# CS 110
# Computer Architecture
# Lecture 17:
# *Performance and Floating Point Arithmetic*

Instructor:
**Sören Schwertfeger**

http://shtech.org/courses/ca/

School of Information Science and Technology SIST

ShanghaiTech University

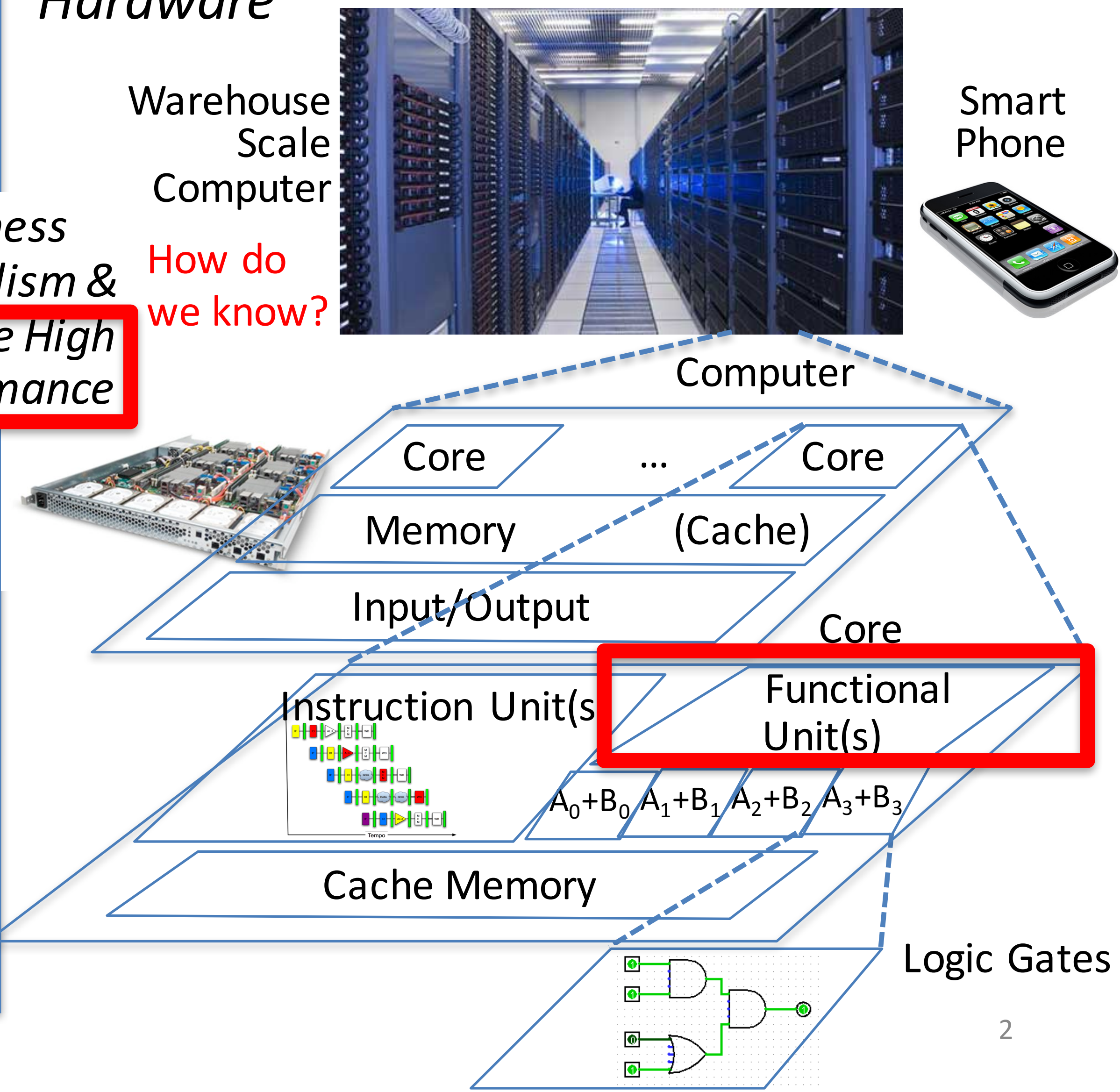
Slides based on UC Berkley's CS61C

# New-School Machine Structures (It’s a bit more complicated!)

*Software* | *Hardware*

- Parallel Requests
  Assigned to computer
  e.g., Search “Katz”
- Parallel Threads
  Assigned to core
  e.g., Lookup, Ads
- Parallel Instructions
  >1 instruction @ one time
  e.g., 5 pipelined instructions
- Parallel Data
  >1 data item @ one time
  e.g., Add of 4 pairs of words
- Hardware descriptions
  All gates @ one time
- Programming Languages

*Harness Parallelism & Achieve High Performance*



# What is Performance?

- *Latency* (or *response time* or *execution time*)
  - Time to complete one task
- *Bandwidth* (or *throughput*)
  - Tasks completed per unit time

# Cloud Performance: Why Application Latency Matters

| Server Delay (ms) | Increased time to next click (ms) | Queries/ user | Any clicks/ user | User satisfac-tion | Revenue/ User |
|---|---|---|---|---|---|
| 50 | -- | -- | -- | -- | -- |
| 200 | 500 | -- | -0.3% | -0.4% | -- |
| 500 | 1200 | -- | -1.0% | -0.9% | -1.2% |
| 1000 | 1900 | -0.7% | -1.9% | -1.6% | -2.8% |
| 2000 | 3100 | -1.8% | -4.4% | -3.8% | -4.3% |

Figure 6.10 Negative impact of delays at Bing search server on user behavior [Brutlag and Schurman 2009].

- Key figure of merit: application responsiveness
  - Longer the delay, the fewer the user clicks, the less the user happiness, and the lower the revenue per user

# Defining CPU Performance

- What does it mean to say X is faster than Y?
- Ferrari vs. School Bus?

- 2013 Ferrari 599 GTB
  - 2 passengers, quarter mile in 10 secs
- 2013 Type D school bus
  - 50 passengers, quarter mile in 20 secs

- *Response Time* (*Latency*): e.g., time to travel ¼ mile
- *Throughput* (*Bandwidth*): e.g., passenger-mi in 1 hour

# Defining Relative CPU Performance

- $\text{Performance}_X = 1/\text{Program Execution Time}_X$
- $\text{Performance}_X > \text{Performance}_Y \Rightarrow$
  $1/\text{Execution Time}_X > 1/\text{Execution Time}_y \Rightarrow$
  $\text{Execution Time}_Y > \text{Execution Time}_X$
- Computer X is N times faster than Computer Y
  $\text{Performance}_X / \text{Performance}_Y = N$ or
  $\text{Execution Time}_Y / \text{Execution Time}_X = N$

- Bus to Ferrari performance:
  - Program: Transfer 1000 passengers for 1 mile
  - Bus: 3,200 sec, Ferrari: 40,000 sec

# Measuring CPU Performance

- Computers use a clock to determine when events takes place within hardware
- *Clock cycles:* discrete time intervals
  - aka clocks, cycles, clock periods, clock ticks
- *Clock rate* or *clock frequency:* clock cycles per second (inverse of clock cycle time)
- 3 GigaHertz clock rate
  => clock cycle time = $1/(3 \times 10^9)$ seconds
  clock cycle time = 333 picoseconds (ps)

# CPU Performance Factors

- To distinguish between processor time and I/O, *CPU time* is time spent in processor
- `CPU Time/Program`
  `= Clock Cycles/Program`
  `x Clock Cycle Time`
- Or

  `CPU Time/Program`
  `= Clock Cycles/Program ÷ Clock Rate`

# Iron Law of Performance

- A program executes instructions
- `CPU Time/Program`

  ```
  = Clock Cycles/Program x Clock Cycle Time
  = Instructions/Program
    x Average Clock Cycles/Instruction
    x Clock Cycle Time
  ```
- 1st term called *Instruction Count*
- 2nd term abbreviated *CPI* for average *__C__lock Cycles __P__er __I__nstruction*
- 3rd term is 1 / Clock rate

# Restating Performance Equation

- $$\text{Time} = \frac{\text{Seconds}}{\text{Program}}$$

$$= \frac{\text{Instructions}}{\text{Program}} \times \frac{\text{Clock cycles}}{\text{Instruction}} \times \frac{\text{Seconds}}{\text{Clock Cycle}}$$

# What Affects Each Component? A)Instruction Count, B)CPI, C)Clock Rate

| | Affects What? |
|---|---|
| Algorithm | |
| Programming Language | |
| Compiler | |
| Instruction Set Architecture | |

# What Affects Each Component? Instruction Count, CPI, Clock Rate

| | Affects What? |
|---|---|
| Algorithm | Instruction Count, CPI |
| Programming Language | Instruction Count, CPI |
| Compiler | Instruction Count, CPI |
| Instruction Set Architecture | Instruction Count, Clock Rate, CPI |

# Clickers

| Computer | Clock frequency | Clock cycles per instruction | #instructions per program | |
|---|---|---|---|---|
| A | 1GHz | 2 | 1000 | |
| B | 2GHz | 5 | 800 | |
| C | 500MHz | 1.25 | 400 | |
| D | 5GHz | 10 | 2000 | |

- Which computer has the highest performance for a given program?

# Clickers

| Computer | Clock frequency | Clock cycles per instruction | #instructions per program | Calculation |
| --- | --- | --- | --- | --- |
| A | 1GHz | 2 | 1000 | 1ns * 2 * 1000 = 2µs |
| B | 2GHz | 5 | 800 | 0.5ns 5 *800 = 2µs |
| C | 500MHz | 1.25 | 400 | 2ns 1.25 * 400 = 1µs |
| D | 5GHz | 10 | 2000 | 0.2ns * 10 * 2000 = 4µs |

- Which computer has the highest performance for a given program?

# Workload and Benchmark

- *Workload:* Set of programs run on a computer
  - Actual collection of applications run or made from real programs to approximate such a mix
  - Specifies programs, inputs, and relative frequencies
- *Benchmark:* Program selected for use in comparing computer performance
  - Benchmarks form a workload
  - Usually standardized so that many use them

# SPEC
# (System Performance Evaluation Cooperative)

- Computer Vendor cooperative for benchmarks, started in 1989
- SPECCPU2006
  - 12 Integer Programs
  - 17 Floating-Point Programs
- Often turn into number where bigger is faster
- *SPECratio*: reference execution time on old reference computer divide by execution time on new computer to get an effective speed-up

# SPECINT2006 on AMD Barcelona

| Description | Instruction Count (B) | CPI | Clock cycle time (ps) | Execution Time (s) | Reference Time (s) | SPECratio |
|---|---|---|---|---|---|---|
| Interpreted string processing | 2,118 | 0.75 | 400 | 637 | 9,770 | 15.3 |
| Block-sorting compression | 2,389 | 0.85 | 400 | 817 | 9,650 | 11.8 |
| GNU C compiler | 1,050 | 1.72 | 400 | 724 | 8,050 | 11.1 |
| Combinatorial optimization | 336 | 10.0 | 400 | 1,345 | 9,120 | 6.8 |
| Go game | 1,658 | 1.09 | 400 | 721 | 10,490 | 14.6 |
| Search gene sequence | 2,783 | 0.80 | 400 | 890 | 9,330 | 10.5 |
| Chess game | 2,176 | 0.96 | 400 | 837 | 12,100 | 14.5 |
| Quantum computer simulation | 1,623 | 1.61 | 400 | 1,047 | 20,720 | 19.8 |
| Video compression | 3,102 | 0.80 | 400 | 993 | 22,130 | 22.3 |
| Discrete event simulation library | 587 | 2.94 | 400 | 690 | 6,250 | 9.1 |
| Games/path finding | 1,082 | 1.79 | 400 | 773 | 7,020 | 9.1 |
| XML parsing | 1,058 | 2.70 | 400 | 1,143 | 6,900 | 6.0 |

# Summarizing Performance …

| System | Rate (Task 1) | Rate (Task 2) |
| --- | --- | --- |
| A | 10 | 20 |
| B | 20 | 10 |

*Clickers: Which system is faster?*

**A: System A**

**B: System B**

**C: Same performance**

**D: Unanswerable question!**

# … Depends Who's Selling

| System | Rate (Task 1) | Rate (Task 2) | Average |
|---|---|---|---|
| A | 10 | 20 | 15 |
| B | 20 | 10 | 15 |

Average throughput

| System | Rate (Task 1) | Rate (Task 2) | Average |
|---|---|---|---|
| A | 0.50 | 2.00 | 1.25 |
| B | 1.00 | 1.00 | 1.00 |

Throughput relative to B

| System | Rate (Task 1) | Rate (Task 2) | Average |
|---|---|---|---|
| A | 1.00 | 1.00 | 1.00 |
| B | 2.00 | 0.50 | 1.25 |

Throughput relative to A

# Summarizing SPEC Performance

- Varies from 6x to 22x faster than reference computer
- *Geometric mean* of ratios: N-th root of product of N ratios

$$\sqrt[n]{\prod_{i=1}^{n} \text{Execution time ratio}_i}$$

  - Geometric Mean gives same relative answer no matter what computer is used as reference
- Geometric Mean for Barcelona is 11.7

# Administrivia

- Proj 2.1 will be posted tomorrow
- Next weeks lab:
  - Lab 8 will be posted tomorrow
  - In parallel: Project checkup!
- HW 6 will come soon, too…

# Review of Numbers

- Computers are made to deal with numbers
- What can we represent in N bits?
  - $2^N$ things, and no more! They could be...
  - Unsigned integers:

    $0$ to $2^N - 1$

    (for N=32, $2^N – 1 = 4{,}294{,}967{,}295$)
  - Signed Integers (Two's Complement)

    $-2^{(N-1)}$ to $2^{(N-1)} - 1$

    (for N=32, $2^{(N-1)} = 2{,}147{,}483{,}648$)

# What about other numbers?

1. Very large numbers? (seconds/millennium)
   => $31,556,926,000_{10}$ ($3.1556926_{10} \times 10^{10}$)
2. Very small numbers? (Bohr radius)
   => $0.0000000000529177_{10}$m ($5.29177_{10} \times 10^{-11}$)
3. Numbers with <u>both</u> integer & fractional parts?
   => 1.5

*First consider #3.*

*…our solution will also help with #1 and #2.*

# Representation of Fractions

**"Binary Point" like decimal point signifies boundary between integer and fractional parts:**

Example 6-bit representation:

xx.yyyy

$2^1$ $2^0$ $2^{-1}$ $2^{-2}$ $2^{-3}$ $2^{-4}$

$$\mathbf{10.1010_{two} = 1x2^1 + 1x2^{-1} + 1x2^{-3} = 2.625_{ten}}$$

If we assume "fixed binary point", range of 6-bit representations with this format:

0 to 3.9375 (almost 4)

# Fractional Powers of 2

| i | $2^{-i}$ | | |
|---|---|---|---|
| 0 | 1.0 | 1 | |
| 1 | 0.5 | | 1/2 |
| 2 | 0.25 | 1/4 | |
| 3 | 0.125 | 1/8 | |
| 4 | 0.0625 | 1/16 | |
| 5 | 0.03125 | 1/32 | |
| 6 | 0.015625 | | |
| 7 | 0.0078125 | | |
| 8 | 0.00390625 | | |
| 9 | 0.001953125 | | |
| 10 | 0.0009765625 | | |
| 11 | 0.00048828125 | | |
| 12 | 0.000244140625 | | |
| 13 | 0.0001220703125 | | |
| 14 | 0.00006103515625 | | |
| 15 | 0.000030517578125 | | |

# Representation of Fractions with Fixed Pt.

**What about addition and multiplication?**

Addition is straightforward:

```
  01.100   1.5ten
+ 00.100   0.5ten
  10.000   2.0ten
```

Multiplication a bit more complex:

```
     01.100   1.5ten
     00.100   0.5ten
     00 000
    000 00
   0110 0
  00000
 00000
0000110000
```

**Where's the answer, `0.11`? (need to remember where point is)**

# Representation of Fractions

**So far, in our examples we used a "fixed" binary point. What we really want is to "float" the binary point. Why?**

Floating binary point most effective use of our limited bits (and thus more accuracy in our number representation):

example: put $0.1640625_{ten}$ into binary. Represent with 5-bits choosing where to put the binary point.

$\ldots\ 000000.00\underbrace{10101}_{}00000\ldots$

Store these bits and keep track of the binary point 2 places to the left of the MSB

Any other solution would lose accuracy!

**With floating-point rep., each numeral carries an exponent field recording the whereabouts of its binary point.**

**The binary point can be outside the stored bits, so very large and small numbers can be represented.**

# Scientific Notation (in Decimal)

**mantissa** **exponent**

$$6.02_{ten} \times 10^{23}$$

**decimal point** **radix (base)**

- Normalized form: no leadings 0s
  (exactly one digit to left of decimal point)
- Alternatives to representing 1/1,000,000,000
  - Normalized: $1.0 \times 10^{-9}$
  - Not normalized: $0.1 \times 10^{-8}$, $10.0 \times 10^{-10}$

# Scientific Notation (in Binary)

**mantissa** **exponent**

$$1.01_{two} \times 2^{-1}$$

**"binary point"** **radix (base)**

- Computer arithmetic that supports it called [floating point](), because it represents numbers where the binary point is not fixed, as it is for integers
  - Declare such variable in C as `float`
    - `double` for double precision.

# Floating-Point Representation (1/2)

- **Normal format: +1.xxx…x$_{two}$ *2$^{yyy…y}$$_{two}$**
- **Multiple of Word Size (32 bits)**

| 31 | 30 23 | 22 0 |
|---|---|---|
| **S** | **Exponent** | **Significand** |
| 1 bit | 8 bits | 23 bits |

- **S represents Sign**
  **Exponent represents y's**
  **Significand represents x's**

- **Represent numbers as small as 2.0$_{ten}$ x 10$^{-38}$ to as large as 2.0$_{ten}$ x 10$^{38}$**

# Floating-Point Representation (2/2)

- What if result too large?

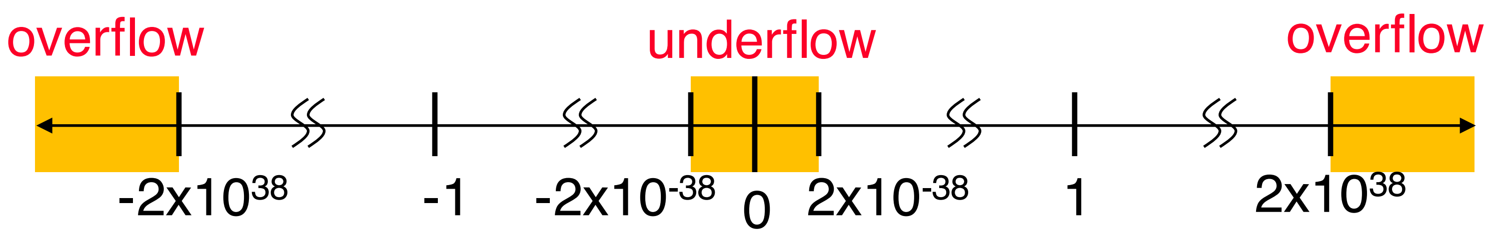
  (> $2.0 \times 10^{38}$ , < $-2.0 \times 10^{38}$ )

  - **Overflow!** => Exponent larger than represented in 8-bit Exponent field

- What if result too small?

  (>0 & < $2.0 \times 10^{-38}$ , <0 & > $-2.0 \times 10^{-38}$ )

  - **Underflow!** => Negative exponent larger than represented in 8-bit Exponent field




- What would help reduce chances of overflow and/or underflow?

# IEEE 754 Floating-Point Standard (1/3)

Single Precision (Double Precision similar):


- Sign bit: 1 means negative 0 means positive
- Significand in *sign-magnitude* format (not 2's complement)
  - To pack more bits, leading 1 implicit for normalized numbers
  - 1 + 23 bits single, 1 + 52 bits double
  - always true: $0 < \text{Significand} < 1$ (for normalized numbers)
- Note: 0 has no leading 1, so reserve exponent value 0 just for number 0

# IEEE 754 Floating Point Standard (2/3)

- IEEE 754 uses “biased exponent” representation
  - Designers wanted FP numbers to be used even if no FP hardware; e.g., sort records with FP numbers using integer compares
  - Wanted bigger (integer) exponent field to represent bigger numbers
  - 2’s complement poses a problem (because negative numbers look bigger)
    - Use just magnitude and offset by half the range

# IEEE 754 Floating Point Standard (3/3)

- Called Biased Notation, where bias is number subtracted to get final number
  - IEEE 754 uses bias of 127 for single prec.
  - Subtract 127 from Exponent field to get actual value for exponent

- **Summary (single precision):**

| 31 | 30 … 23 | 22 … 0 |
|---|---|---|
| **S** | **Exponent** | **Significand** |
| **1 bit** | **8 bits** | **23 bits** |

- $(-1)^S \times (1 + \text{Significand}) \times 2^{(\text{Exponent}-127)}$

  - Double precision identical, except with exponent bias of 1023 (half, quad similar)

# Question

- Guess this Floating Point number:

1 1000 0000  1000 0000 0000 0000 0000 000

A: $-1x\ 2^{128}$

B: $+1x\ 2^{-128}$

C: $-1x\ 2^{1}$

D: $+1.5x\ 2^{-1}$

E: $-1.5x\ 2^{1}$

# Representation for $\pm\infty$

- In FP, divide by 0 should produce $\pm\infty$, not overflow.
- Why?
  - OK to do further computations with $\infty$
    E.g., $X/0 > Y$ may be a valid comparison
- IEEE 754 represents $\pm\infty$
  - Most positive exponent reserved for $\infty$
  - Significands all zeroes

# Representation for 0

- Represent 0?
  - exponent all zeroes
  - significand all zeroes
  - What about sign?  Both cases valid

```
+0: 0 00000000 00000000000000000000000
-0: 1 00000000 00000000000000000000000
```

# Special Numbers

- **What have we defined so far? (Single Precision)**

| Exponent | Significand | Object |
|---|---|---|
| 0 | 0 | 0 |
| 0 | nonzero | ??? |
| 1-254 | anything | +/- fl. pt. # |
| 255 | 0 | +/- ∞ |
| 255 | nonzero | ??? |

- **Clever idea:**
  - Use exp=0,255 & Sig!=0

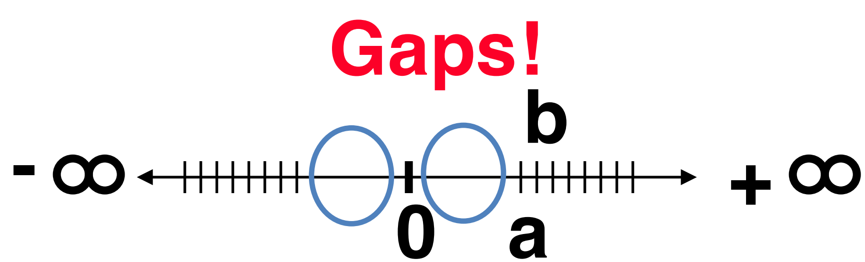
# Representation for Not a Number

- What do I get if I calculate `sqrt(-4.0)` or `0/0`?
  - If ∞ not an error, these shouldn't be either
  - Called Not a Number (NaN)
  - Exponent = 255, Significand nonzero
- Why is this useful?
  - Hope NaNs help with debugging?
  - They contaminate: op(NaN, X) = NaN
  - Can use the significand to identify which!

# Representation for Denorms (1/2)

- Problem: There's a gap among representable FP numbers around 0
  - Smallest representable pos num:
    - a = 1.0… 2 * 2-126 = 2-126
  - Second smallest representable pos num:
    - b = 1.000……1 2 * 2-126
      = (1 + 0.00…12) * 2-126
      = (1 + 2-23) * 2-126
      = 2-126 + 2-149
  - a - 0 = 2-126
  - b - a = 2-149

**Normalization and implicit 1 is to blame!**

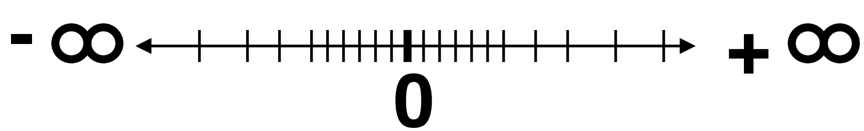


# Representation for Denorms (2/2)

- **Solution:**
  - **We still haven't used Exponent = 0, Significand nonzero**
  - **DEnormalized number: no (implied) leading 1, implicit exponent = -126.**
  - **Smallest representable pos num:**

    **a = $2^{-149}$**
  - **Second smallest representable pos num:**

    **b = $2^{-148}$**



# Special Numbers Summary

- **Reserve exponents, significands:**

| Exponent | Significand | Object |
|---|---|---|
| 0 | 0 | 0 |
| 0 | nonzero | Denorm |
| 1-254 | anything | +/- fl. pt. # |
| 255 | 0 | +/- ∞ |
| 255 | nonzero | NaN |

www.h-schmidt.net/FloatApplet/IEEE754.html

# Conclusion

Exponent tells Significand how much ($2^i$) to count by (…, 1/4, 1/2, 1, 2, …)

- Floating Point lets us:
  - Represent numbers containing both integer and fractional parts; makes efficient use of available bits.
  - Store approximate values for very large and very small #s.

Can store NaN, $\pm\infty$

- IEEE 754 Floating-Point Standard is most widely accepted attempt to standardize interpretation of such numbers (Every desktop or server computer sold since ~1997 follows these conventions)

- Summary (single precision):

| 31 | 30 … 23 | 22 … 0 |
|---|---|---|
| S | Exponent | Significand |
| 1 bit | 8 bits | 23 bits |

- $(-1)^S \times (1 + \text{Significand}) \times 2^{(\text{Exponent}-127)}$

  - Double precision identical, except with exponent bias of 1023 (half, quad similar)

# And In Conclusion, …

- Time (seconds/program) is measure of performance

$$= \frac{\text{Instructions}}{\text{Program}} \times \frac{\text{Clock cycles}}{\text{Instruction}} \times \frac{\text{Seconds}}{\text{Clock Cycle}}$$

- Floating-point representations hold approximations of real numbers in a finite number of bits