}^{G}_{A}\check{q} \end{pmatrix} $$

$$ {}^{G}_{A}\Theta \triangleq \left(\theta_r, \theta_p, \theta_y\right)^T $$

In ROS: Quaternions! (w, x, y, z)
Uses Bullet library for Transforms

Rotation Matrix 3x3

$$ R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} $$

$$ R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} $$

$$ R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

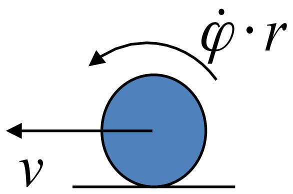
$$ R = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma) $$

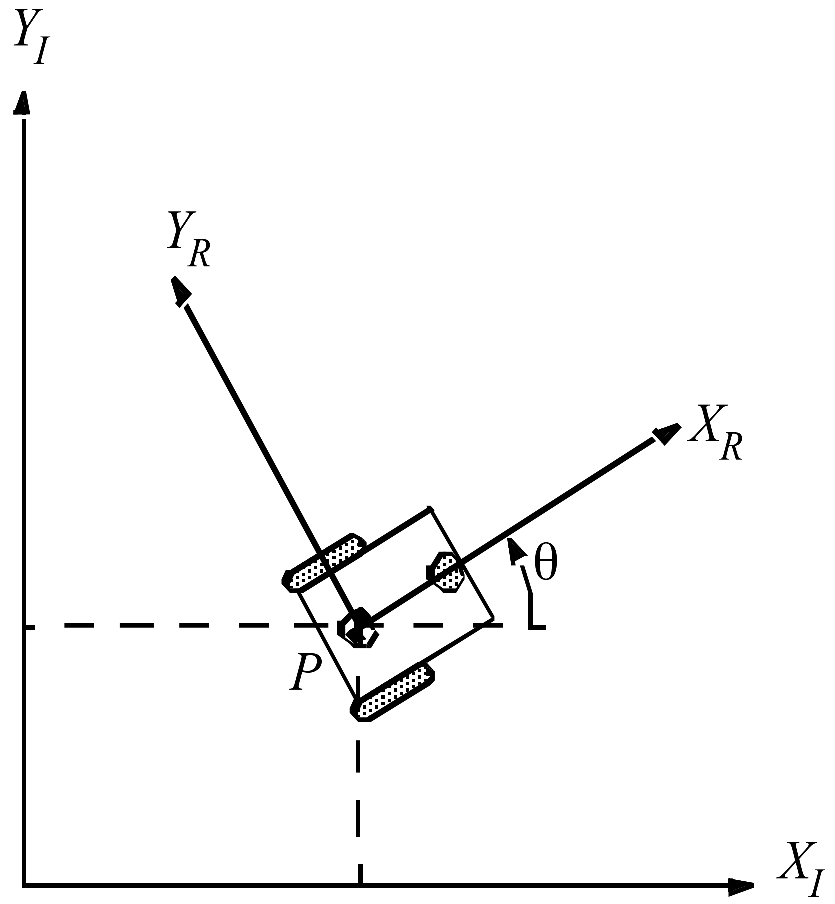
yaw = α, pitch = β, roll = γ

KINEMATICS

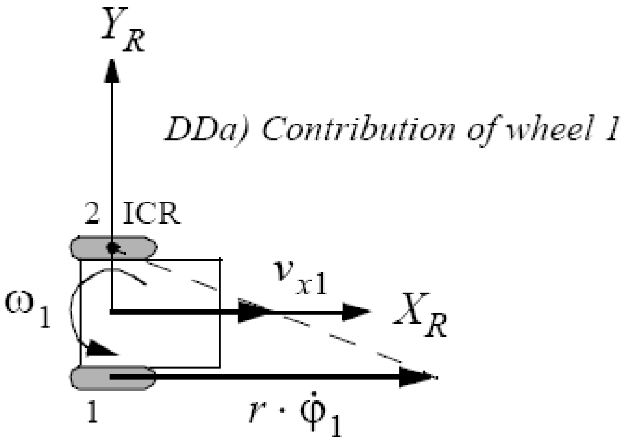
Wheel Kinematic Constraints: Assumptions

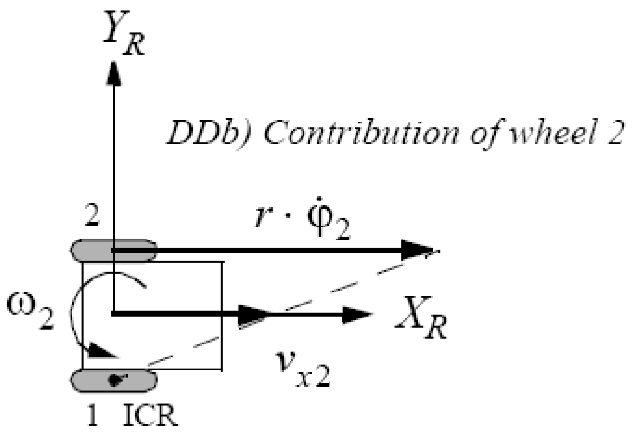
- Movement on a horizontal plane
- Point contact of the wheels
- Wheels not deformable
- Pure rolling
  - $v_c = 0$ at contact point
- No slipping, skidding or sliding
- No friction for rotation around contact point
- Steering axes orthogonal to the surface
- Wheels connected by rigid frame (chassis)





Forward Kinematic Model: Geometric Approach





Differential-Drive:

DDa) $v_{x1} = \frac{1}{2} r\dot{\varphi}_1 \quad ; \quad v_{y1} = 0 \quad ; \quad \omega_1 = \frac{1}{2l} r\dot{\varphi}_1$

DDb) $v_{x2} = \frac{1}{2} r\dot{\varphi}_2 \quad ; \quad v_{y2} = 0 \quad ; \quad \omega_2 = -\frac{1}{2l} r\dot{\varphi}_2$

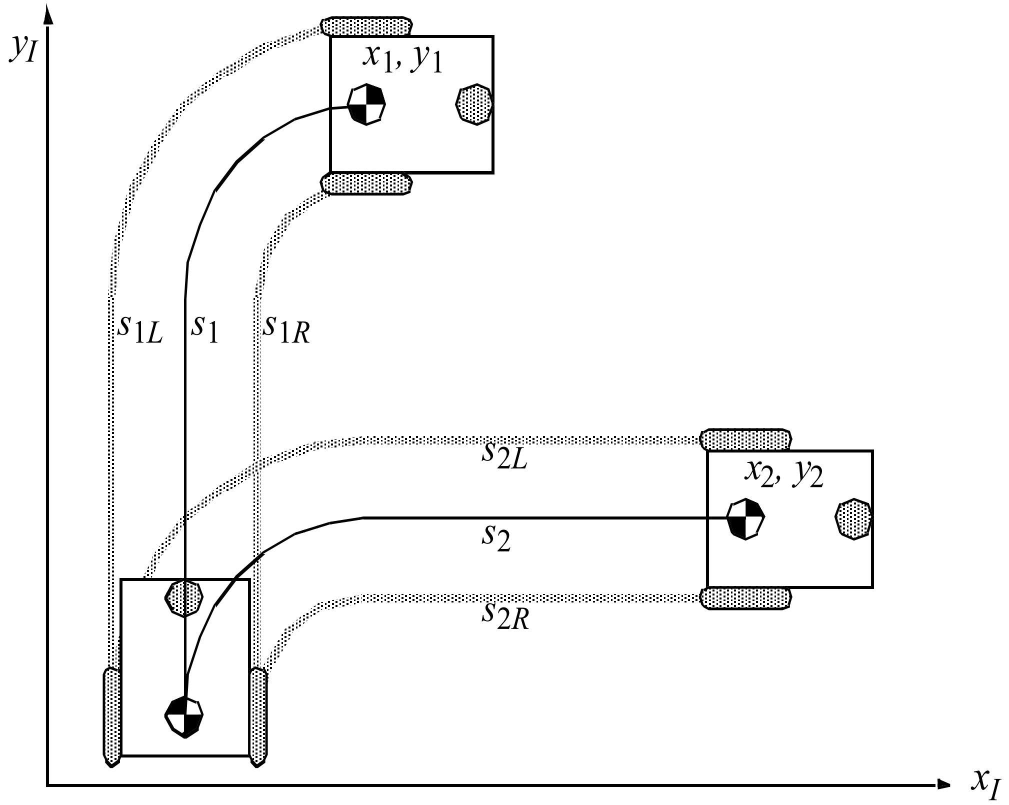
$$\text{->}\ \dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_I = R(\theta)^{-1} \begin{bmatrix} v_{x1} + v_{x2} \\ v_{y1} + v_{y2} \\ \omega_1 + \omega_2 \end{bmatrix} = R(\theta)^{-1} \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ 0 & 0 \\ \frac{r}{2l} & -\frac{r}{2l} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix}$$

Inverse of R => Active and Passive Transform:
http://en.wikipedia.org/wiki/Active_and_passive_transformation

Mobile Robot Kinematics: Non-Holonomic Systems

$s_1 = s_2\,;\ s_{1R} = s_{2R}\,;\ s_{1L} = s_{2L}$

but: $x_1 \neq x_2\,;\ y_1 \neq y_2$



- Non-holonomic systems
  - differential equations are not integrable to the final position.
  - the measure of the traveled distance of each wheel is not sufficient to calculate the final position of the robot. One has also to know how this movement was executed as a function of time.