CS 110
Computer Architecture
Lecture 8:
Synchronous Digital Systems

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http://shtech.org/courses/ca/

School of Information Science and Technology SIST

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Slides based on UC Berkley's CS61C

Levels of Representation/Interpretation

High Level Language Program (e.g., C)

Compiler

Assembly Language Program (e.g., MIPS)

Assembler

Machine Language Program (MIPS)

Machine Interpretation

Hardware Architecture Description (e.g., block diagrams)

Architecture Implementation

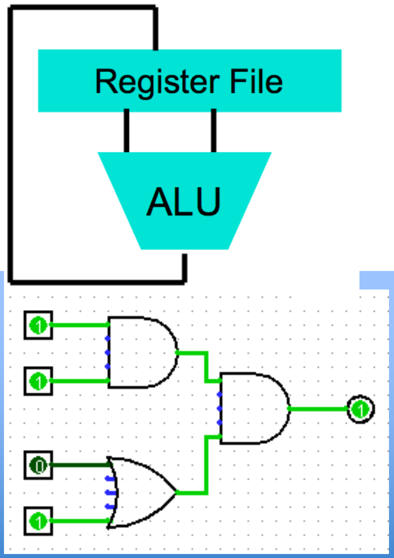
Logic Circuit Description (Circuit Schematic Diagrams)

```
temp = v[k];
v[k] = v[k+1];
v[k+1] = temp;
```

```
lw    $t0, 0($2)
lw    $t1, 4($2)
sw    $t1, 0($2)
sw    $t0, 4($2)
```

Anything can be represented as a *number*, i.e., data or instructions

```
0000 1001 1100 0110 1010 1111 0101 1000
1010 1111 0101 1000 0000 1001 1100 0110
1100 0110 1010 1111 0101 1000 0000 1001
0101 1000 0000 1001 1100 0110 1010 1111
```

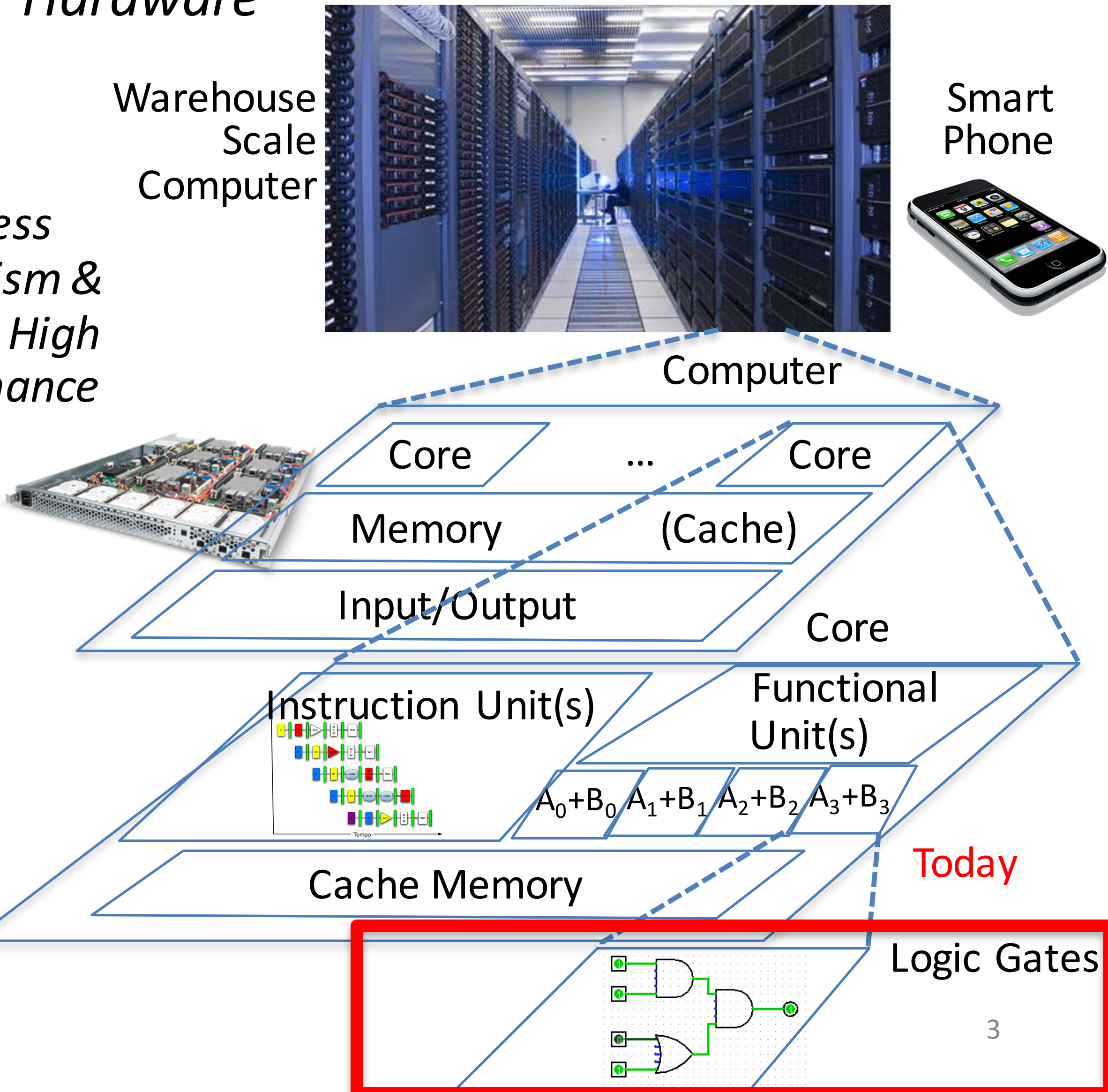


You are Here!

Software | *Hardware*

- Parallel Requests
  Assigned to computer
  e.g., Search "Katz"
- Parallel Threads
  Assigned to core
  e.g., Lookup, Ads
- Parallel Instructions
  >1 instruction @ one time
  e.g., 5 pipelined instructions
- Parallel Data
  >1 data item @ one time
  e.g., Add of 4 pairs of words
- Hardware descriptions
  All gates @ one time
- Programming Languages

Harness Parallelism & Achieve High Performance



Hardware Design

- Next several weeks: how a modern processor is built, starting with basic elements as building blocks
- Why study hardware design?
  - Understand capabilities and limitations of HW in general and processors in particular
  - What processors can do fast and what they can't do fast (avoid slow things if you want your code to run fast!)
  - Background for more in-depth HW courses
  - Hard to know what you'll need for next 30 years
  - There is only so much you can do with standard processors: you may need to design own custom HW for extra performance
    - Even some commercial processors today have customizable hardware!

Synchronous Digital Systems

Hardware of a processor, such as the MIPS, is an example of a Synchronous Digital System

Synchronous:

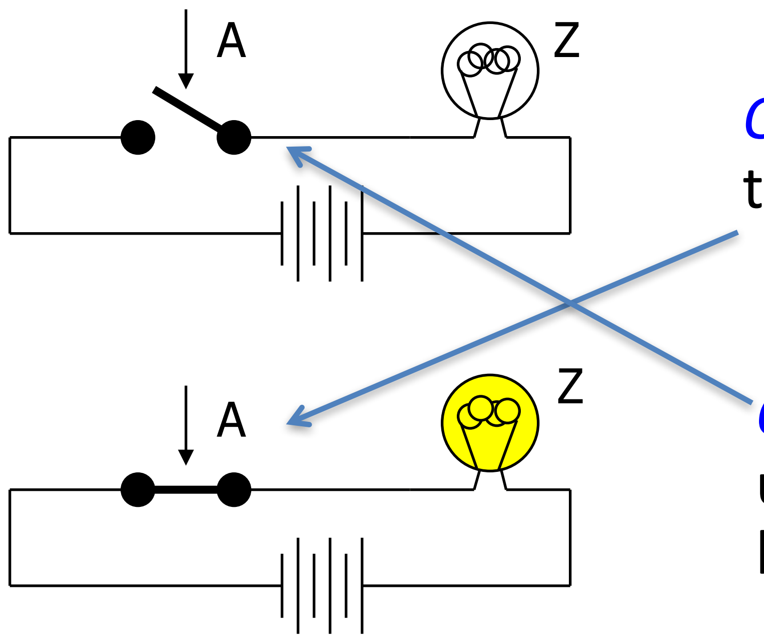
- All operations coordinated by a central clock
  - "Heartbeat" of the system!

Digital:

- Represent all values by discrete values
- Two binary digits: 1 and 0
- Electrical signals are treated as 1's and 0's
  - 1 and 0 are complements of each other
- High /low voltage for true / false, 1 / 0

Switches: Basic Element of Physical Implementations

- Implementing a simple circuit (arrow shows action if wire changes to "1" or is *asserted*):



On-switch (if A is "1" or asserted) turns-on light bulb (Z)

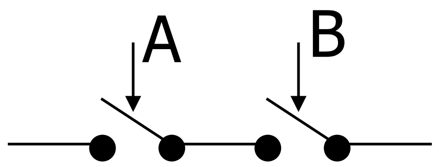
Off-switch (if A is "0" or unasserted) turns-off light bulb (Z)

$$Z \equiv A$$

Switches (cont'd)

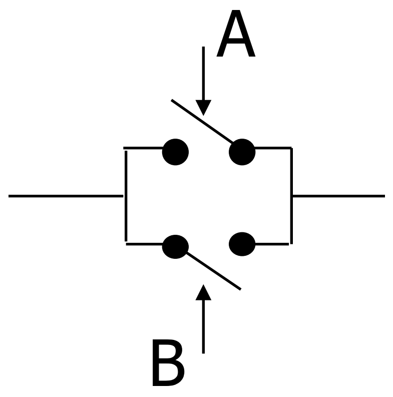
- Compose switches into more complex ones (Boolean functions):

AND



Z ≡ A <u>and</u> B

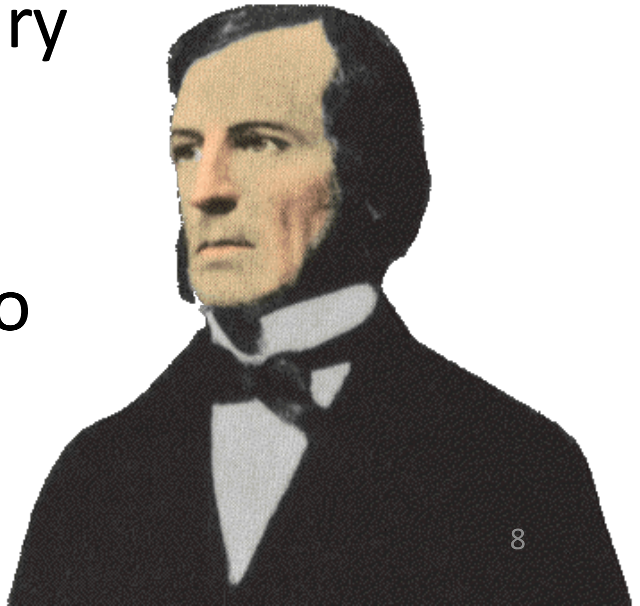
OR



Z ≡ A <u>or</u> B

Historical Note

- Early computer designers built ad hoc circuits from switches
- Began to notice common patterns in their work: ANDs, ORs, …
- Master's thesis (by Claude Shannon, 1940) made link between work and 19th Century Mathematician George Boole
  - Called it "Boolean" in his honor
- Could apply math to give theory to hardware design, minimization, …



Transistors

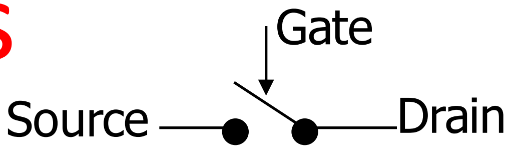
- High voltage (V_{dd}) represents 1, or true
  - In modern microprocessors, Vdd ~ 1.0 Volt
- Low voltage (0 Volt or Ground) represents 0, or false
- Pick a midpoint voltage to decide if a 0 or a 1
  - Voltage greater than midpoint = 1
  - Voltage less than midpoint = 0
  - This removes noise as signals propagate – a big advantage of digital systems over analog systems
- If one switch can control another switch, we can build a computer!
- Our switches: CMOS transistors

CMOS Transistor Networks

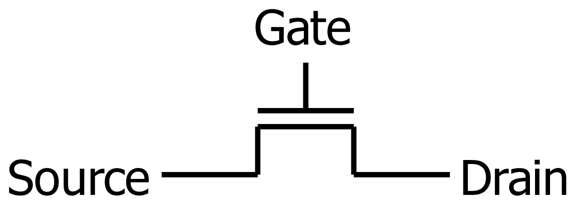
- Modern digital systems designed in CMOS
  - MOS: Metal-Oxide on Semiconductor
  - C for complementary: use *pairs* of normally-*on* and normally-*off* switches

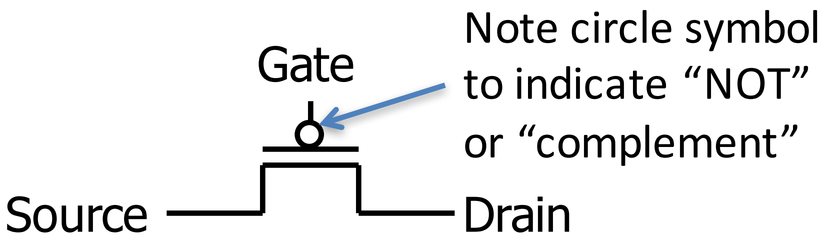
- CMOS transistors act as voltage-controlled switches
  - Similar, though easier to work with, than electro-mechanical relay switches from earlier era
  - Use energy primarily when switching

CMOS Transistors



- Three terminals: source, gate, and drain
  - Switch action:
    if voltage on gate terminal is (some amount) higher/lower than source terminal then conducting path established between drain and source terminals (switch is closed)



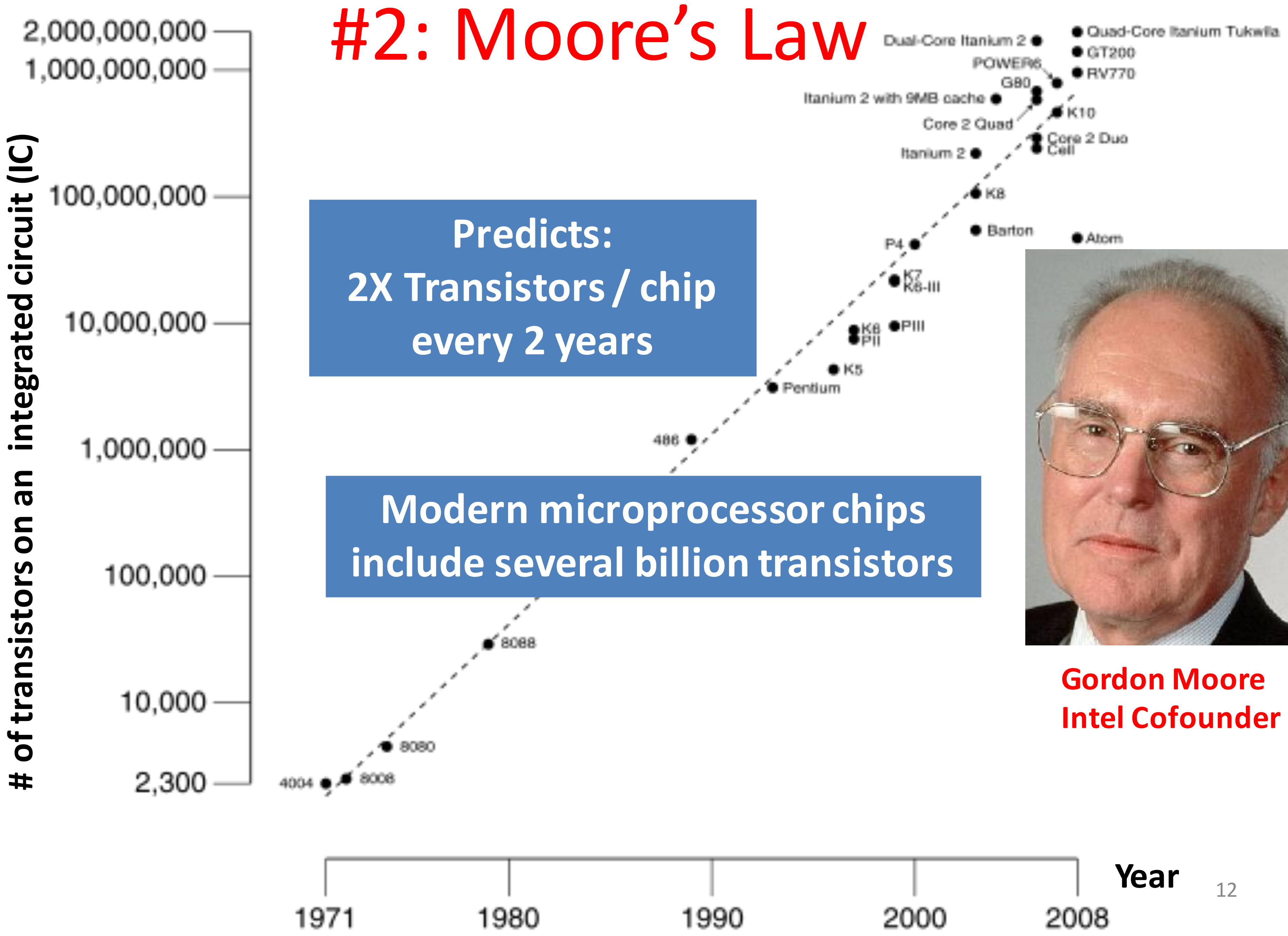


n-channel transitor
off when voltage at Gate is low
on when:
voltage(Gate) > voltage (Threshold)

p-channel transistor
on when voltage at Gate is low
off when:
voltage(Gate) > voltage (Threshold)

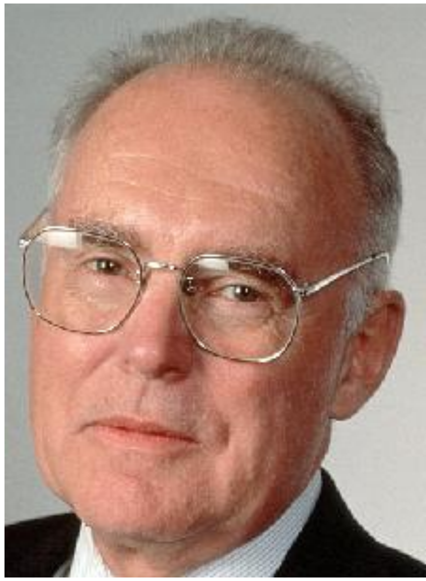
field-effect transistor (FET) => CMOS circuits use a combination of p-type and n-type metal–oxide–semiconductor field-effect transistors => MOSFET

#2: Moore's Law



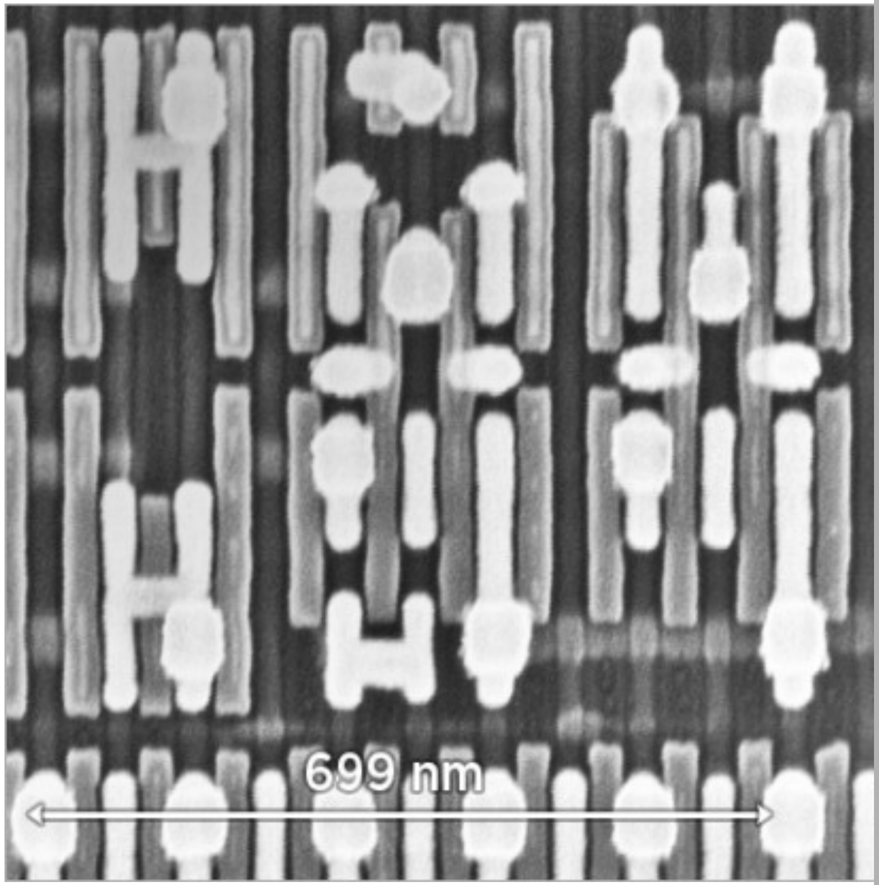
Predicts:
2X Transistors / chip every 2 years

Modern microprocessor chips include several billion transistors

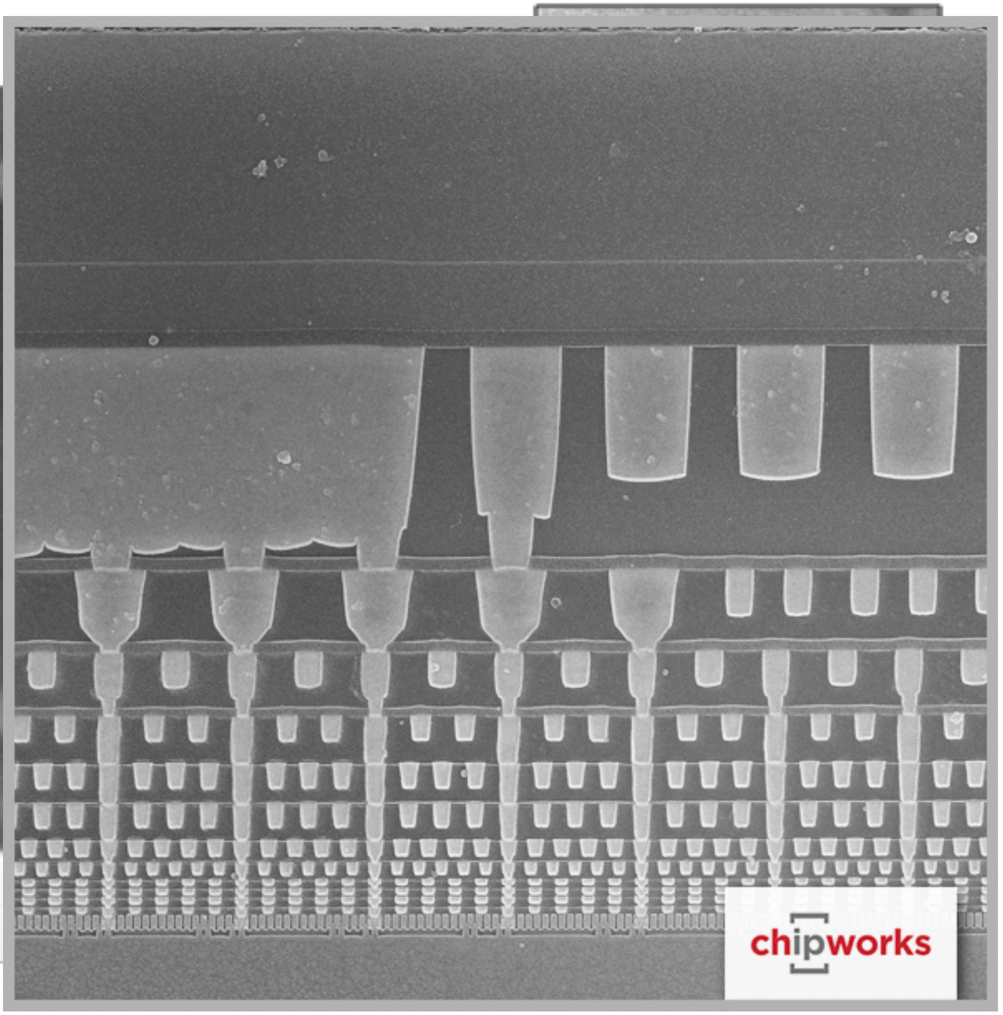


Gordon Moore
Intel Cofounder

Intel 14nm Technology

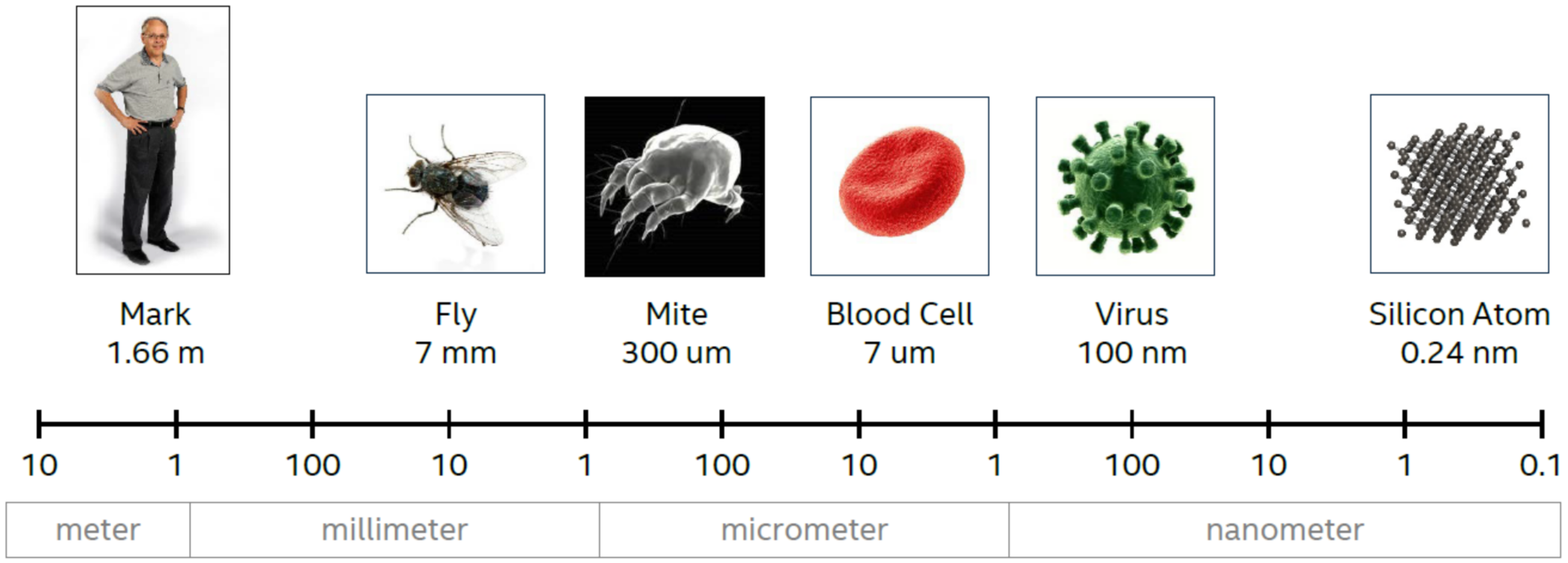


Plan view of transistors



Side view of wiring layers

Sense of Scale



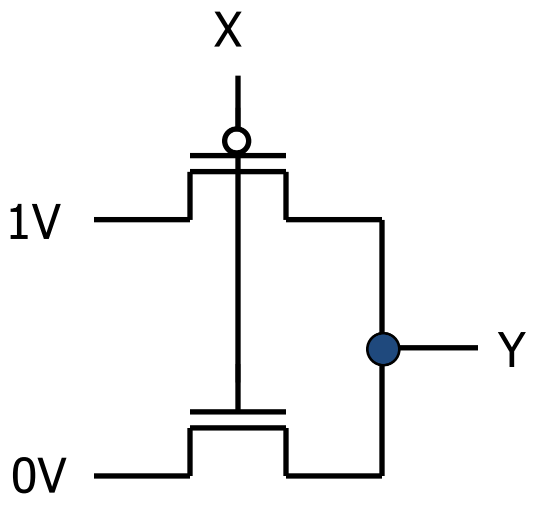
Source: Mark Bohr, IDF14

CMOS Circuit Rules

- Don't pass weak values => Use Complementary Pairs
  - N-type transistors pass weak 1's (V_{dd} - V_{th})
  - N-type transistors pass strong 0's (ground)
  - Use N-type transistors only to pass 0's (N for negative)
  - Converse for P-type transistors: Pass weak 0s, strong 1s
    - Pass weak 0's (V_{th}), strong 1's (V_{dd})
    - Use P-type transistors only to pass 1's (P for positive)
  - Use pairs of N-type and P-type to get strong values
- Never leave a wire undriven
  - Make sure there's always a path to V_{dd} or GND
- Never create a path from V_{dd} to GND (ground)
  - This would short-circuit the power supply!

CMOS Networks

p-channel transistor
on when voltage at Gate is low
off when:
voltage(Gate) > voltage (Threshold)



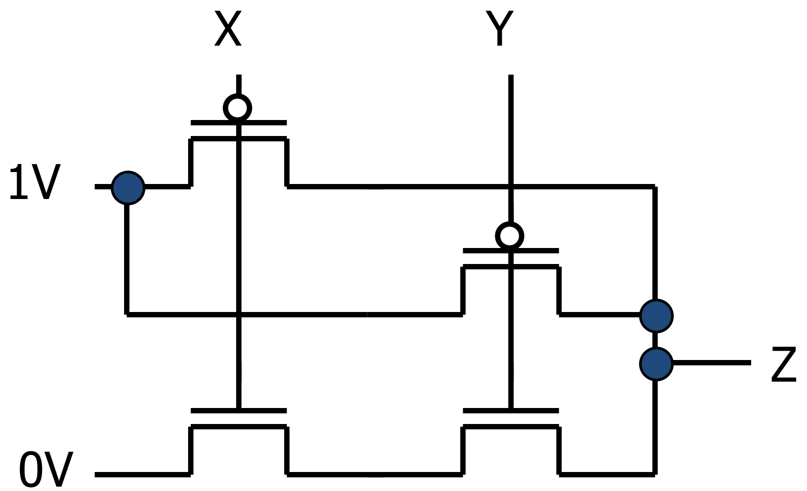
n-channel transitor
off when voltage at Gate is low
on when:
voltage(Gate) > voltage (Threshold)

what is the relationship between x and y?

x	y
0 Volt (GND)	1 Volt (Vdd)
1 Volt (Vdd)	0 Volt (GND)

Called an *inverter* or *not gate*

Two-Input Networks

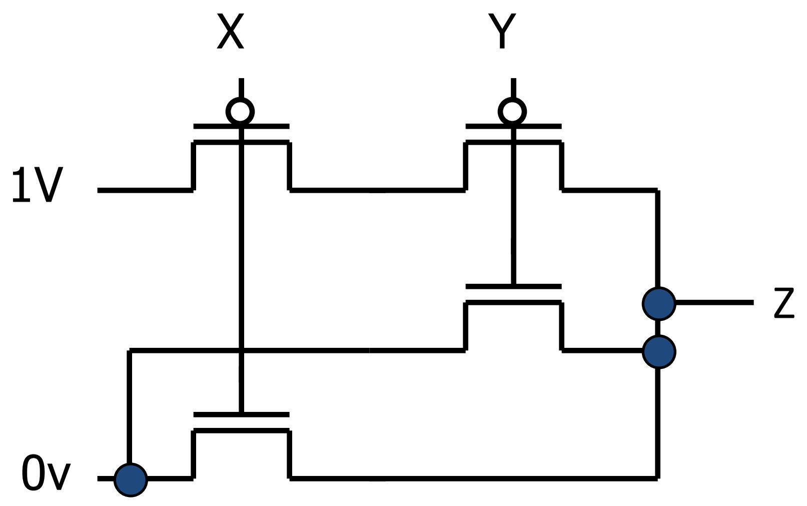


what is the
relationship between x, y and z?

x	y	z
0 Volt	0 Volt	1 Volt
0 Volt	1 Volt	1 Volt
1 Volt	0 Volt	1 Volt
1 Volt	1 Volt	0 Volt

Called a *NAND gate*
(NOT AND)

Clickers/Peer Instruction



x	y	z: A	B	C	D	
0 Volt	0 Volt	0	0	1	1	Volts
0 Volt	1 Volt	0	1	0	1	Volts
1 Volt	0 Volt	0	1	0	1	Volts
1 Volt	1 Volt	1	1	0	0	Volts

Administrivia

- Final HW1 scores will be published next week:
  - We will by hand take a quick look and deduct points for bugs, e.g.
    - Memory leaks
    - reverse_list changing the provided list
    - Empty or meaningless comments
- Project 1.1 test cases will be updated latest Monday – grading similar to above
  - 11 groups did not register their group e-mail yet!
  - 8 groups have registrations pending in gradebot!

Administrivia

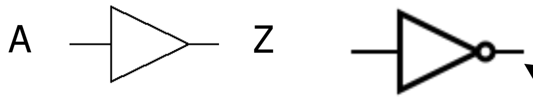
- Bug in HW3 grading script …
- In Lab 8:
  - Your project team explains the TA and Prof your projects 1.1 and 1.2
  - Both of you should know your software well
  - If we find one of you clearly did contribute much less, we will reduce that students points a little
- Check out the additional material provided by UC Berkeley:
  - http://inst.eecs.berkeley.edu/~cs61c/resources/sds.pdf
  - http://inst.eecs.berkeley.edu/~cs61c/resources/boolean.pdf

Policy on Assignments and Independent Work

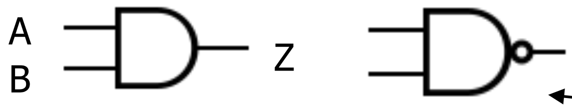
- **ALL PROJECTS WILL BE DONE WITH A PARTNER**
- With the exception of laboratories and assignments that explicitly permit you to work in groups, all homework and projects are to be YOUR work and your work ALONE.
- PARTNER TEAMS MAY NOT WORK WITH OTHER PARTNER TEAMS
- You are encouraged to discuss your assignments with other students, and credit will be assigned to students who help others, particularly by answering questions on Piazza, but we expect that what you hand in is yours.
- It is NOT acceptable to copy solutions from other students.
- It is NOT acceptable to copy (or start your) solutions from the Web.
- It is NOT acceptable to use PUBLIC github archives (giving your answers away)
- We have tools and methods, developed over many years, for detecting this. You WILL be caught, and the penalties WILL be severe.
- **At the minimum F in the course**, and a letter to your university record documenting the incidence of cheating.
- **Both Giver and Receiver are equally culpable and suffer equal penalties**

Combinational Logic Symbols

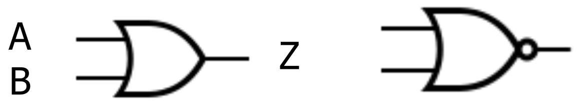
- Common combinational logic systems have standard symbols called logic gates
  - Buffer, NOT

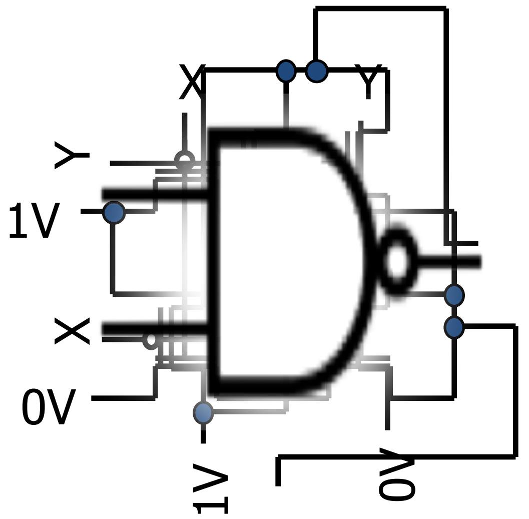


  - AND, NAND



  - OR, NOR

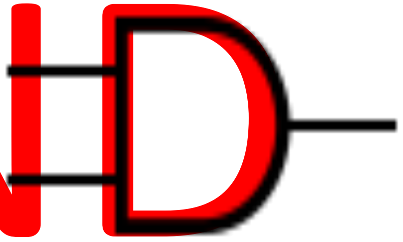




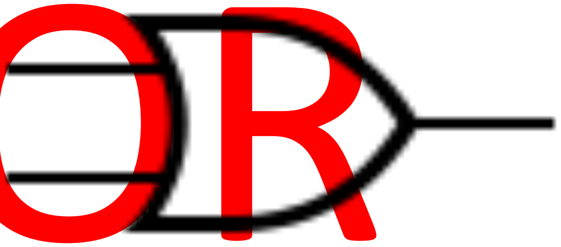
Inverting versions (NOT, NAND, NOR) easiest to implement with CMOS transistors (the switches we have available and use most)

Remember…

- AND



- OR



Boolean Algebra

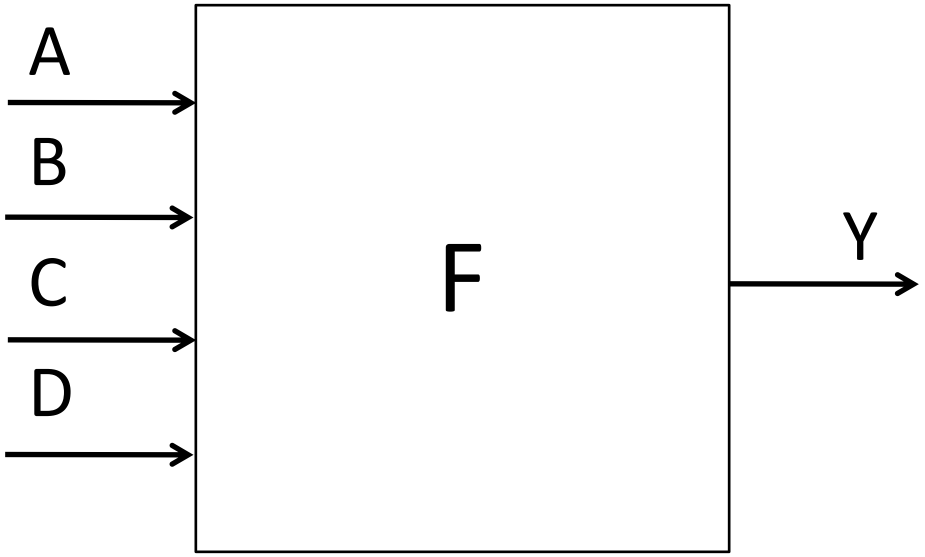
- Use plus "+" for OR
  - "logical sum" 1+0 = 0+1 = 1 (True); 1+1=2 (True); 0+0 = 0 (False)
- Use product for AND (a•b or implied via ab)
  - "logical product" 0*0 = 0*1 = 1*0 = 0 (False); 1*1 = 1 (True)
- "Hat" to mean complement (NOT)
- Thus

$ab + a + \overline{c}$

$= a \bullet b + a + \overline{c}$

= (a AND b) OR a OR (NOT c)

Truth Tables for Combinational Logic



Exhaustive list of the output value generated for each combination of inputs

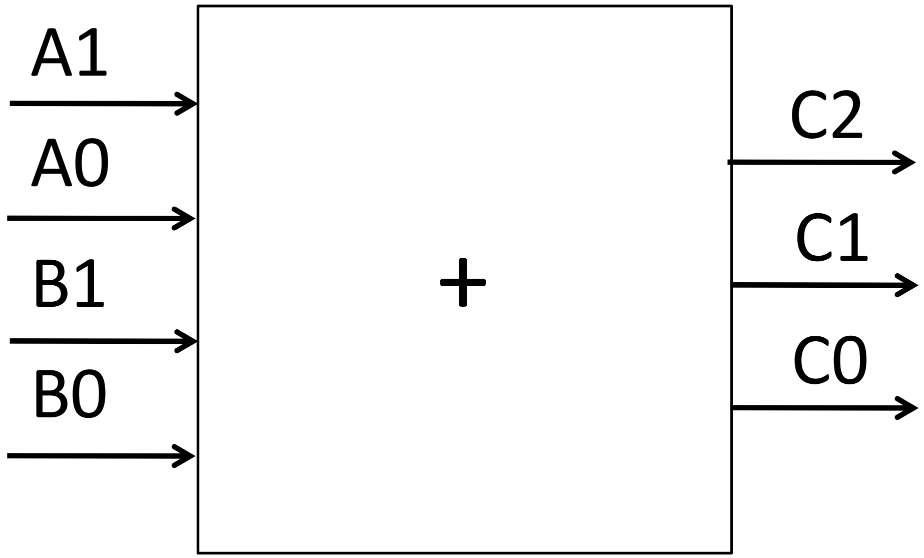
How many logic functions can be defined with N inputs?

a	b	c	d	y
0	0	0	0	F(0,0,0,0)
0	0	0	1	F(0,0,0,1)
0	0	1	0	F(0,0,1,0)
0	0	1	1	F(0,0,1,1)
0	1	0	0	F(0,1,0,0)
0	1	0	1	F(0,1,0,1)
0	1	1	0	F(0,1,1,0)
0	1	1	1	F(0,1,1,1)
1	0	0	0	F(1,0,0,0)
1	0	0	1	F(1,0,0,1)
1	0	1	0	F(1,0,1,0)
1	0	1	1	F(1,0,1,1)
1	1	0	0	F(1,1,0,0)
1	1	0	1	F(1,1,0,1)
1	1	1	0	F(1,1,1,0)
1	1	1	1	F(1,1,1,1)

Truth Table Example #1: y= F(a,b): 1 iff a ≠ b

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

Truth Table Example #2: 2-bit Adder



How Many Rows?

A	B	C
a_1a_0	b_1b_0	$c_2c_1c_0$

Truth Table Example #3: 32-bit Unsigned Adder

A	B	C
000 ... 0	000 ... 0	000 ... 00
000 ... 0	000 ... 1	000 ... 01
.	.	.
.	.	.
.	.	.
111 ... 1	111 ... 1	111 ... 10

How Many Rows?

Truth Table Example #4: 3-input Majority Circuit

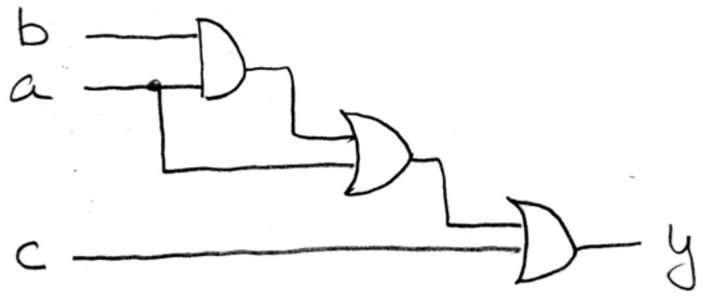
Y =

This is called *Sum of Products* form; Just another way to represent the TT as a logical expression

More simplified forms (fewer gates and wires)

a	b	c	y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Boolean Algebra: Circuit & Algebraic Simplification



original circuit

$$\downarrow$$

$$y = ((ab) + a) + c$$

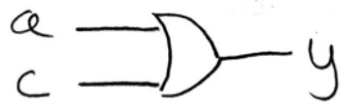
equation derived from original circuit

$$\downarrow$$

$$\begin{aligned} &= ab + a + c \\ &= a(b+1) + c \\ &= a(1) + c \\ &= a + c \end{aligned}$$

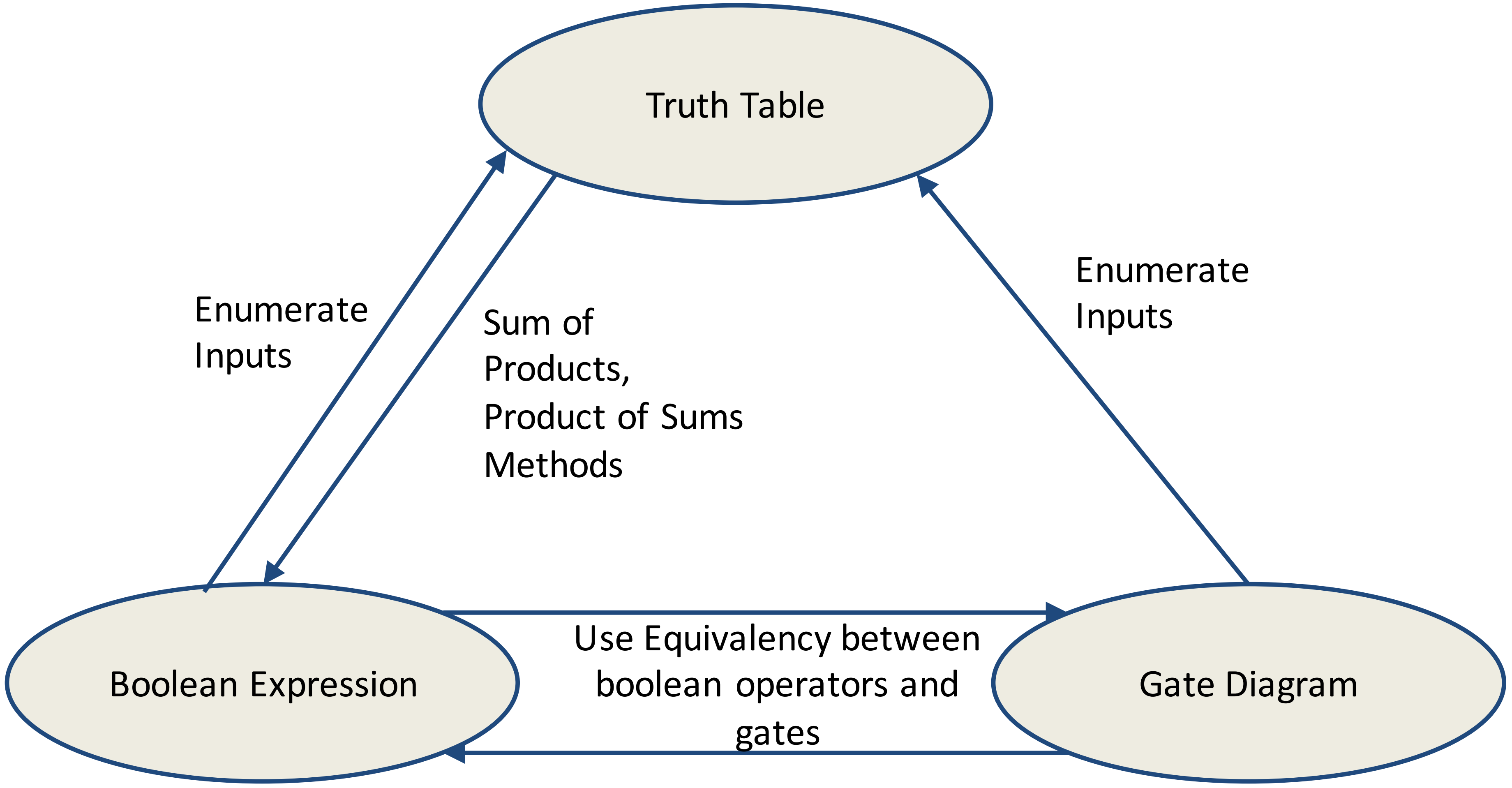
algebraic simplification

$$\downarrow$$



simplified circuit

Representations of Combinational Logic (groups of logic gates)


Truth Table
Enumerate Inputs
Sum of Products, Product of Sums Methods
Enumerate Inputs
Boolean Expression
Use Equivalency between boolean operators and gates
Gate Diagram

Laws of Boolean Algebra

$X \overline{X} = 0$	$X + \overline{X} = 1$	Complementarity
$X\ 0 = 0$	$X + 1 = 1$	Laws of 0's and 1's
$X\ 1 = X$	$X + 0 = X$	Identities
$X\ X = X$	$X + X = X$	Idempotent Laws
$X\ Y = Y\ X$	$X + Y = Y + X$	Commutativity
$(X\ Y)\ Z = Z\ (Y\ Z)$	$(X + Y) + Z = Z + (Y + Z)$	Associativity
$X\ (Y + Z) = X\ Y + X\ Z$	$X + Y\ Z = (X + Y)\ (X + Z)$	Distribution
$X\ Y + X = X$	$(X + Y)\ X = X$	Uniting Theorem
$\overline{X}\ Y + X = X + Y$	$(\overline{X} + Y)\ X = X\ Y$	Uniting Theorem v. 2
$\overline{X\ Y} = \overline{X} + \overline{Y}$	$\overline{X + Y} = \overline{X}\ \overline{Y}$	DeMorgan's Law

Boolean Algebraic Simplification Example

$$y = ab + a + c$$

Boolean Algebraic Simplification Example

$$\begin{aligned} y &= ab + a + c \\ &= a(b+1) + c && \textit{distribution, identity} \\ &= a(1) + c && \textit{law of 1's} \\ &= a + c && \textit{identity} \end{aligned}$$

a	b	c	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

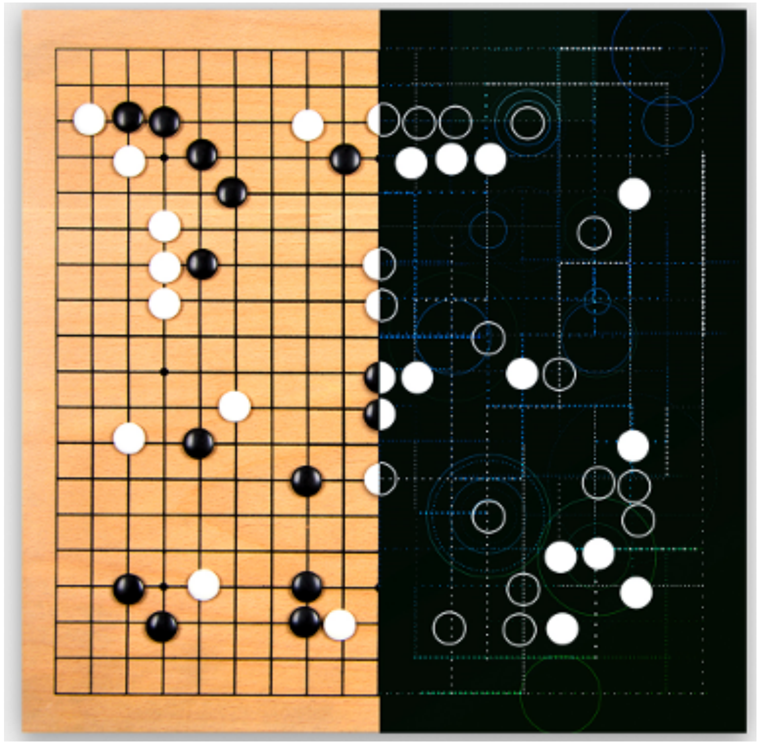
Question

- Simplify $Z = A+BC + \overline{A}(\overline{BC})$

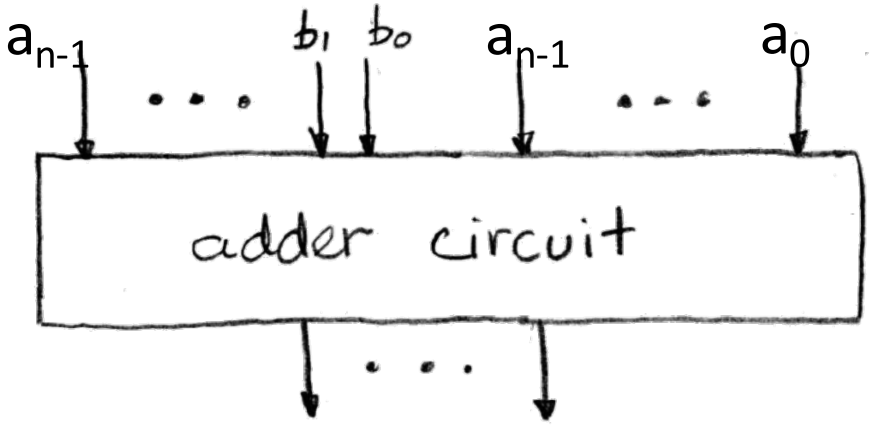
- A: $Z = 0$
- B: $Z = \overline{A(1+ BC)}$
- C: $Z = (A + BC)$
- D: $Z = BC$
- E: $Z = 1$

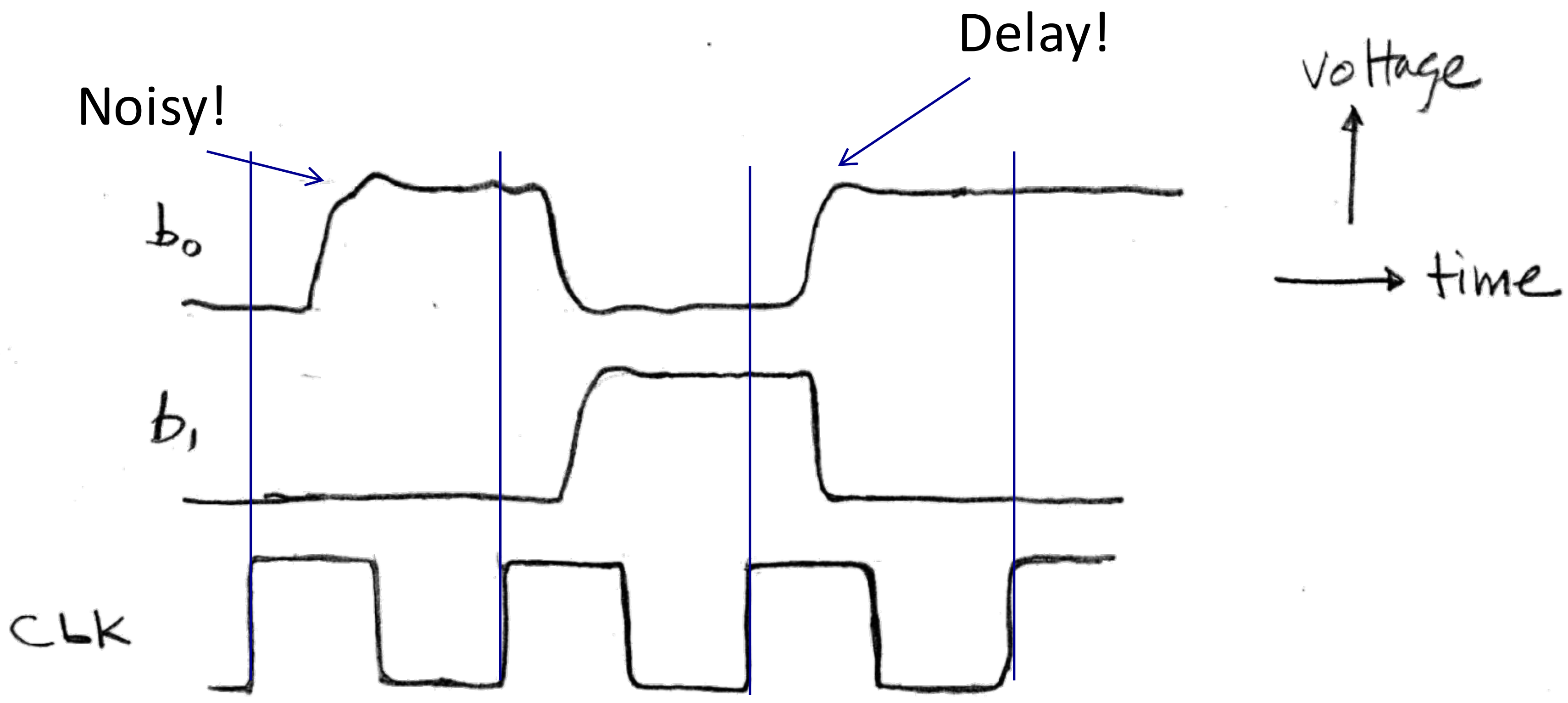
In the News: Googles AlphaGo beats world champion Lee Sedol

- Google wins 4:1
- Go: maybe 200 possible moves per turn
  - 250 or more turns
  - even 200^{150} is about infinite (chess: about 35^{80})
- Monte Carlo Tree Search (MCTS) and Machine Learning (Neural Networks)
- 1202 CPUs und 176 GPU (or more)
- One million USD prize

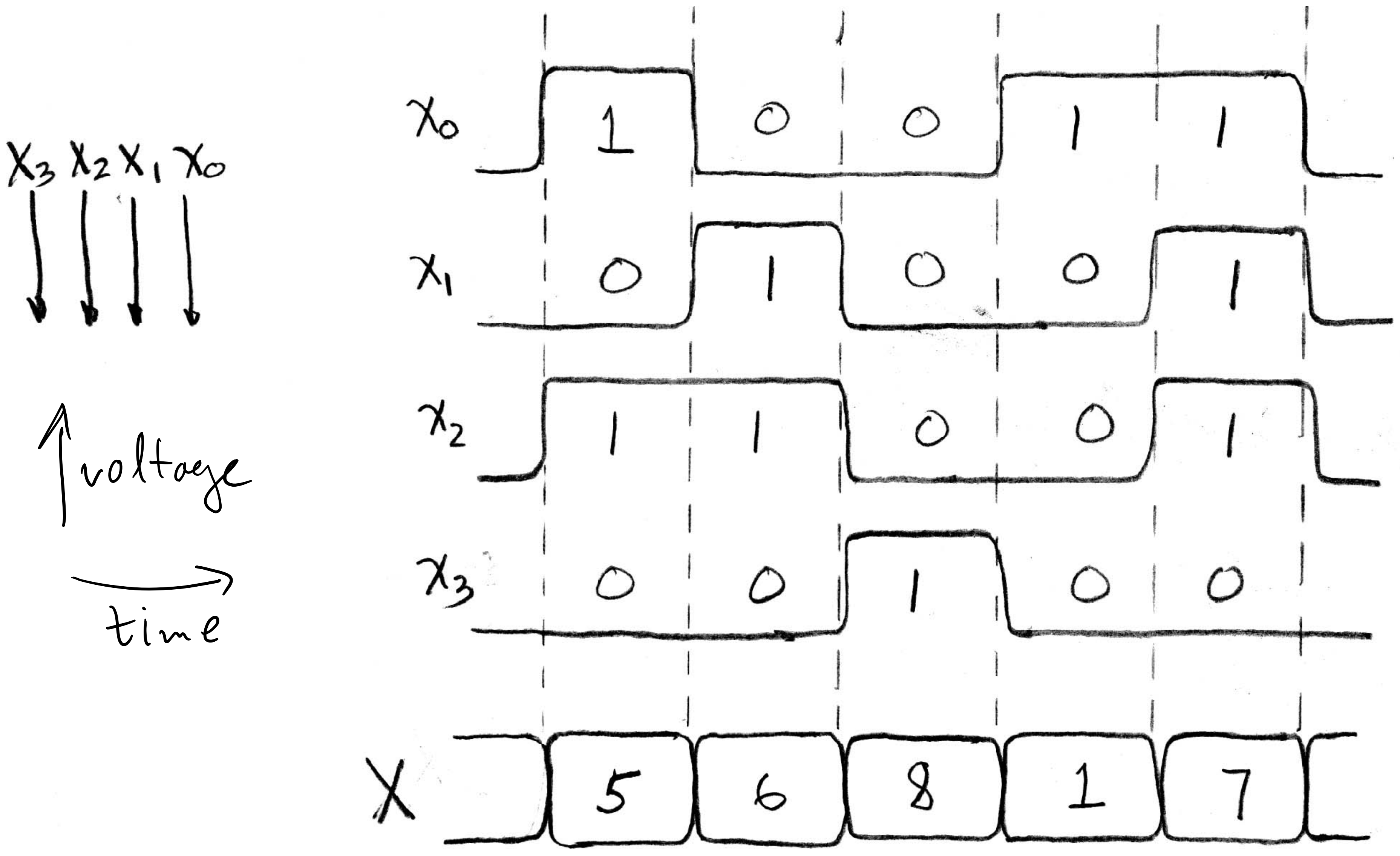


Signals and Waveforms

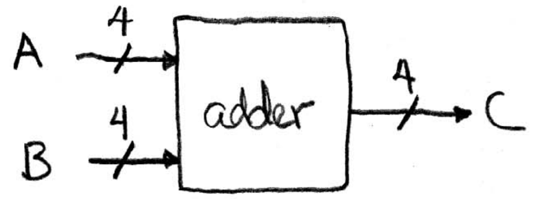




Signals and Waveforms: Grouping

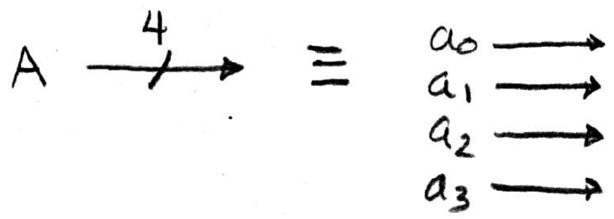


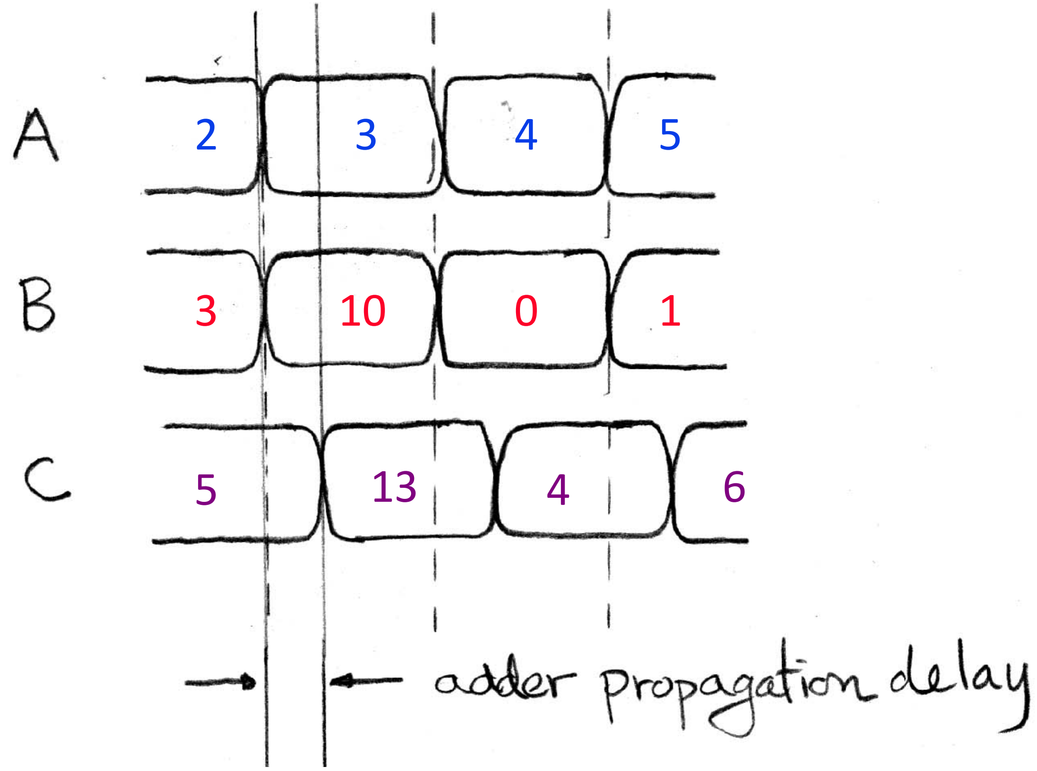
Signals and Waveforms: Circuit Delay



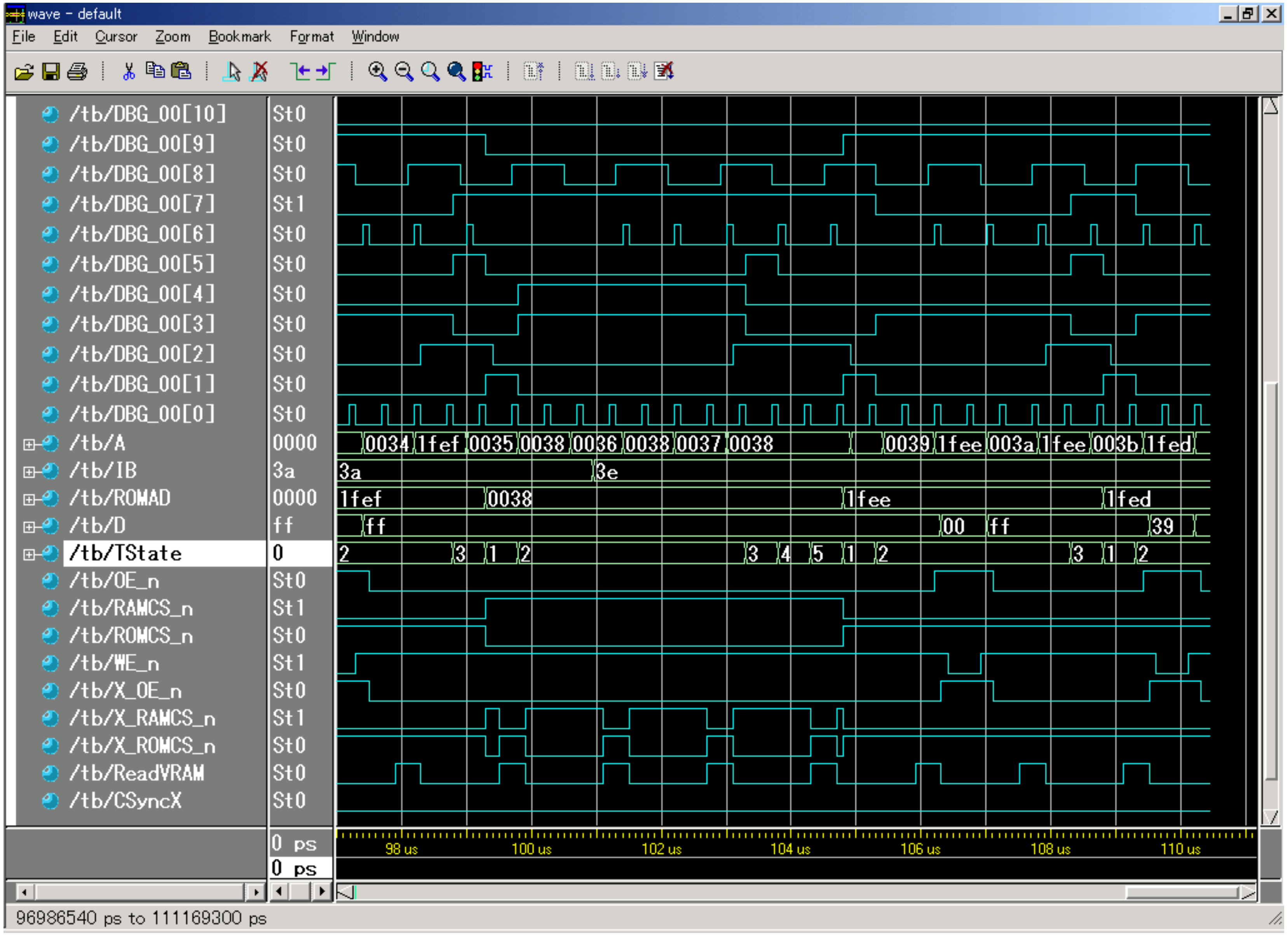
$A = [a_3, a_2, a_1, a_0]$

$B = [b_3, b_2, b_1, b_0]$





Sample Debugging Waveform


wave - default
File Edit Cursor Zoom Bookmark Format Window
0034 1fef 0035 0038 0036 0038 0037 0038 0039 1fee 003a 1fee 003b 1fed
3a 3e
1fef 0038 1fee 1fed
ff 00 ff 39
2 3 1 2 3 4 5 1 2 3 1 2
98 us 100 us 102 us 104 us 106 us 108 us 110 us

```
/tb/DBG_00[10]   St0
/tb/DBG_00[9]    St0
/tb/DBG_00[8]    St0
/tb/DBG_00[7]    St1
/tb/DBG_00[6]    St0
/tb/DBG_00[5]    St0
/tb/DBG_00[4]    St0
/tb/DBG_00[3]    St0
/tb/DBG_00[2]    St0
/tb/DBG_00[1]    St0
/tb/DBG_00[0]    St0
/tb/A            0000
/tb/IB           3a
/tb/ROMAD        0000
/tb/D            ff
/tb/TState       0
/tb/OE_n         St0
/tb/RAMCS_n      St1
/tb/ROMCS_n      St0
/tb/WE_n         St1
/tb/X_OE_n       St0
/tb/X_RAMCS_n    St1
/tb/X_ROMCS_n    St0
/tb/ReadVRAM     St0
/tb/CSyncX       St0
                 0 ps
                 0 ps
96986540 ps to 111169300 ps
```

Type of Circuits

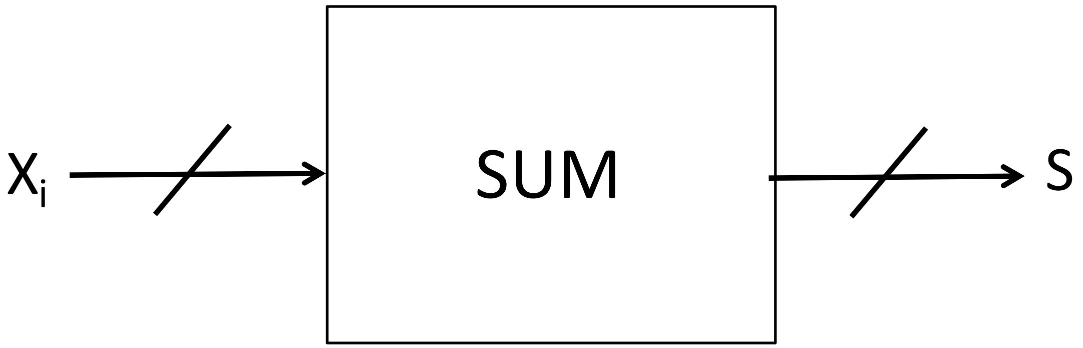
- *Synchronous Digital Systems* consist of two basic types of circuits:
  - Combinational Logic (CL) circuits
    - – Output is a function of the inputs only, not the history of its execution
    - – E.g., circuits to add A, B (ALUs)
  - Sequential Logic (SL)
    - Circuits that "remember" or store information
    - aka "State Elements"
    - E.g., memories and registers (Registers)

Uses for State Elements

- Place to store values for later re-use:
  - Register files (like \$1-\$31 in MIPS)
  - Memory (caches and main memory)
- *Help control flow of information between combinational logic blocks*
  - State elements hold up the movement of information at input to combinational logic blocks to allow for orderly passage

Accumulator Example

Why do we need to control the flow of information?

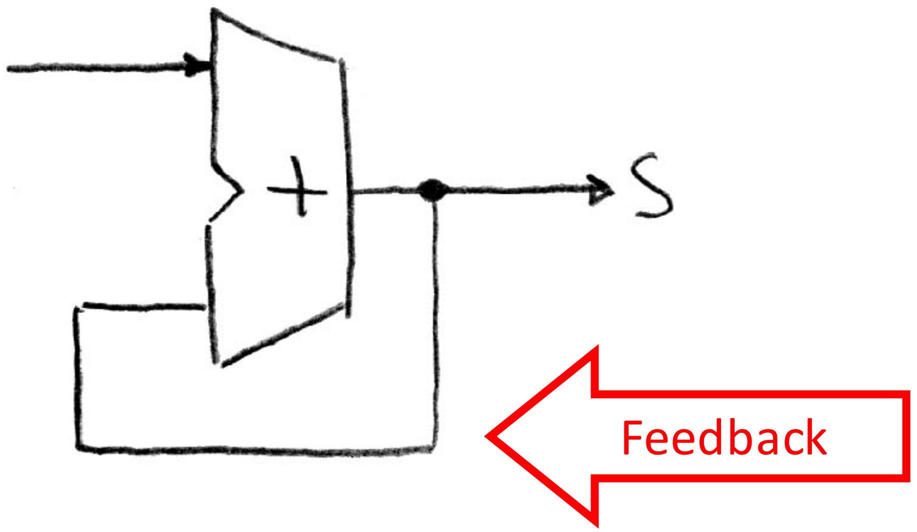


Want:
```
S=0;
for (i=0;i<n;i++)
          S = S + Xi
```

Assume:

- Each X value is applied in succession, one per cycle
- After n cycles the sum is present on S

First Try: Does this work?

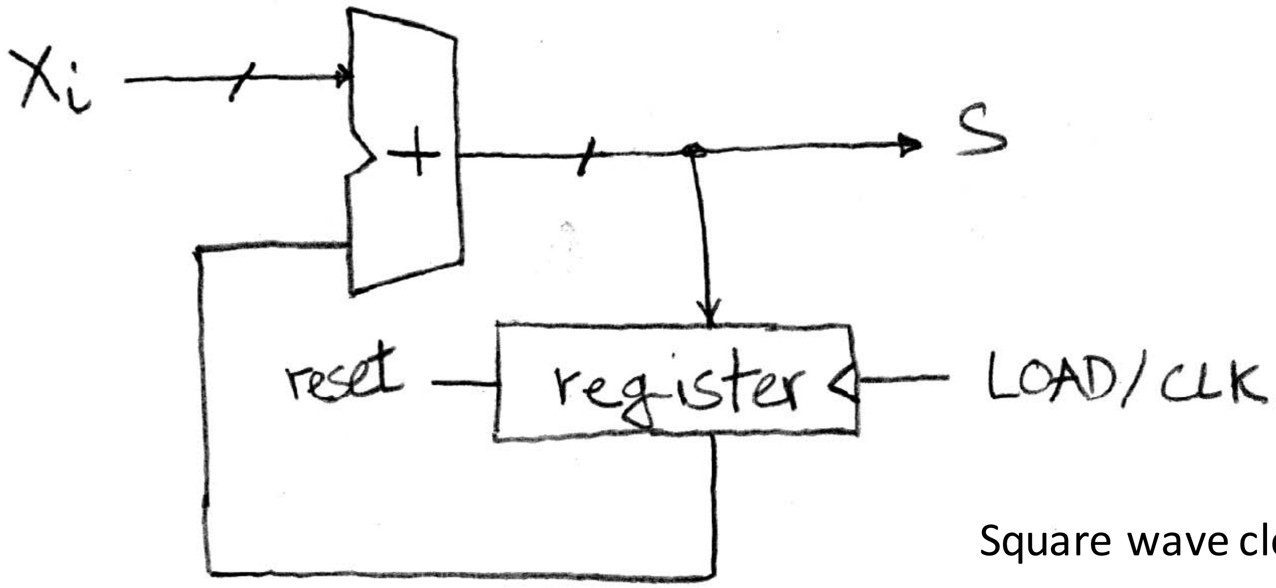


No!

Reason #1: How to control the next iteration of the ‘for’ loop?

Reason #2: How do we say: ‘S=0’?

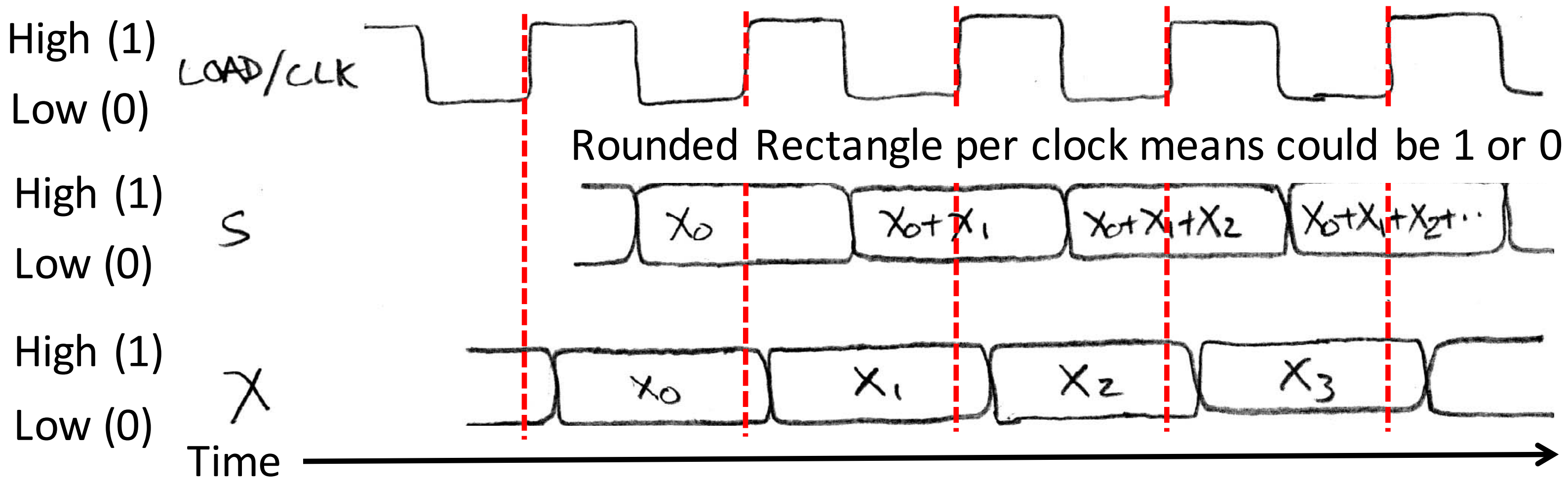
Second Try: How About This?



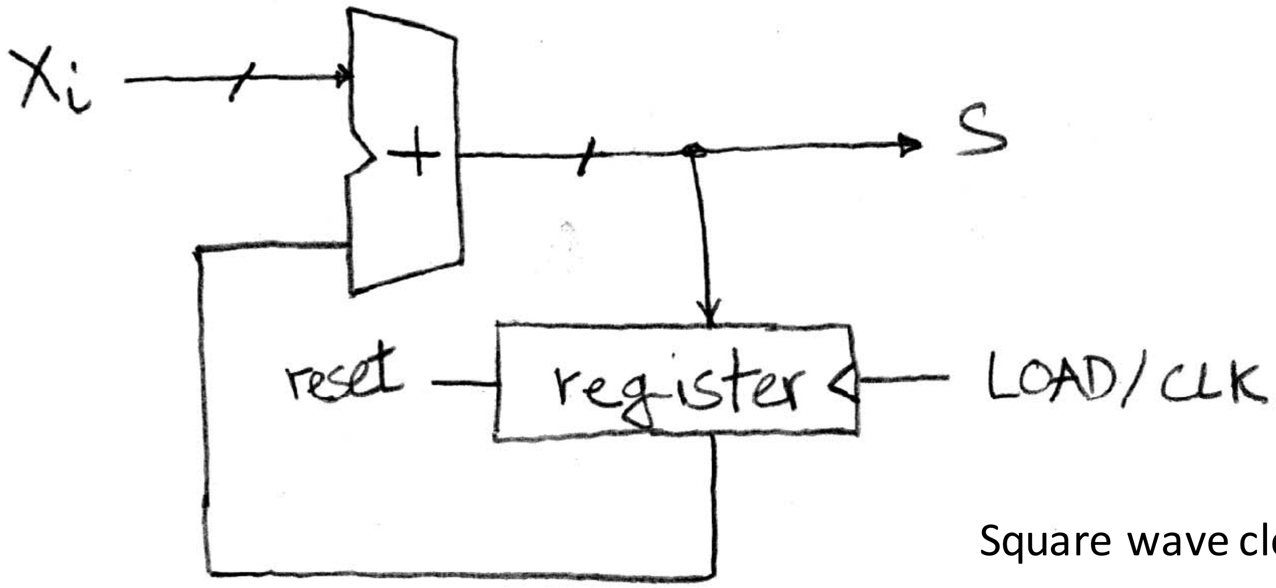
Register is used to hold up the transfer of data to adder

Square wave clock sets when things change

Rough timing …



Second Try: How About This?



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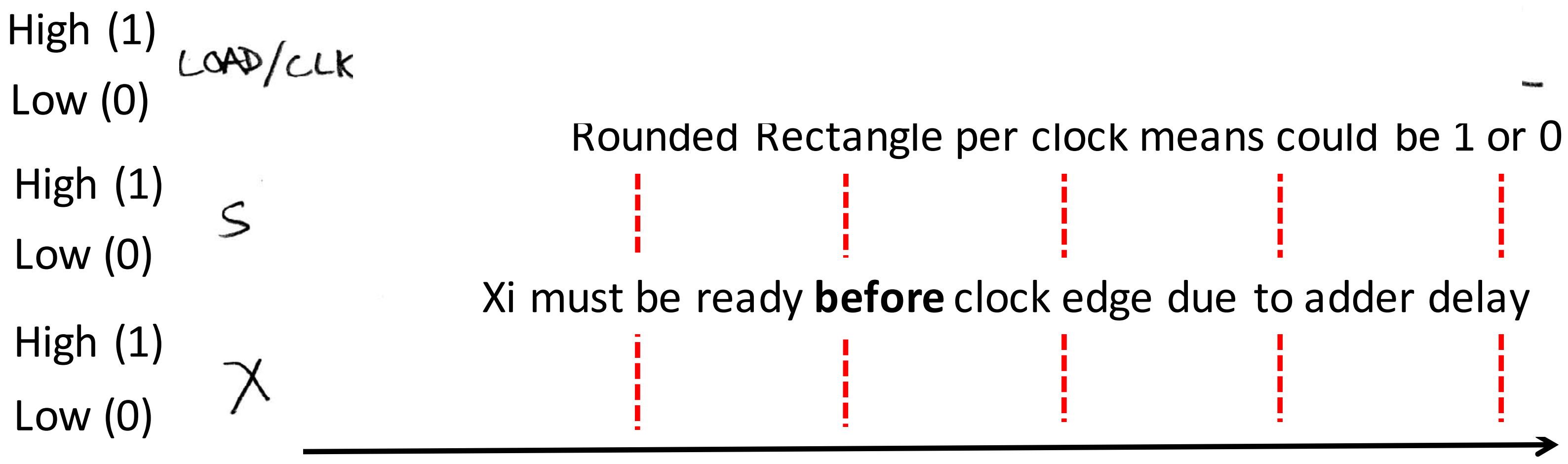
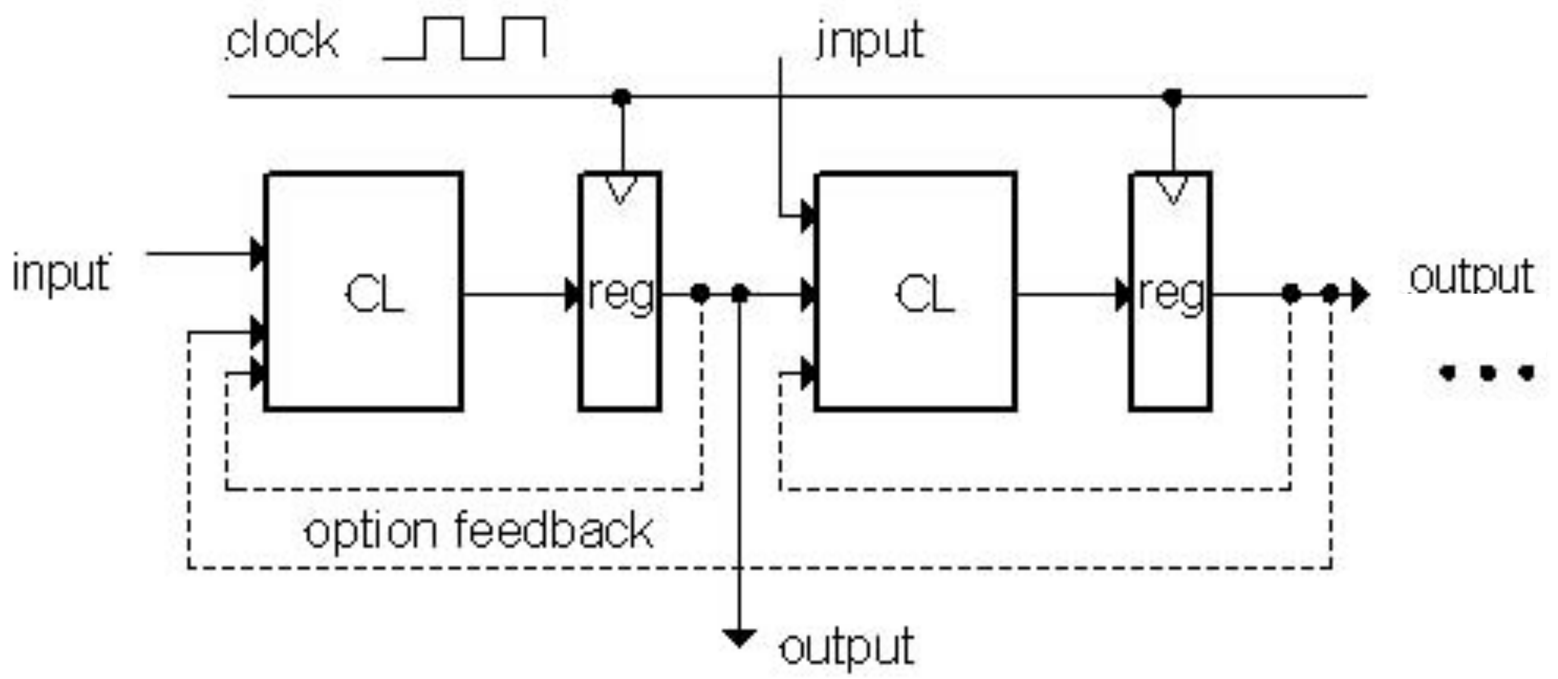


Rounded Rectangle per clock means could be 1 or 0

Xi must be ready **before** clock edge due to adder delay

Model for Synchronous Systems



- Collection of Combinational Logic blocks separated by registers
- Feedback is optional
- Clock signal(s) connects only to clock input of registers
- Clock (CLK): steady square wave that synchronizes the system
- Register: several bits of state that samples on rising edge of CLK (positive edge-triggered) or falling edge (negative edge-triggered)