CS 110 Computer Architecture Lecture 18:

Amdahl's Law, Data-level Parallelism

Instructor:

Sören Schwertfeger

http://shtech.org/courses/ca/

School of Information Science and Technology SIST

ShanghaiTech University

Slides based on UC Berkley's CS61C

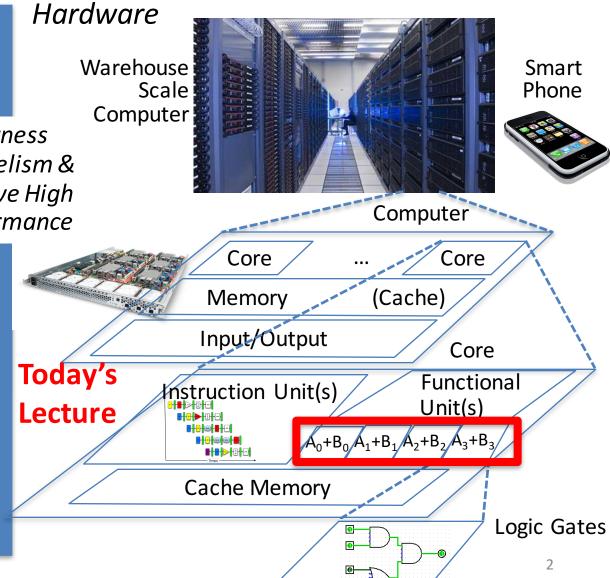
New-School Machine Structures (It's a bit more complicated!)

Software

- Parallel Requests Assigned to computer e.g., Search "Katz"
- Parallel Threads Assigned to core e.g., Lookup, Ads

Harness Parallelism & Achieve High **Performance**

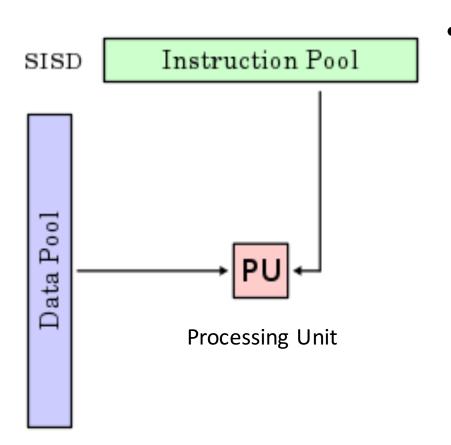
- Parallel Instructions >1 instruction @ one time e.g., 5 pipelined instructions
- Parallel Data >1 data item @ one time e.g., Add of 4 pairs of words
- Hardware descriptions All gates @ one time
- Programming Languages



Using Parallelism for Performance

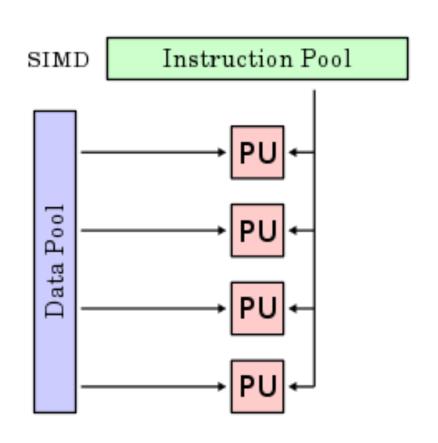
- Two basic ways:
 - Multiprogramming
 - run multiple independent programs in parallel
 - "Easy"
 - Parallel computing
 - run one program faster
 - "Hard"
- We'll focus on parallel computing for next few lectures

Single-Instruction/Single-Data Stream (SISD)



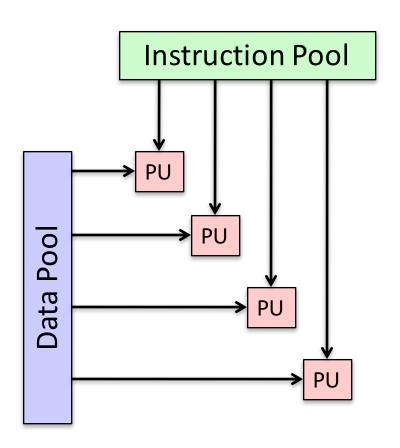
 Sequential computer that exploits no parallelism in either the instruction or data streams. Examples of SISD architecture are traditional uniprocessor machines

Single-Instruction/Multiple-Data Stream (SIMD or "sim-dee")



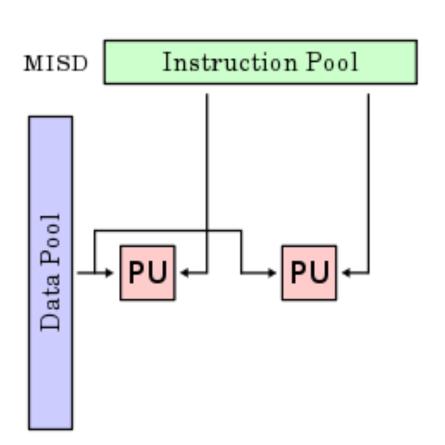
 SIMD computer exploits multiple data streams against a single instruction stream to operations that may be naturally parallelized, e.g., Intel SIMD instruction extensions or NVIDIA Graphics Processing Unit (GPU)

Multiple-Instruction/Multiple-Data Streams (MIMD or "mim-dee")



- Multiple autonomous processors simultaneously executing different instructions on different data.
 - MIMD architectures include multicore and Warehouse-Scale Computers

Multiple-Instruction/Single-Data Stream (MISD)



- Multiple-Instruction,
 Single-Data stream
 computer that exploits
 multiple instruction
 streams against a single
 data stream.
 - Rare, mainly of historical interest only

Flynn* Taxonomy, 1966

		Data Streams		
		Single	Multiple	
Instruction Streams	Single	SISD: Intel Pentium 4	SIMD: SSE instructions of x86	
	Multiple	MISD: No examples today	MIMD: Intel Xeon e5345 (Clovertown)	

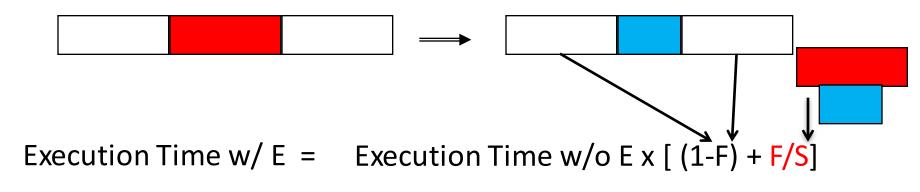
- In 2013, SIMD and MIMD most common parallelism in architectures – usually both in same system!
- Most common parallel processing programming style: Single Program Multiple Data ("SPMD")
 - Single program that runs on all processors of a MIMD
 - Cross-processor execution coordination using synchronization primitives
- SIMD (aka hw-level data parallelism): specialized function units, for handling lock-step calculations involving arrays
 - Scientific computing, signal processing, multimedia (audio/video processing)

*Prof. Michael Flynn, Stanford

Big Idea: Amdahl's (Heartbreaking) Law

Speedup due to enhancement E is

Suppose that enhancement E accelerates a fraction F (F < 1)
 of the task by a factor S (S>1) and the remainder of the task is
 unaffected



Speedup w/ E = 1/[(1-F) + F/S]

Big Idea: Amdahl's Law

Speedup =
$$\frac{1}{(1-F) + \frac{F}{S}}$$

Non-speed-up part

Example: the execution time of half of the program can be accelerated by a factor of 2.

What is the program speed-up overall?

$$\frac{1}{0.5 + 0.5} = \frac{1}{0.5 + 0.25} = 1.33$$

Example #1: Amdahl's Law

Speedup w/
$$E = 1/[(1-F) + F/S]$$

 Consider an enhancement which runs 20 times faster but which is only usable 25% of the time

Speedup w/ E =
$$1/(.75 + .25/20) = 1.31$$

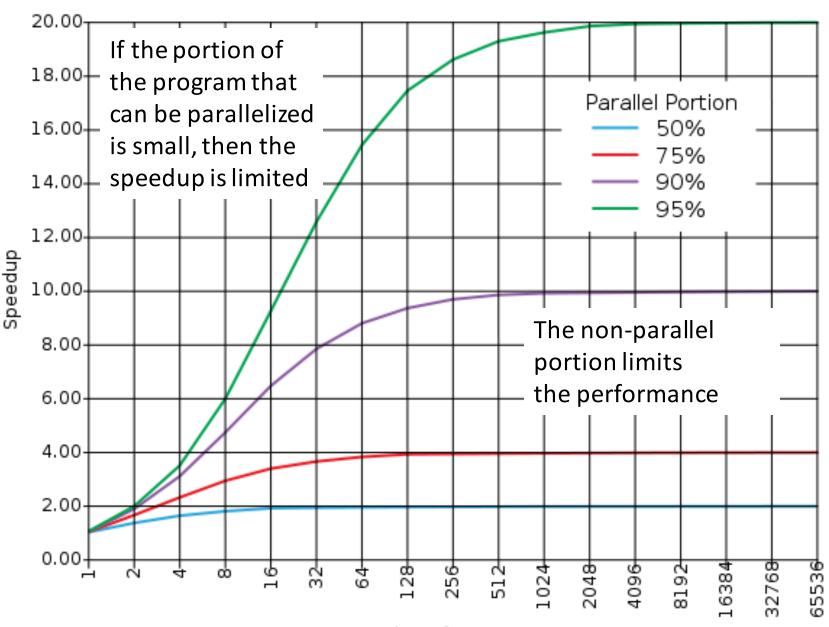
What if its usable only 15% of the time?

Speedup w/E =
$$1/(.85 + .15/20) = 1.17$$

- Amdahl's Law tells us that to achieve linear speedup with 100 processors, none of the original computation can be scalar!
- To get a speedup of 90 from 100 processors, the percentage of the original program that could be scalar would have to be 0.1% or less

Speedup w/E =
$$1/(.001 + .999/100) = 90.99$$

Amdahl's Law



Number of Processors

Strong and Weak Scaling

- To get good speedup on a parallel processor while keeping the problem size fixed is harder than getting good speedup by increasing the size of the problem.
 - Strong scaling: when speedup can be achieved on a parallel processor without increasing the size of the problem
 - Weak scaling: when speedup is achieved on a parallel processor by increasing the size of the problem proportionally to the increase in the number of processors
- Load balancing is another important factor: every processor doing same amount of work
 - Just one unit with twice the load of others cuts speedup almost in half

Question

Suppose a program spends 80% of its time in a square root routine. How much must you speedup square root to make the program run 5 times faster?

Speedup w/
$$E = 1 / [(1-F) + F/S]$$

A: 5

B: 16

C: 20

D: 100

E: None of the above

SIMD Architectures

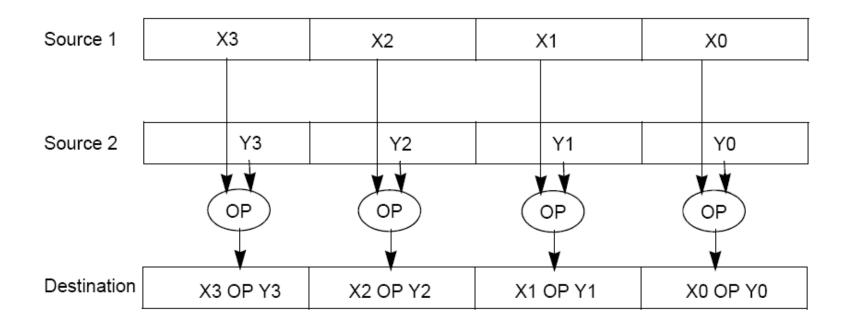
- Data parallelism: executing same operation on multiple data streams
- Example to provide context:
 - Multiplying a coefficient vector by a data vector (e.g., in filtering)

$$y[i] := c[i] \times x[i], 0 \le i < n$$

- Sources of performance improvement:
 - One instruction is fetched & decoded for entire operation
 - Multiplications are known to be independent
 - Pipelining/concurrency in memory access as well

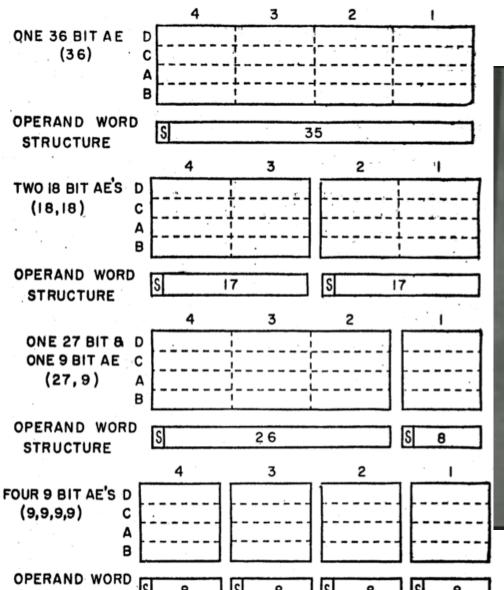
Intel "Advanced Digital Media Boost"

- To improve performance, Intel's SIMD instructions
 - Fetch one instruction, do the work of multiple instructions



First SIMD Extensions:

MIT Lincoln Labs TX-2, 1957



STRUCTURE



Admin

- Project Checkup:
 - 2 groups from lab 1 missing: today after lecture!
 - Lab 2 and 3: everybody needs to attend
- Project 2.x:
 - Use git to share your work and collaborate
 - New rule enforced:
 - Every group member needs to have at least 2 commits and at least 25% of the # commits from his own PC.
 - Penalty for violation: 10% off project score for violator!
 - Will be posted also on piazza

Intel SIMD Extensions

- MMX 64-bit registers, reusing floating-point registers [1992]
- SSE2/3/4, new 128-bit registers [1999]
- AVX, new 256-bit registers [2011]
 - Space for expansion to 1024-bit registers

XMM Registers

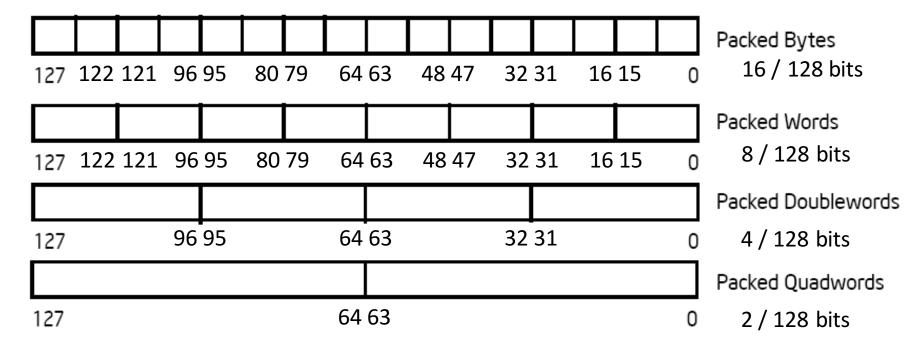
127 0 XMM7 XMM6 XMM5 XMM4 XMM3 XMM2 XMM1 XMM0

- Architecture extended with eight 128-bit data registers:
 XMM registers
 - x86 64-bit address architecture adds 8 additional registers (XMM8 – XMM15)

Intel Architecture SSE2+ 128-Bit SIMD Data Types

- Note: in Intel Architecture (unlike MIPS) a word is 16 bits
 - Single-precision FP: Double word (32 bits)
 - Double-precision FP: Quad word (64 bits)

Fundamental 128-Bit Packed SIMD Data Types



SSE/SSE2 Floating Point Instructions

Move does both load and store

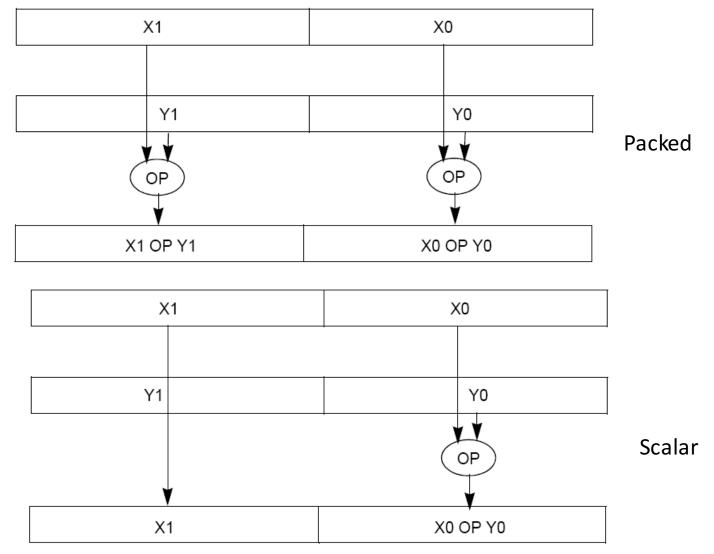
Data transfer	Arithmetic	Compare
MOV{A/U}{SS/PS/SD/PD} xmm, mem/xmm	ADD{SS/PS/SD/PD} xmm, mem/xmm	CMP{SS/PS/SD/ PD}
	SUB{SS/PS/SD/PD} xmm, mem/xmm	
MOV {H/L} {PS/PD} xmm, mem/xmm	<pre>MUL{SS/PS/SD/PD} xmm, mem/xmm</pre>	
	<pre>DIV{SS/PS/SD/PD} xmm, mem/xmm</pre>	
	SQRT{SS/PS/SD/PD} mem/xmm	
	MAX {SS/PS/SD/PD} mem/xmm	
	MIN{SS/PS/SD/PD} mem/xmm	

xmm: one operand is a 128-bit SSE2 register

mem/xmm: other operand is in memory or an SSE2 register

- {SS} Scalar Single precision FP: one 32-bit operand in a 128-bit register
- {PS} Packed Single precision FP: four 32-bit operands in a 128-bit register
- {SD} Scalar Double precision FP: one 64-bit operand in a 128-bit register
- {PD} Packed Double precision FP, or two 64-bit operands in a 128-bit register
- {A} 128-bit operand is aligned in memory
- {U} means the 128-bit operand is unaligned in memory
- {H} means move the high half of the 128-bit operand
- {L} means move the low half of the 128-bit operand

Packed and Scalar Double-Precision Floating-Point Operations



Example: SIMD Array Processing

```
for each f in array
    f = sqrt(f)
for each f in array
    load f to the floating-point register
   calculate the square root
   write the result from the register to memory
for each 4 members in array
    load 4 members to the SSE register
   calculate 4 square roots in one operation
    store the 4 results from the register to memory
                   SIMD style
```

Data-Level Parallelism and SIMD

- SIMD wants adjacent values in memory that can be operated in parallel
- Usually specified in programs as loops

```
for(i=1000; i>0; i=i-1)
x[i] = x[i] + s;
```

- How can reveal more data-level parallelism than available in a single iteration of a loop?
- Unroll loop and adjust iteration rate

Looping in MIPS

Assumptions:

- \$t1 is initially the address of the element in the array with the highest address
- \$f0 contains the scalar value s
- 8(\$t2) is the address of the last element to operate on

CODE:

```
Loop: 1. l.d $f2,0($t1) ; $f2=array element
2. add.d $f10,$f2,$f0 ; add s to $f2
3. s.d $f10,0($t1) ; store result
4. addiu $t1,$t1,#-8 ; decrement pointer 8 byte
5. bne $t1,$t2,Loop ; repeat loop if $t1 != $t2
```

Loop Unrolled

```
Loop:
       I.d
               $f2,0($t1)
       add.d
               $f10,$f2,$f0
               $f10,0($t1)
       s.d
               $f4,-8($t1)
       I.d
       add.d
              $f12,$f4,$f0
               $f12,-8($t1)
       s.d
       I.d
               $f6,-16($t1)
              $f14,$f6,$f0
       add.d
       s.d
               $f14,-16($t1)
       I.d
               $f8,-24($t1)
       add.d $f16,$f8,$f0
               $f16,-24($t1)
       s.d
       addiu
              $t1,$t1,#-32
               $t1,$t2,Loop
       bne
```

NOTE:

- 1. Only 1 Loop Overhead every 4 iterations
- 2. This unrolling works if

$$loop_limit(mod 4) = 0$$

3. Using different registers for each iteration eliminates data hazards in pipeline

Loop Unrolled Scheduled

```
Loop:I.d
             $f2,0($t1)
     I.d
             $f4,-8($t1)
                              4 Loads side-by-side: Could replace with 4-wide SIMD
     I.d
             $f6,-16($t1)
                               Load
             $f8,-24($t1)
     I.d
     add.d $f10,$f2,$f0
     add.d $f12,$f4,$f0
                               4 Adds side-by-side: Could replace with 4-wide SIMD Add
     add.d $f14,$f6,$f0
     add.d $f16,$f8,$f0
             $f10,0($t1)
     s.d
             $f12,-8($t1)
     s.d
                             4 Stores side-by-side: Could replace with 4-wide SIMD Store
             $f14,-16($t1)
     s.d
             $f16,-24($t1)
     s.d
     addiu $t1,$t1,#-32
             $t1,$t2,Loop
     bne
```

Loop Unrolling in C

Instead of compiler doing loop unrolling, could do it yourself in C

```
for(i=1000; i>0; i=i-1)
x[i] = x[i] + s;
```

Could be rewritten
 What is downside of doing it in C?

```
for(i=1000; i>0; i=i-4) {
  x[i] = x[i] + s;
  x[i-1] = x[i-1] + s;
  x[i-2] = x[i-2] + s;
  x[i-3] = x[i-3] + s;
}
```

Generalizing Loop Unrolling

- A loop of n iterations
- k copies of the body of the loop
- Assuming (n mod k) ≠ 0

Then we will run the loop with 1 copy of the body (n mod k) times and with k copies of the body floor(n/k) times

Example: Add Two Single-Precision Floating-Point Vectors

Computation to be performed:

```
vec_res.x = v1.x + v2.x;
vec_res.y = v1.y + v2.y;
vec_res.z = v1.z + v2.z;
vec_res.w = v1.w + v2.w;
```

mov a ps: **mov**e from mem to XMM register, mem ory **a**ligned, **p**acked **s**ingle precision add ps: **add** from mem to XMM register,

add ps: add from mem to XMM register, packed single precision

mov a ps: **mov**e from XMM register to mem, memory **a**ligned, **p**acked **s**ingle precision

SSE Instruction Sequence:

(Note: Destination on the fight in x86 assembly)

Intel SSE Intrinsics

- Intrinsics are C functions and procedures for inserting assembly language into C code, including SSE instructions
 - With intrinsics, can program using these instructions indirectly
 - One-to-one correspondence between SSE instructions and intrinsics

Example SSE Intrinsics

Intrinsics:

Corresponding SSE instructions:

• Vector data type:

```
_m128d
```

Load and store operations:

```
_mm_load_pd
_mm_store_pd
_mm_loadu_pd
_mm_storeu_pd
```

MOVAPD/aligned, packed double MOVAPD/aligned, packed double MOVUPD/unaligned, packed double MOVUPD/unaligned, packed double

Load and broadcast across vector

• Arithmetic:

Definition of Matrix Multiply:
$$C_{i,j} = (A \times B)_{i,j} = \sum_{k=1}^{2} A_{i,k} \times B_{k,j}$$

$$C_{i,j} = (A \times B)_{i,j} = \sum_{k=1}^{\infty} A_{i,k} \times B_{k,j}$$

$$K = 1$$

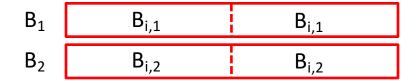
$$\begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \times \begin{bmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{bmatrix} = \begin{bmatrix} C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1} & C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2} \\ C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1} & C_{2,2} = A_{2,1}B_{1,2} + A_{2,2}B_{2,2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} C_{1,1} = 1*1 + 0*2 = 1 & C_{1,2} = 1*3 + 0*4 = 3 \\ C_{2,1} = 0*1 + 1*2 = 2 & C_{2,2} = 0*3 + 1*4 = 4 \end{bmatrix}$$

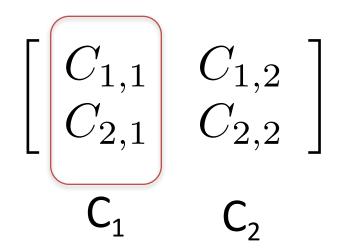
- Using the XMM registers
 - 64-bit/double precision/two doubles per XMM reg

$$\begin{array}{c|cccc} C_1 & C_{1,1} & C_{2,1} \\ C_2 & C_{1,2} & C_{2,2} \end{array}$$





Stored in memory in Column order



Initialization



$$\begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \times \begin{bmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{bmatrix} = \begin{bmatrix} C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1} & C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2} \\ C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1} & C_{2,2} = A_{2,1}B_{1,2} + A_{2,2}B_{2,2} \end{bmatrix}$$

Initialization



• | = 1



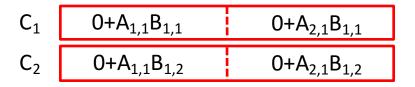
_mm_load_pd: Load 2 doubles into XMM
reg, Stored in memory in Column order

$$B_1$$
 $B_{1,1}$ $B_{1,1}$ $B_{1,2}$ $B_{1,2}$

_mm_load1_pd: SSE instruction that loads a double word and stores it in the high and low double words of the XMM register (duplicates value in both halves of XMM)

$$\begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \times \begin{bmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{bmatrix} = \begin{bmatrix} C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1} & C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2} \\ C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1} & C_{2,2} = A_{2,1}B_{1,2} + A_{2,2}B_{2,2} \end{bmatrix}$$

• First iteration intermediate result



c1 = _mm_add_pd(c1,_mm_mul_pd(a,b1));
c2 = _mm_add_pd(c2,_mm_mul_pd(a,b2));
SSE instructions first do parallel multiplies
and then parallel adds in XMM registers



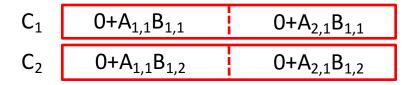
_mm_load_pd: Stored in memory in Column order

$$B_1$$
 $B_{1,1}$ $B_{1,1}$ $B_{1,2}$ $B_{1,2}$

_mm_load1_pd: SSE instruction that loads a double word and stores it in the high and low double words of the XMM register (duplicates value in both halves of XMM)

$$\begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \times \begin{bmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{bmatrix} = \begin{bmatrix} C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1} \\ C_{2,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1} \end{bmatrix} \quad C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2}$$

First iteration intermediate result



c1 = _mm_add_pd(c1,_mm_mul_pd(a,b1));
c2 = _mm_add_pd(c2,_mm_mul_pd(a,b2));
SSE instructions first do parallel multiplies
and then parallel adds in XMM registers



_mm_load_pd: Stored in memory in Column order

$$B_1$$
 $B_{2,1}$ $B_{2,1}$ $B_{2,2}$ $B_{2,2}$

_mm_load1_pd: SSE instruction that loads a double word and stores it in the high and low double words of the XMM register (duplicates value in both halves of XMM)

Second iteration intermediate result

c1 = _mm_add_pd(c1,_mm_mul_pd(a,b1));
c2 = _mm_add_pd(c2,_mm_mul_pd(a,b2));
SSE instructions first do parallel multiplies
and then parallel adds in XMM registers



_mm_load_pd: Stored in memory in Column order



_mm_load1_pd: SSE instruction that loads a double word and stores it in the high and low double words of the XMM register (duplicates value in both halves of XMM)

Example: 2 x 2 Matrix Multiply (Part 1 of 2)

```
#include <stdio.h>
// header file for SSE compiler intrinsics
#include <emmintrin.h>
// NOTE: vector registers will be represented in
    // comments as v1 = [a | b]
// where v1 is a variable of type m128d and
    // a, b are doubles
int main(void) {
  // allocate A,B,C aligned on 16-byte boundaries
  double A[4] attribute ((aligned (16)));
  double B[4] attribute ((aligned (16)));
  double C[4] attribute ((aligned (16)));
  int Ida = 2;
  int i = 0;
  // declare several 128-bit vector variables
  m128d c1,c2,a,b1,b2;
```

```
// Initialize A, B, C for example
/* A =
                     (note column order!)
    10
   01
   */
  A[0] = 1.0; A[1] = 0.0; A[2] = 0.0; A[3] = 1.0;
/* B =
                      (note column order!)
    13
    24
   */
  B[0] = 1.0; B[1] = 2.0; B[2] = 3.0; B[3] = 4.0;
/* C =
                       (note column order!)
    00
    00
   */
  C[0] = 0.0; C[1] = 0.0; C[2] = 0.0; C[3] = 0.0;
```

Example: 2 x 2 Matrix Multiply (Part 2 of 2)

```
// used aligned loads to set
  //c1 = [c \ 11 \ | \ c \ 21]
  c1 = mm load pd(C+0*lda);
  //c2 = [c 12 | c 22]
  c2 = mm load pd(C+1*lda);
  for (i = 0; i < 2; i++) {
    /* a =
     i = 0: [a 11 | a 21]
     i = 1: [a_12 | a 22]
     a = mm load pd(A+i*lda);
    /* b1 =
     i = 0: [b 11 | b 11]
     i = 1: [b 21 | b 21]
     */
     b1 = mm load1 pd(B+i+0*lda);
    /* b2 =
     i = 0: [b 12 | b 12]
     i = 1: [b 22 | b 22]
     b2 = mm load1 pd(B+i+1*lda);
```

```
/* c1 =
   i = 0: [c 11 + a 11*b 11 | c 21 + a 21*b 11]
   i = 1: [c 11 + a 21*b 21 | c 21 + a 22*b 21]
  */
  c1 = mm add pd(c1, mm mul pd(a,b1));
  /* c2 =
   i = 0: [c 12 + a 11*b 12 | c 22 + a 21*b 12]
   i = 1: [c 12 + a 21*b_22 | c_22 + a_22*b_22]
  c2 = mm add pd(c2, mm mul pd(a,b2));
// store c1,c2 back into C for completion
mm store pd(C+0*lda,c1);
mm store pd(C+1*lda,c2);
// print C
printf("%g,%g\n%g,%g\n",C[0],C[2],C[1],C[3]);
return 0;
```

Inner loop from gcc –O -S

```
(%rax,%rsi), %xmm1 //Load aligned A[i,i+1]->m1
L2: movapd
                                //Load B[j], duplicate->m0
   movddup
             (%rdx), %xmm0
                                //Multiply m0*m1->m0
   mulpd
             %xmm1, %xmm0
   addpd
                                //Add m0+m3->m3
             %xmm0, %xmm3
                                //Load B[j+1], duplicate->m0
   movddup
             16(%rdx), %xmm0
                                //Multiply m0*m1->m1
   mulpd
             %xmm0, %xmm1
   addpd
             %xmm1, %xmm2
                                //Add m1+m2->m2
                                // rax + 16 -> rax (i += 2)
   addq
             $16, %rax
                                // rdx + 8 -> rdx (j+=1)
   addq
             $8, %rdx
             $32, %rax
                                // rax == 32?
   cmpq
                                // jump to L2 if not equal
   ine
             L2
                                //store aligned m3 into C[k,k+1]
             %xmm3, (%rcx)
   movapd
                                //store aligned m2 into C[I,I+1]
             %xmm2, (%rdi)
   movapd
```

And in Conclusion, ...

- Amdahl's Law: Serial sections limit speedup
- Flynn Taxonomy
- Intel SSE SIMD Instructions
 - Exploit data-level parallelism in loops
 - One instruction fetch that operates on multiple operands simultaneously
 - 128-bit XMM registers
- SSE Instructions in C
 - Embed the SSE machine instructions directly into C programs through use of intrinsics
 - Achieve efficiency beyond that of optimizing compiler