CS 110
Computer Architecture
Lecture 9:
Finite State Machines, Functional Units

Instructors:
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https://robotics.shanghaitech.edu.cn/courses/ca/20s/

School of Information Science and Technology SIST

ShanghaiTech University

Slides based on UC Berkley's CS61C

Admin

- HW 3 is due tomorrow!
- HW 4 published
  - No programming – still 2 weeks time!
- Project 1.1 due in about 1 week.

Levels of Representation/Interpretation

High Level Language Program (e.g., C)

Compiler

Assembly Language Program (e.g., RISC-V)

Machine Language Program (RISC-V)

```
temp = v[k];
v[k] = v[k+1];
v[k+1] = temp;
```

```
lw    xt0, 0(x2)
lw    xt1, 4(x2)
sw    xt1, 0(x2)
sw    xt0, 4(x2)
```

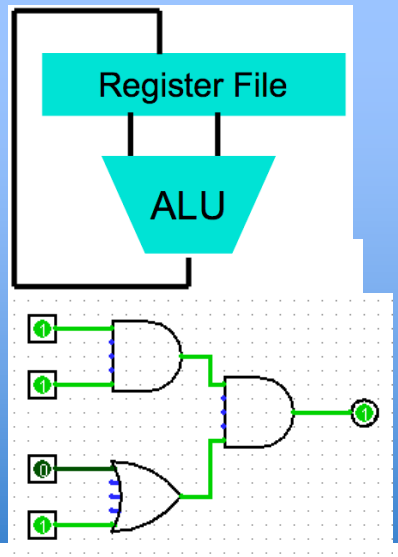
```
0000 1001 1100 0110 1010 1111 0101 1000
1010 1111 0101 1000 0000 1001 1100 0110
1100 0110 1010 1111 0101 1000 0000 1001
0101 1000 0000 1001 1100 0110 1010 1111
```

Machine Interpretation

Hardware Architecture Description (e.g., block diagrams)

Architecture Implementation

Logic Circuit Description (Circuit Schematic Diagrams)



Review

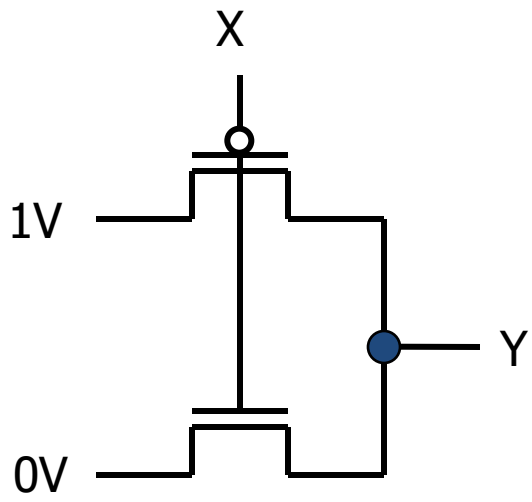
CMOS Networks

p-channel transistor

on when voltage at Gate is low

off when:

voltage(Gate) > voltage (Threshold)



n-channel transitor

off when voltage at Gate is low

on when:

voltage(Gate) > voltage (Threshold)

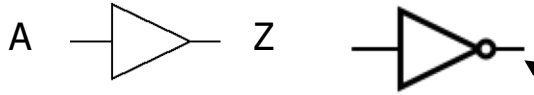
what is the relationship between x and y?

X	Y
0 Volt (GND)	1 Volt (Vdd)
1 Volt (Vdd)	0 Volt (GND)

Called an *inverter* or *not gate*

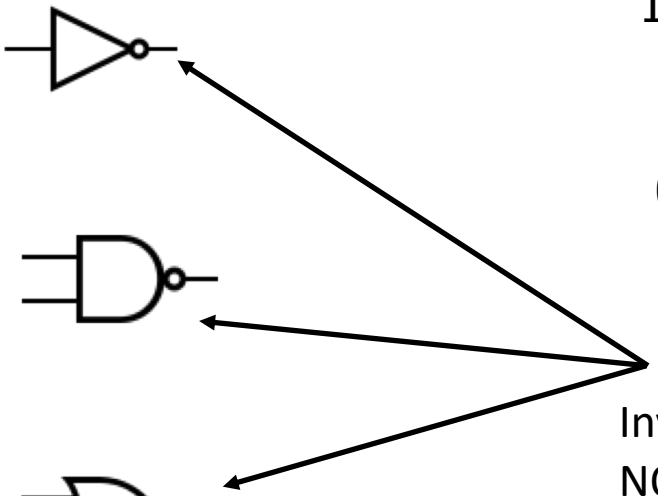
Combinational Logic Symbols

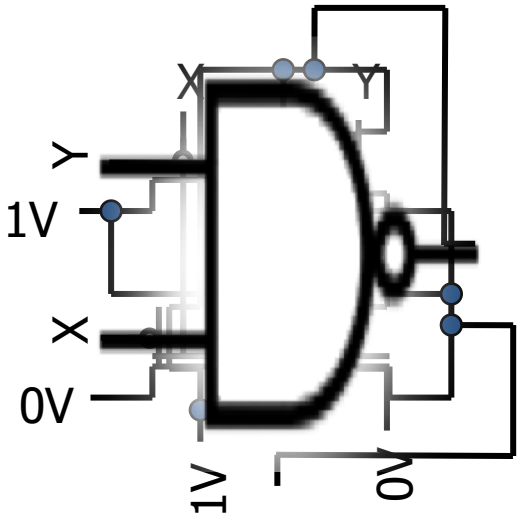
- Common combinational logic systems have standard symbols called logic gates
  - Buffer, NOT



  - AND, NAND

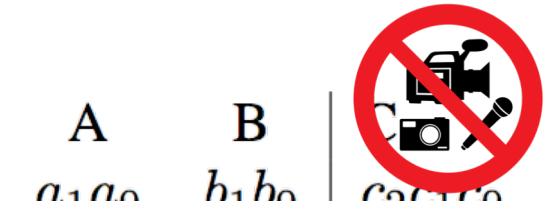
  - OR, NOR





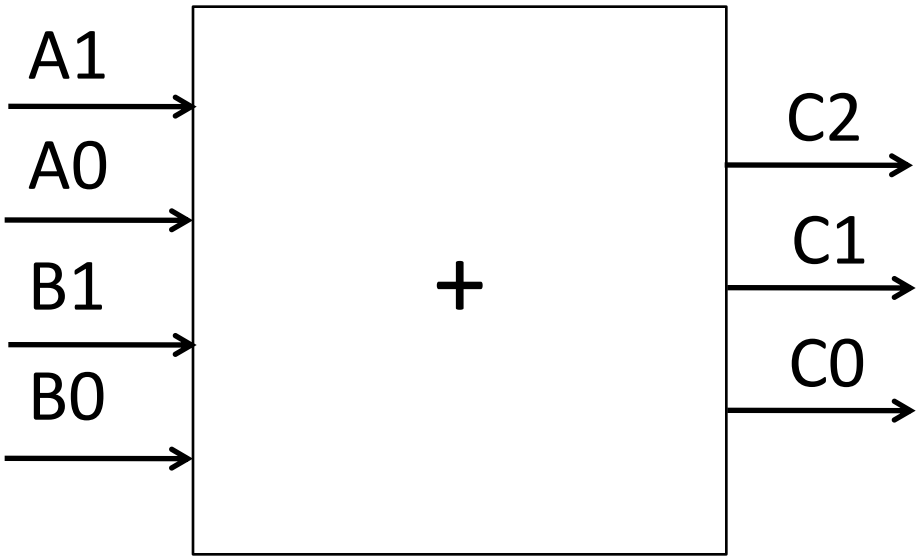
Inverting versions (NOT, NAND, NOR) easiest to implement with CMOS transistors (the switches we have available and use most)

Truth Table Example #2: 2-bit Adder



A	B	C
a_1a_0	b_1b_0	$c_2c_1c_0$

How Many Rows?

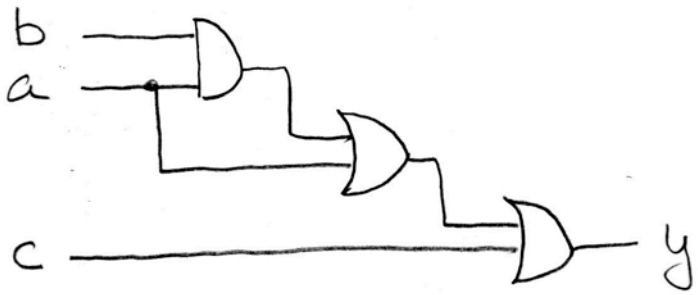


Truth Table Example #3: 32-bit Unsigned Adder

A	B	C
000 ... 0	000 ... 0	000 ... 00
000 ... 0	000 ... 1	000 ... 01
.	.	.
.	.	.
.	.	.
111 ... 1	111 ... 1	111 ... 10

How Many Rows?

Boolean Algebra: Circuit & Algebraic Simplification



original circuit

$$\downarrow$$

$$y = ((ab) + a) + c$$

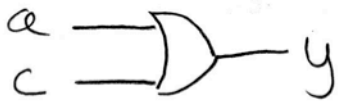
equation derived from original circuit

$$\downarrow$$

$$\begin{aligned} &= ab + a + c \\ &= a(b + 1) + c \\ &= a(1) + c \\ &= a + c \end{aligned}$$

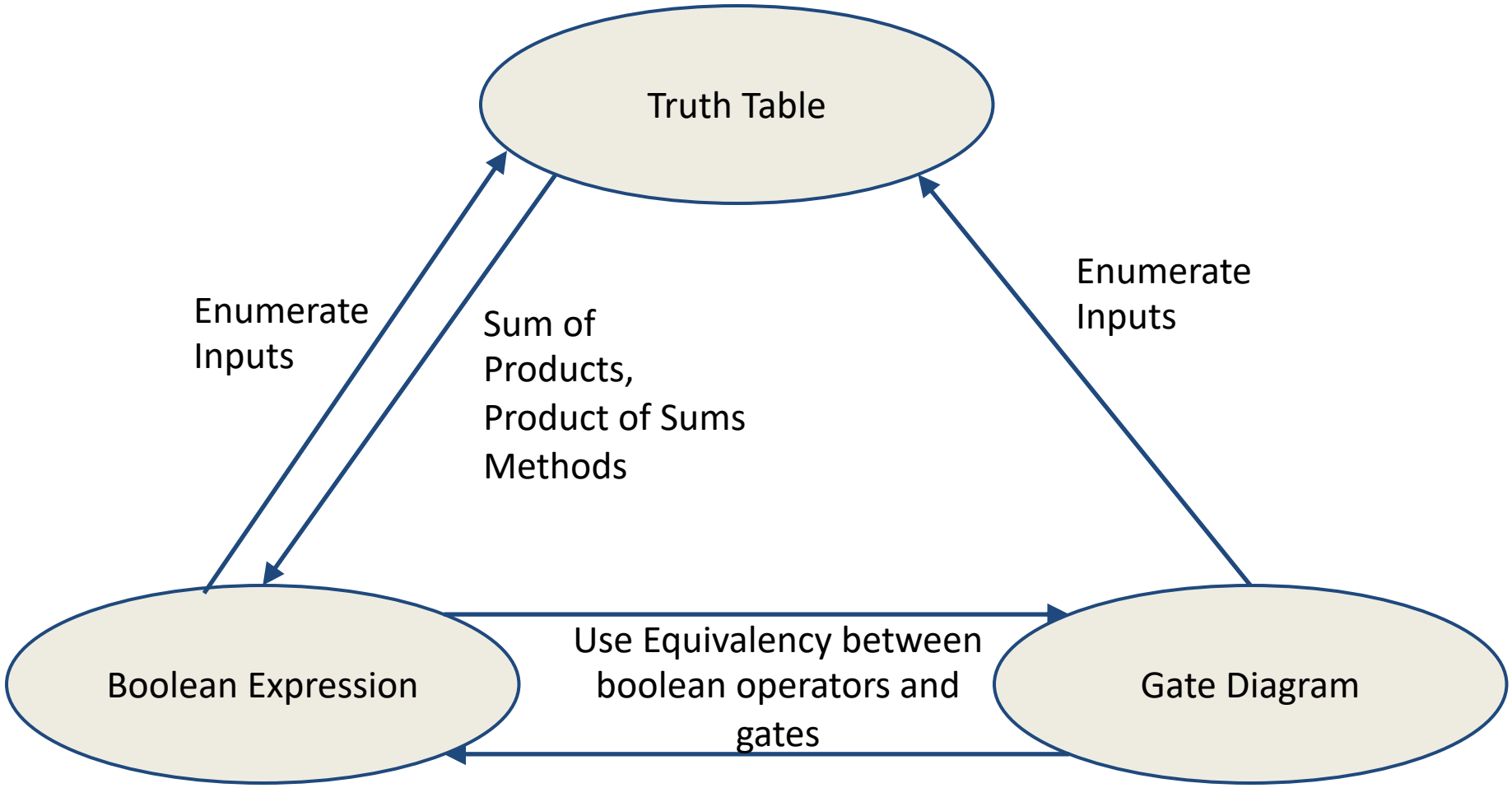
algebraic simplification

$$\downarrow$$

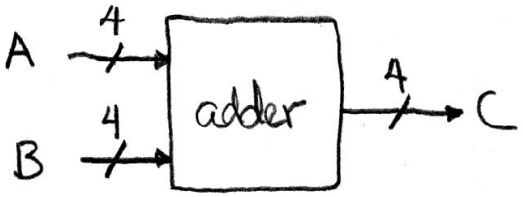


simplified circuit

Representations of Combinational Logic (groups of logic gates)

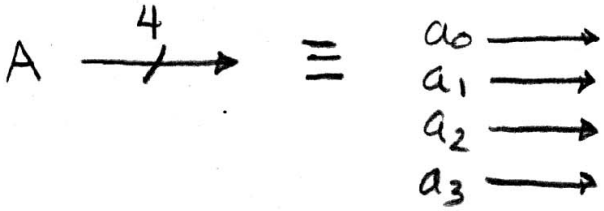


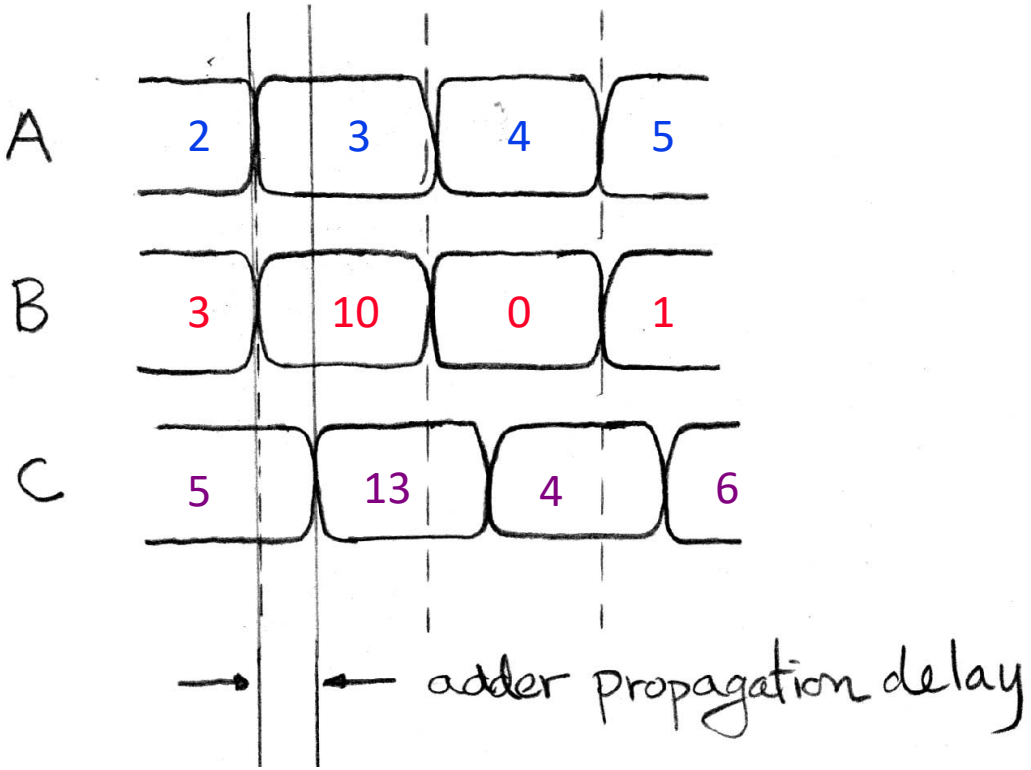
Signals and Waveforms: Circuit Delay



$A = [a_3, a_2, a_1, a_0]$

$B = [b_3, b_2, b_1, b_0]$



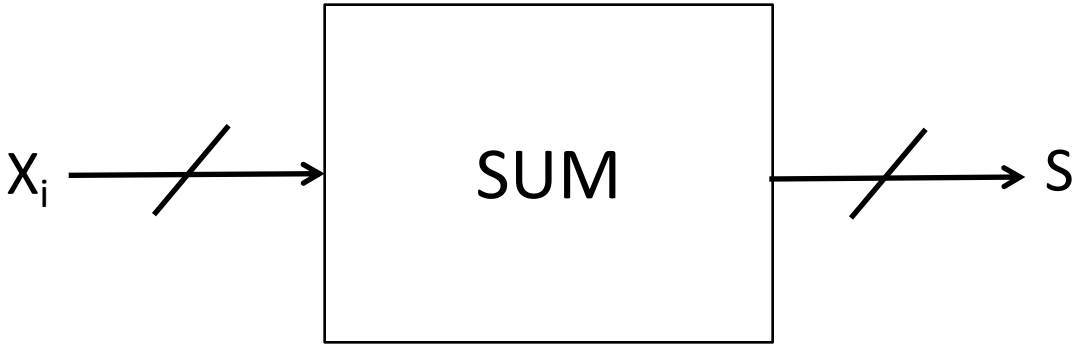


Type of Circuits

- *Synchronous Digital Systems* consist of two basic types of circuits:
  - Combinational Logic (CL) circuits
    - Output is a function of the inputs only, not the history of its execution
    - E.g., circuits to add A, B (ALUs)
  - Sequential Logic (SL)
    - Circuits that "remember" or store information
    - aka "State Elements"
    - E.g., memories and registers (Registers)

Accumulator Example

Why do we need to control the flow of information?



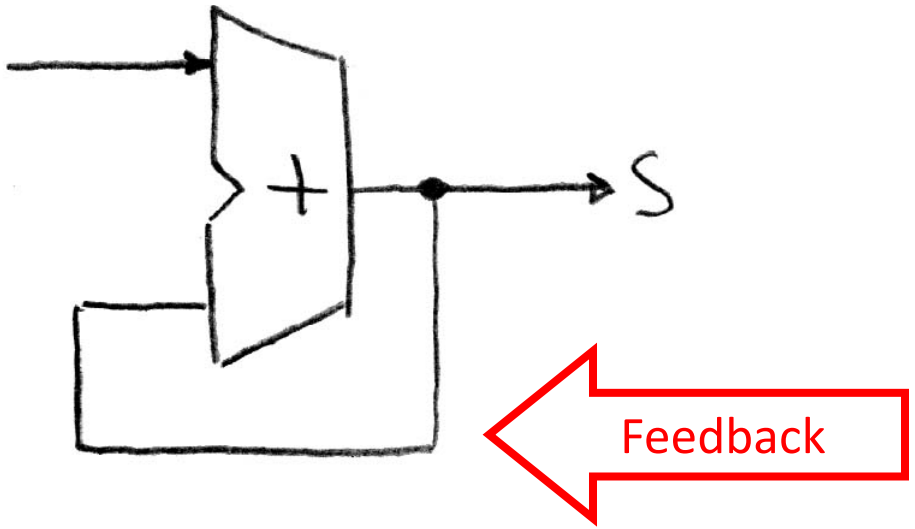
Want:

```
S=0;
for (i=0;i<n;i++)
    S = S + Xi
```

Assume:

- Each X value is applied in succession, one per cycle
- After n cycles the sum is present on S

First Try: Does this work?

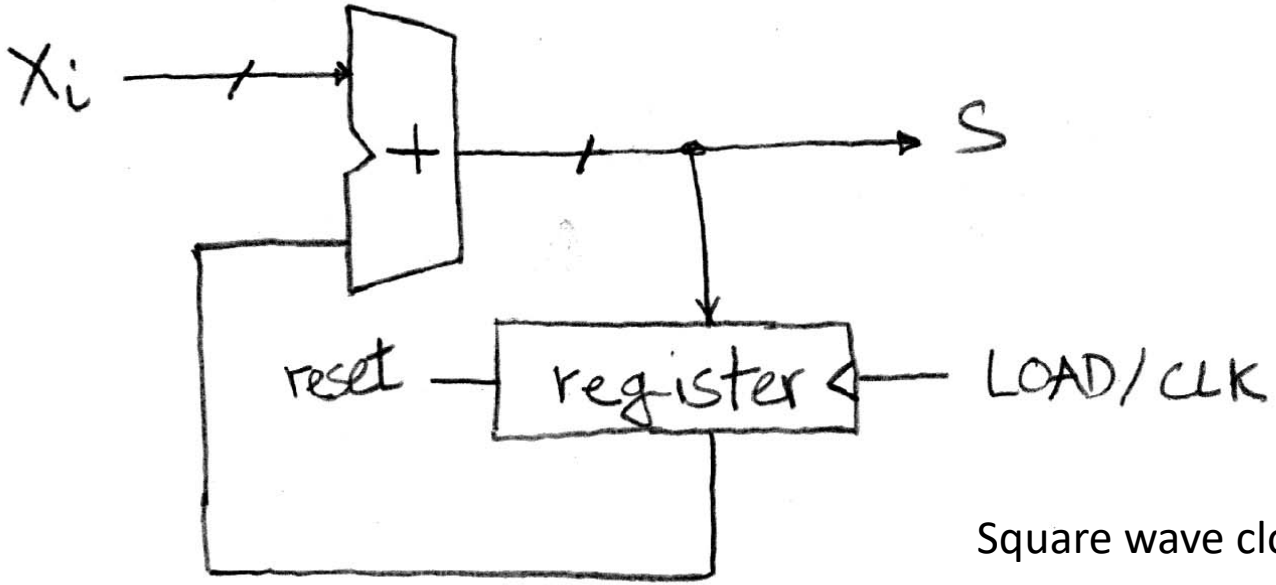


No!

Reason #1: How to control the next iteration of the ‘for’ loop?

Reason #2: How do we say: ‘S=0’?

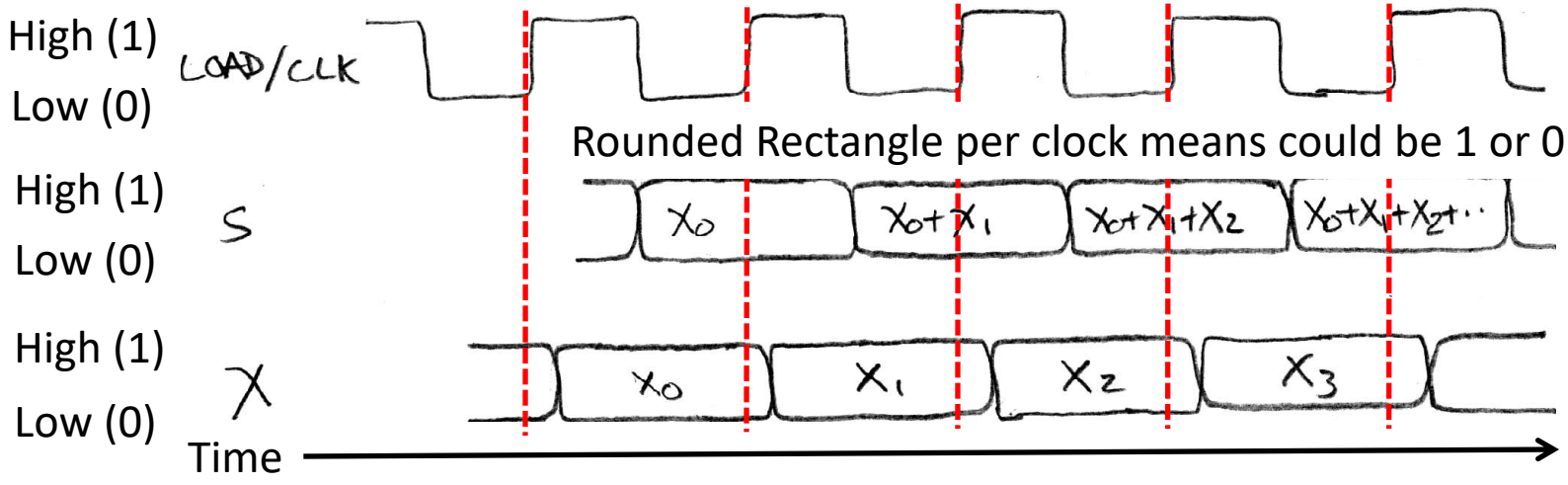
Second Try: How About This?



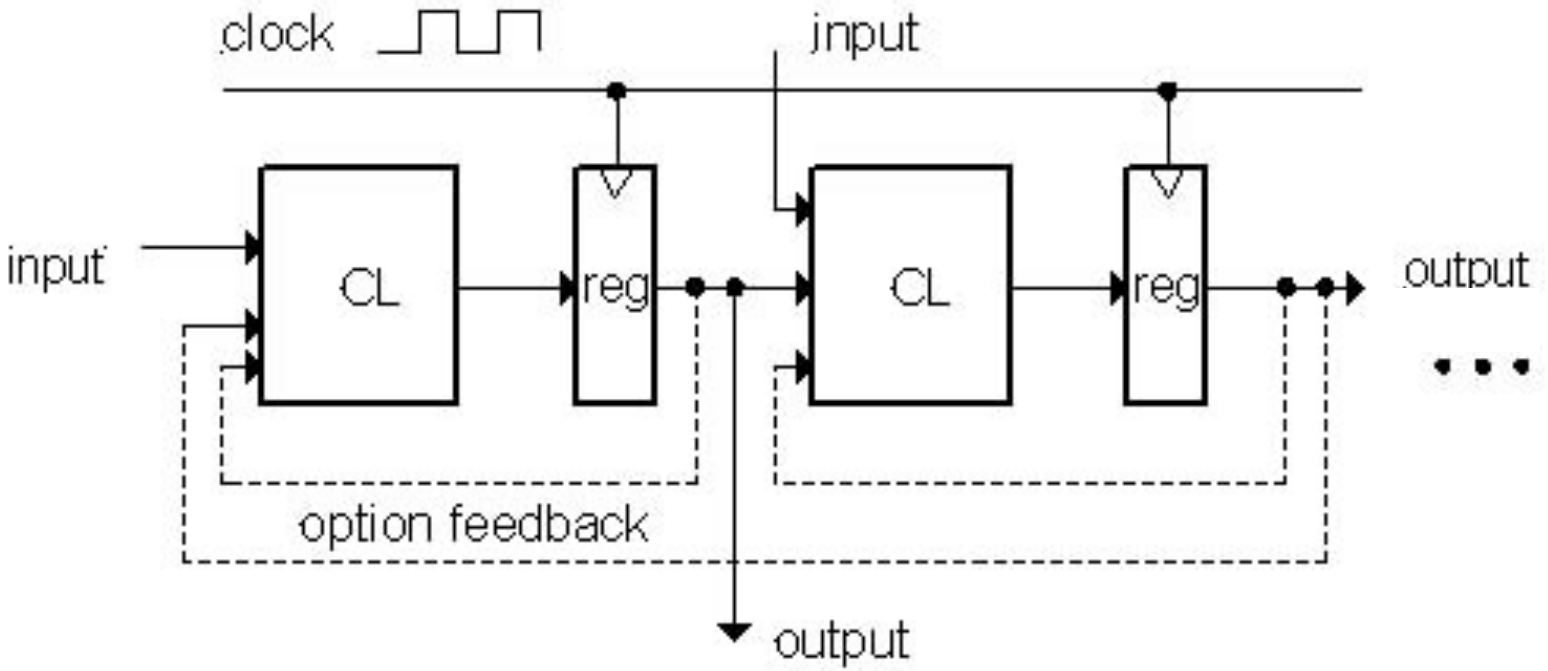
Register is used to hold up the transfer of data to adder

Square wave clock sets when things change

Rough timing …

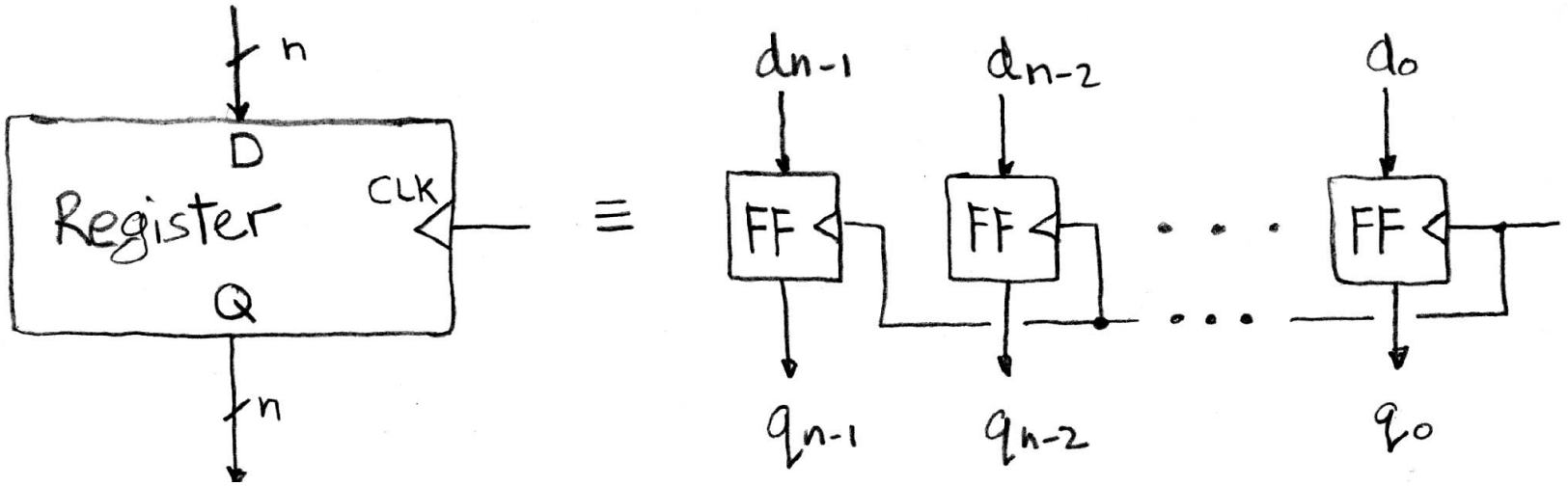


Model for Synchronous Systems



- Collection of Combinational Logic blocks separated by registers
- Feedback is optional
- Clock signal(s) connects only to clock input of registers
- Clock (CLK): steady square wave that synchronizes the system
- Register: several bits of state that samples on rising edge of CLK (positive edge-triggered) or falling edge (negative edge-triggered)

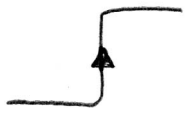
Register Internals



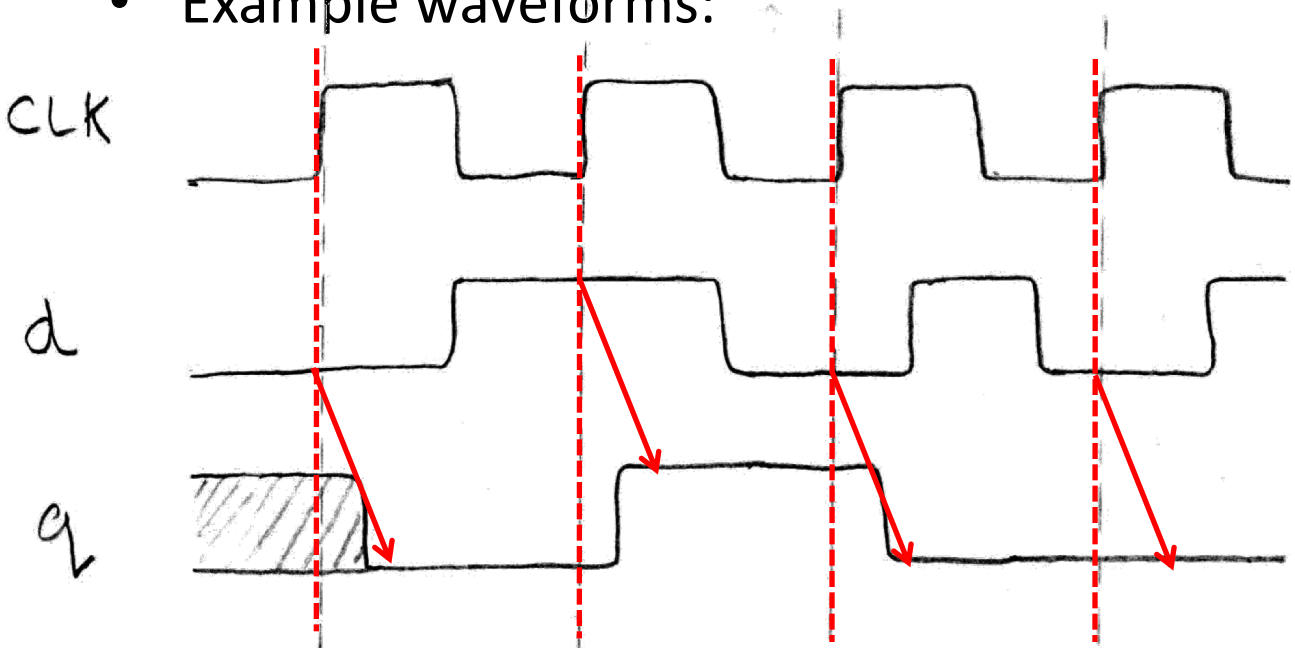
- n instances of a "Flip-Flop"
- Flip-flop name because the output flips and flops between 0 and 1
- D is "data input", Q is "data output"
- Also called "D-type Flip-Flop"

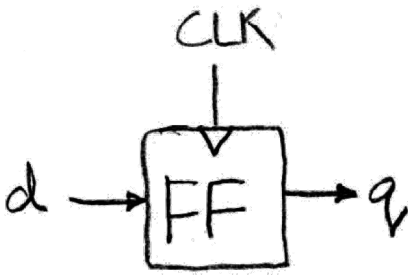
Flip-Flop Operation

- Edge-triggered d-type flip-flop
  - This one is "positive edge-triggered"



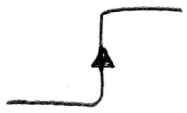
- "On the rising edge of the clock, the input d is sampled and transferred to the output. At all other times, the input d is ignored."
- Example waveforms:



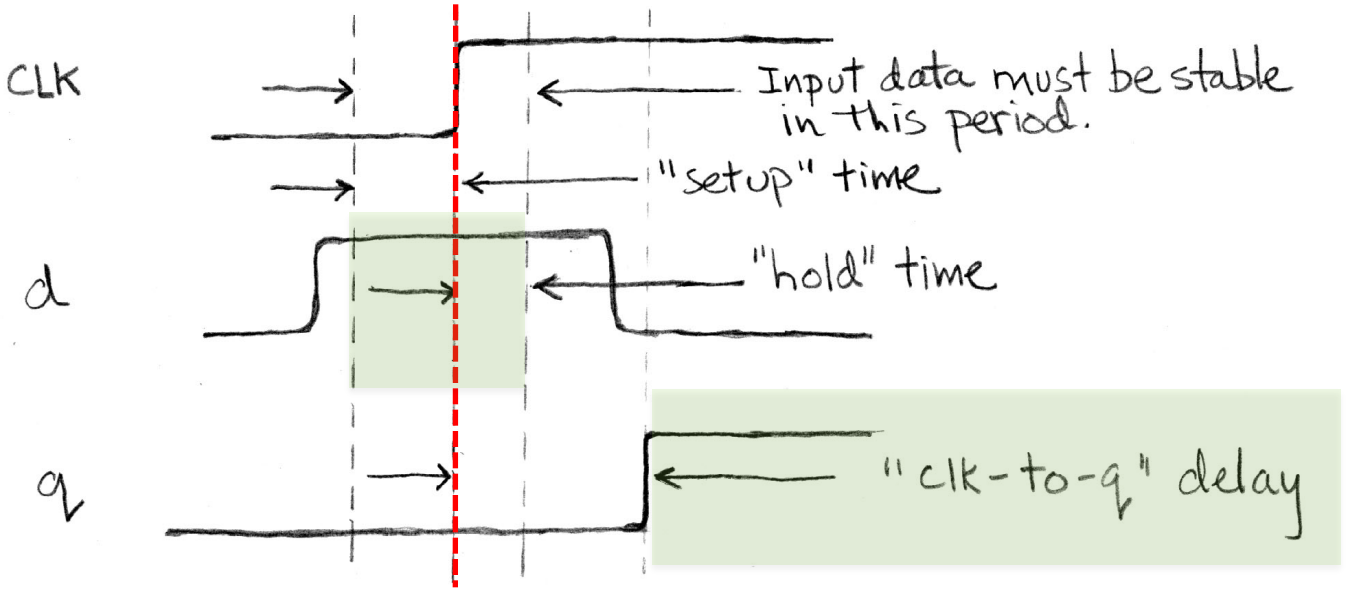


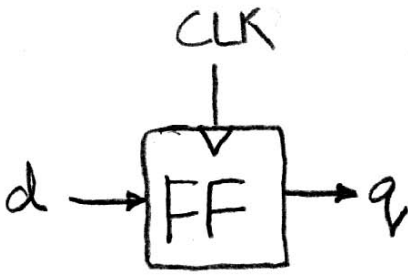
Flip-Flop Timing

- Edge-triggered d-type flip-flop
  - This one is “positive edge-triggered”



- “On the rising edge of the clock, the input d is sampled and transferred to the output. At all other times, the input d is ignored.”
- Example waveforms (more detail):

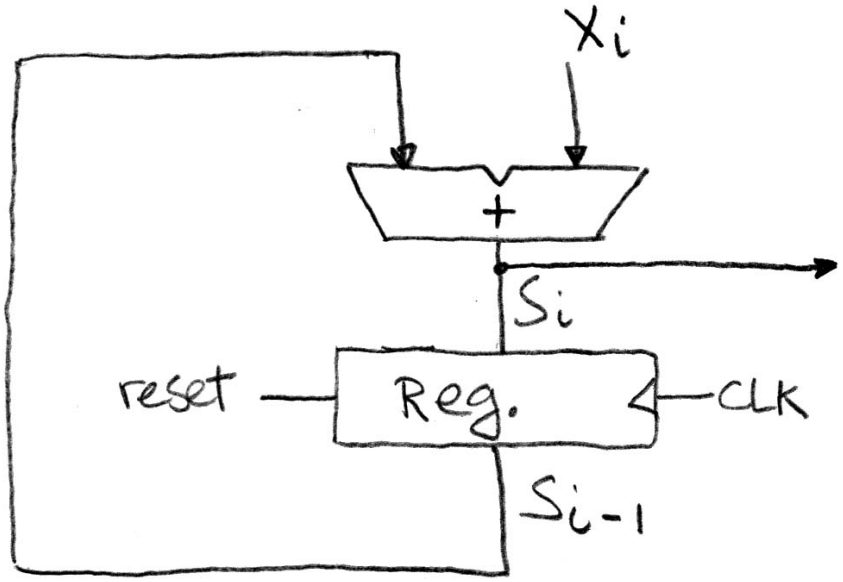




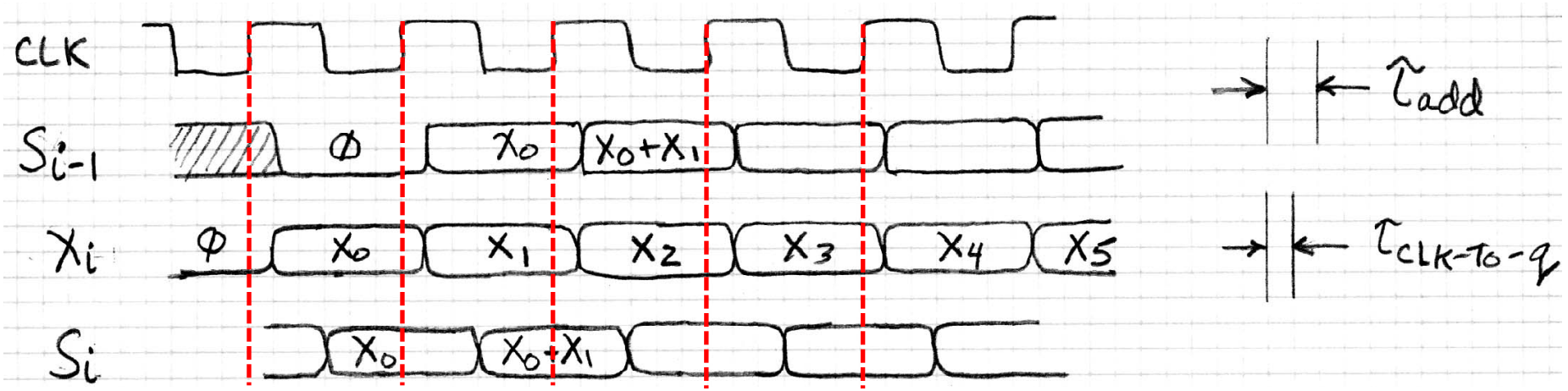
Hardware Timing Terms

- Setup Time: when the input must be stable *before* the edge of the CLK
- Hold Time: when the input must be stable *after* the edge of the CLK
- "CLK-to-Q" Delay: how long it takes the output to change, measured from the edge of the CLK

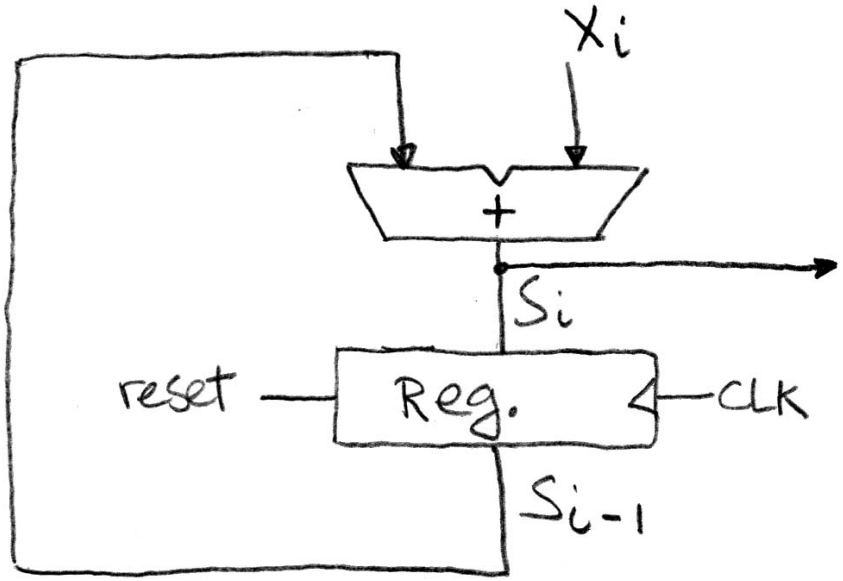
Accumulator Timing 1/2



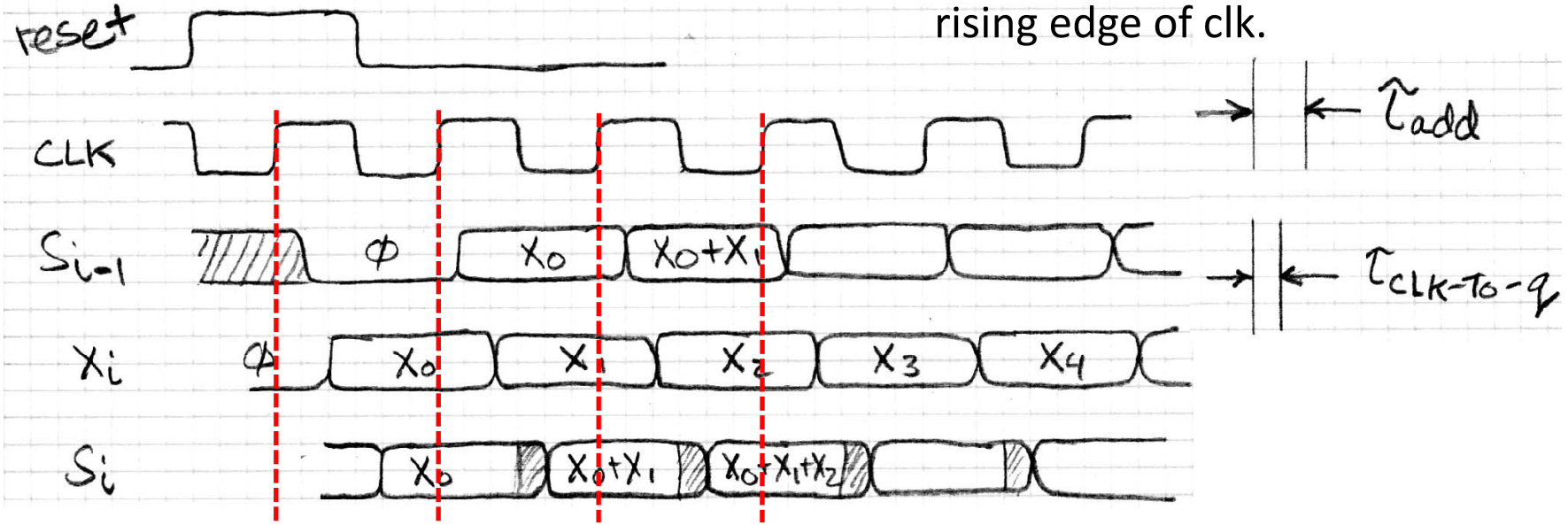
- Reset input to register is used to force it to all zeros (takes priority over D input).
- S_{i-1} holds the result of the i^{th}-1 iteration.
- Analyze circuit timing starting at the output of the register.



Accumulator Timing 2/2

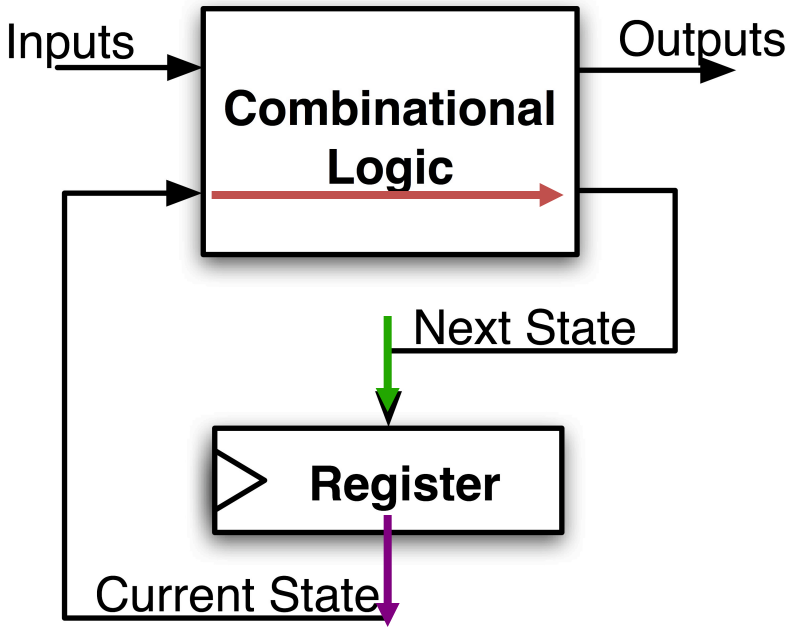


- reset signal shown.
- Also, in practice X might not arrive to the adder at the same time as S_{i-1}
- S_i temporarily is wrong, but register always captures correct value.
- In good circuits, instability never happens around rising edge of clk.



Maximum Clock Frequency

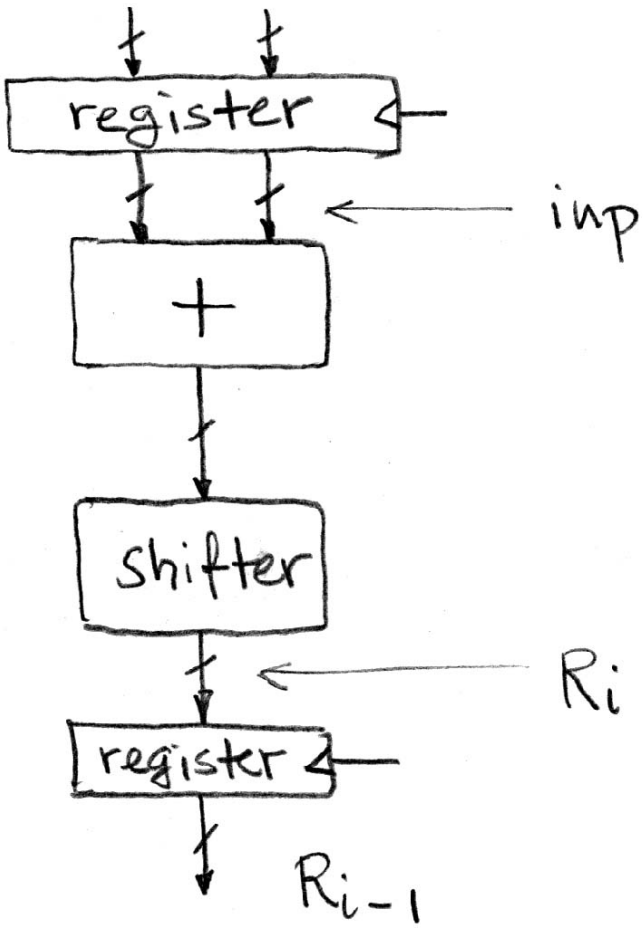
- What is the maximum frequency of this circuit?



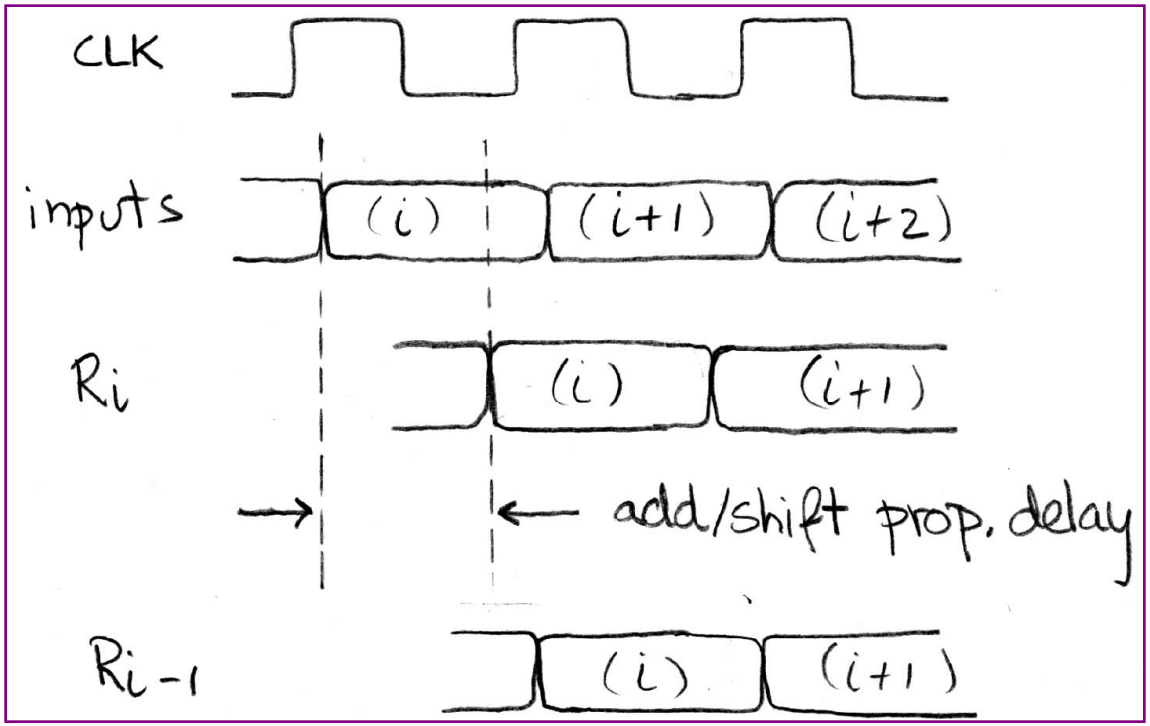
Hint:
Frequency = 1/Period

Max Delay = CLK-to-Q Delay + CL Delay + Setup Time

Critical Paths



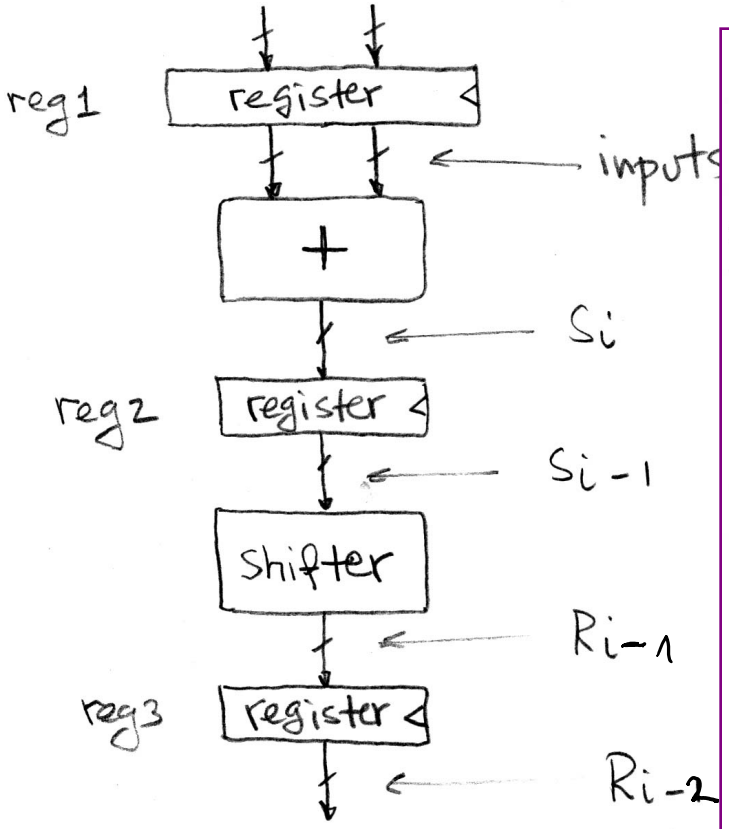
Timing…

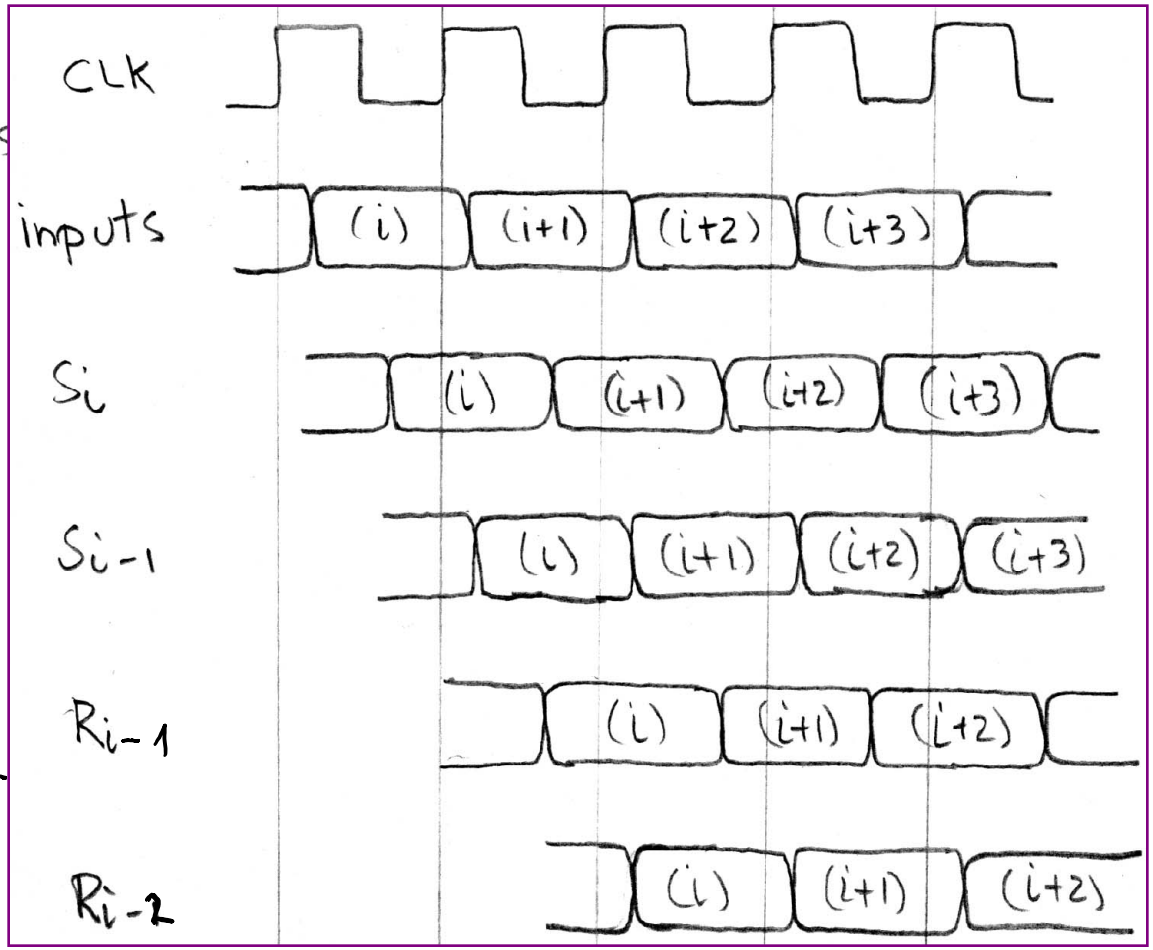


Note: delay of 1 clock cycle from input to output.
Clock period limited by propagation delay of adder/shifter.

Pipelining to improve performance

Timing…





- Insertion of register allows higher clock frequency.
- More outputs per second (higher bandwidth)
- But each individual result takes longer (greater latency)

Recap of Timing Terms

- Clock (CLK) - steady square wave that synchronizes system
- Setup Time - when the input must be stable before the rising edge of the CLK
- Hold Time - when the input must be stable after the rising edge of the CLK
- "CLK-to-Q" Delay - how long it takes the output to change, measured from the rising edge of the CLK

- Flip-flop - one bit of state that samples every rising edge of the CLK (positive edge-triggered)
- Register - several bits of state that samples on rising edge of CLK or on LOAD (positive edge-triggered)

Problems with Clocking

- The clock period ***must be*** longer than the critical path
  - Otherwise, you will get the wrong answers
  - But it can be even longer than that
- Critical path:
  - clk->q time
  - Necessary to get the output of the registers
  - ***worst case*** combinational logic delay
  - ***Setup time*** for the next register
- Must meet all of these to be correct

Hold-Time Violations...

- An alternate problem can occur...
  - Clk->Q + ***best case*** combinational delay < Hold time...
- What happens?
  - Clk->Q + data propagates...
  - And now you don't hold the input to the flip flop long enough
- Solution:
  - ***Add*** delay on the best-case path (e.g. two inverters)

TA Discussion

Zhongyi Cai

Q & A

Quiz

Quiz

- No Quiz Today ☺
- Will not happen again ☹

CS 110
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Lecture 9:
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Video 2: FSM

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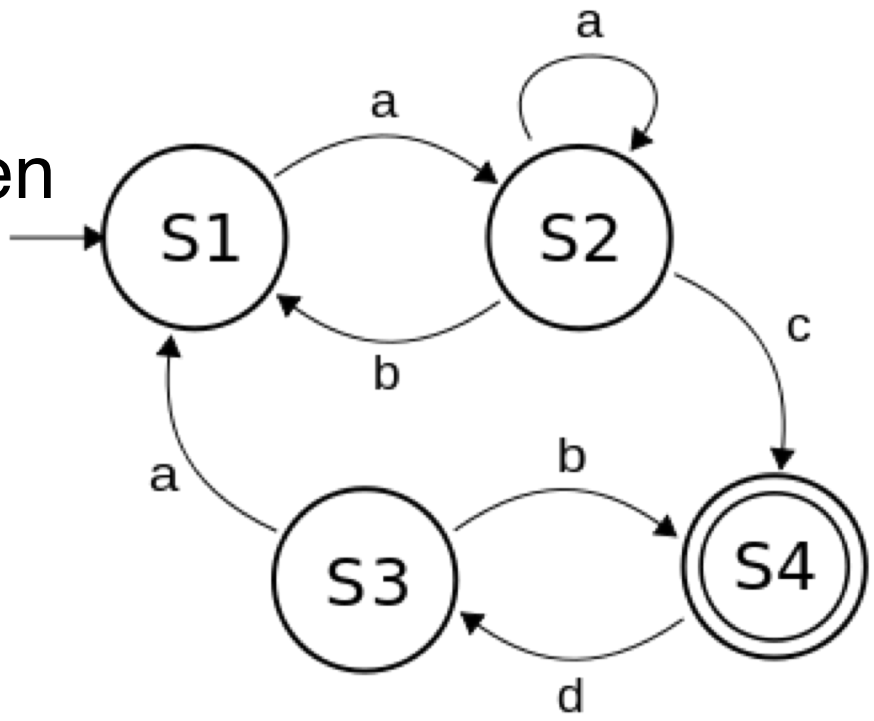
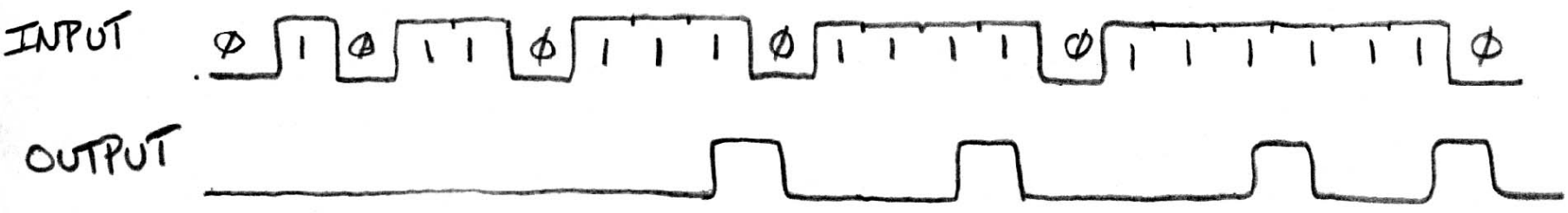
Finite State Machines (FSM) Intro

- A convenient way to conceptualize computation over time
- We start at a state and given an input, we follow some edge to another (or the same) state
- The function can be represented with a “state transition diagram”.
- With combinational logic and registers, any FSM can be implemented in hardware.

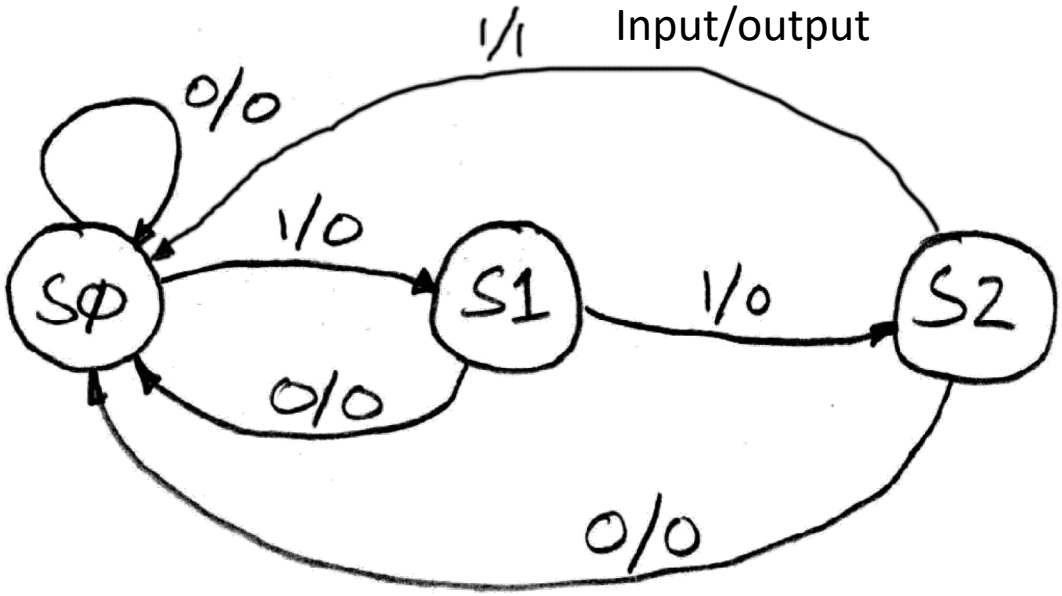


FSM Example: 3 ones…

FSM to detect the occurrence of 3 consecutive 1's in the input.



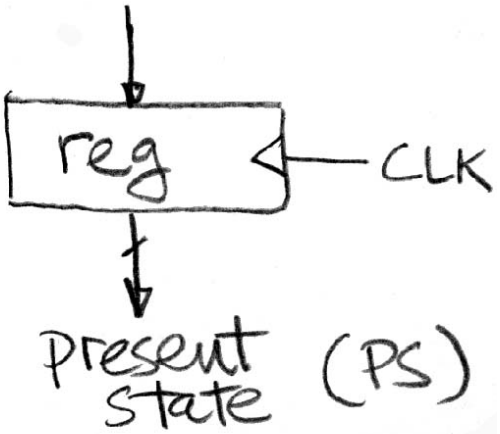
Draw the FSM…



Assume state transitions are controlled by the clock: on each clock cycle the machine checks the inputs and moves to a new state and produces a new output…

Hardware Implementation of FSM

… Therefore a register is needed to hold the a representation of which state the machine is in. Use a unique bit pattern for each state.



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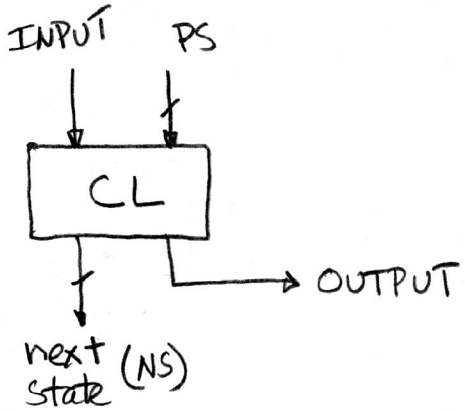

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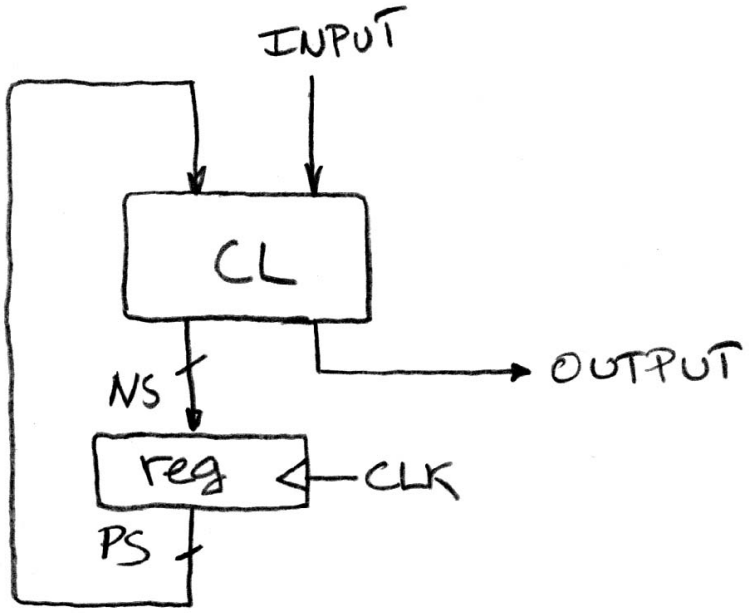
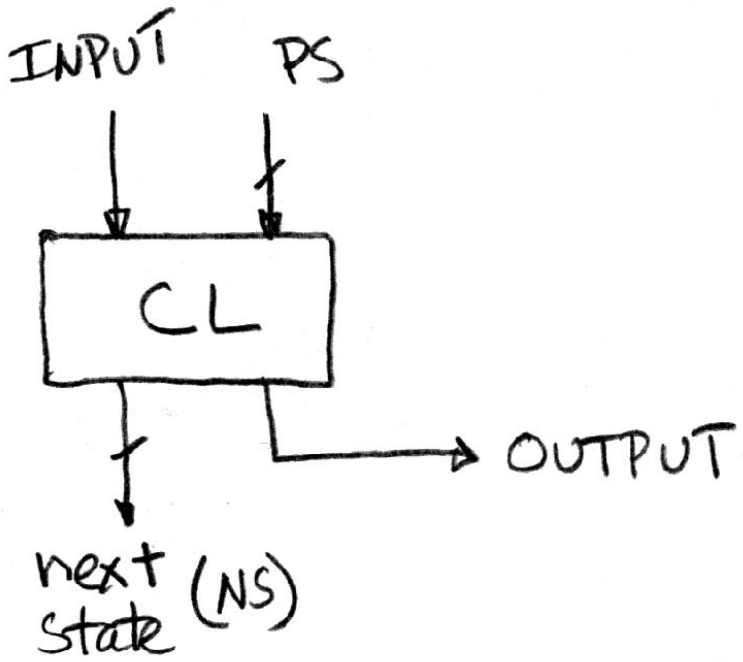


Combinational logic circuit is used to implement a function that maps from *present state and input* to *next state and output.*

FSM Combinational Logic

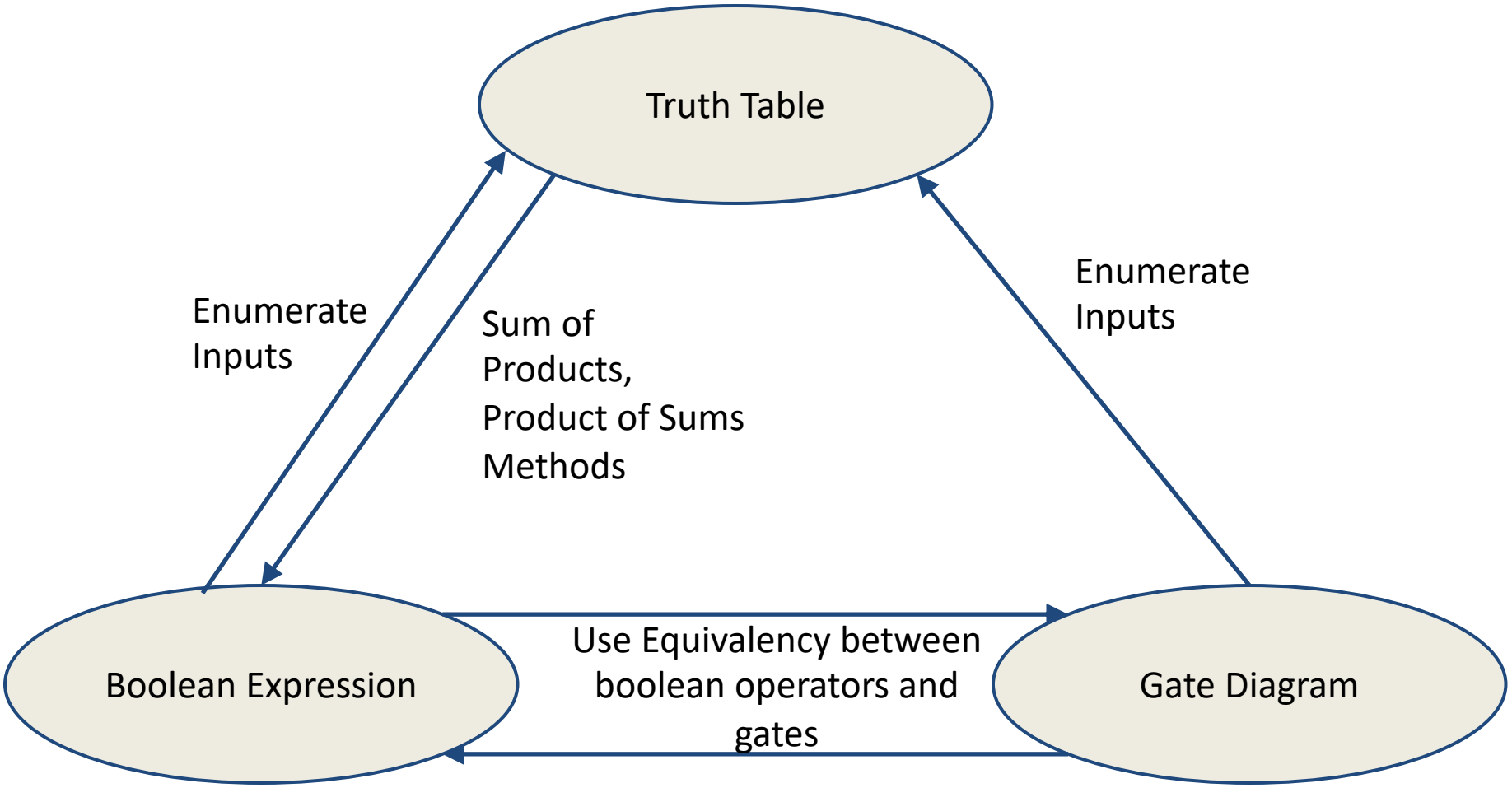
Specify CL using a truth table



Truth table…

PS	Input	NS	Output
00	0	00	0
00	1	01	0
01	0	00	0
01	1	10	0
10	0	00	0
10	1	00	1

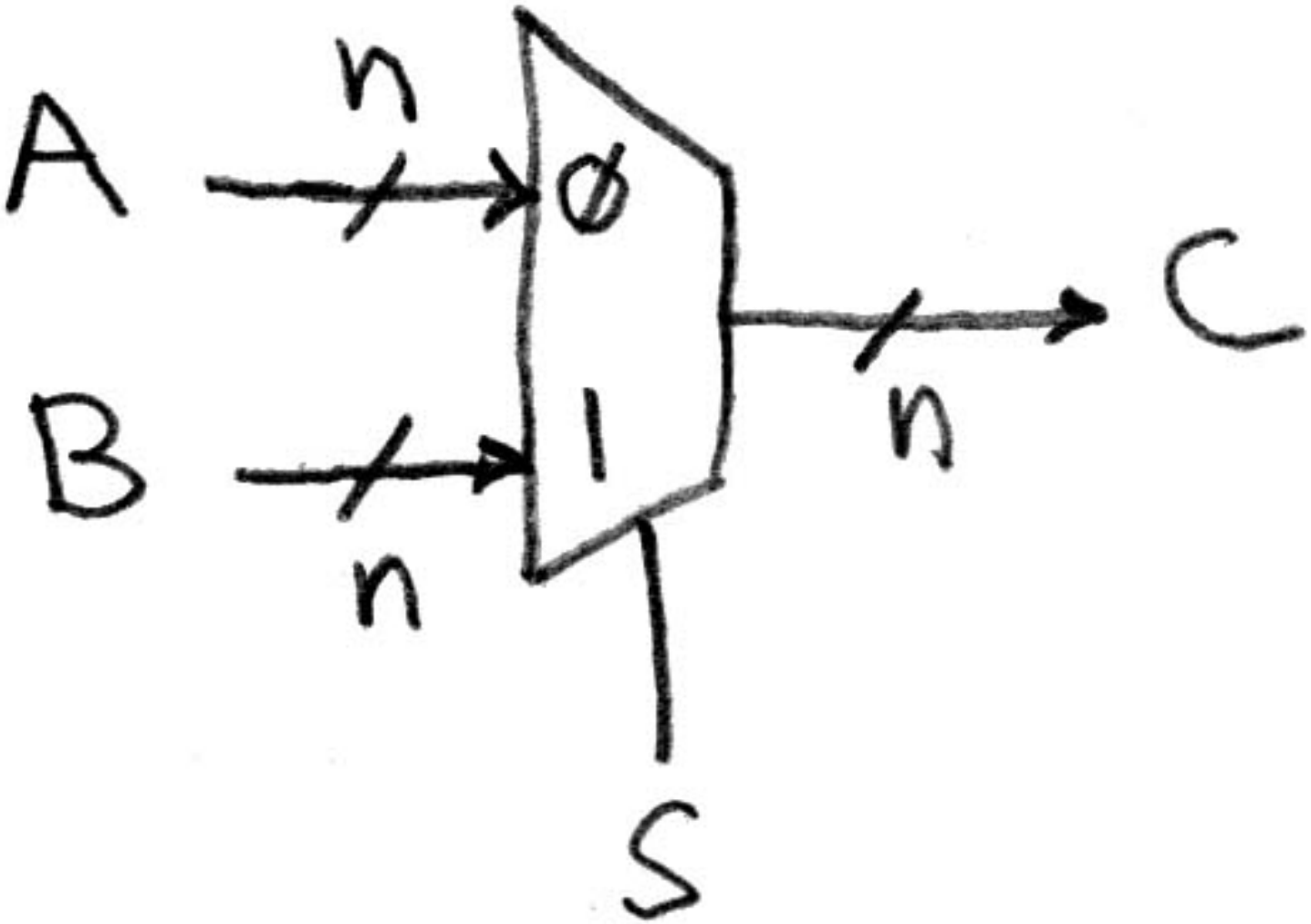
Representations of Combinational Logic (groups of logic gates)



Building Standard Functional Units

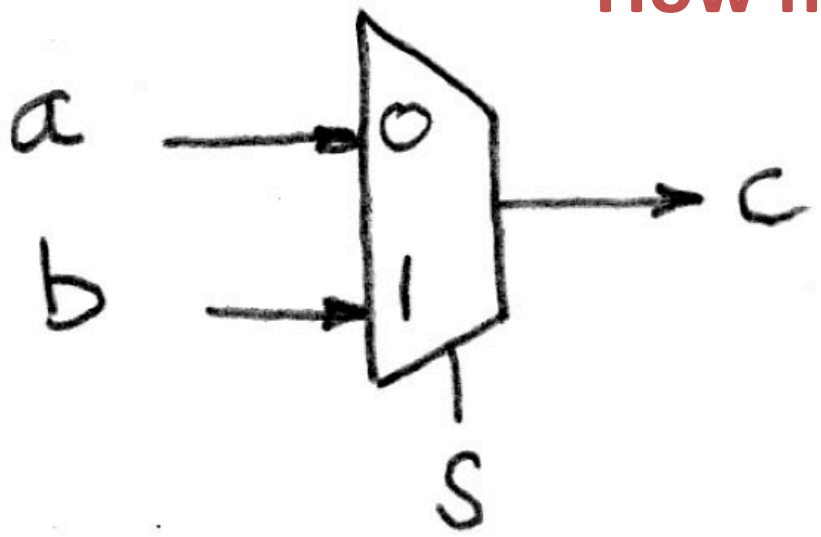
- Data multiplexers
- Arithmetic and Logic Unit
- Adder/ Subtractor

Data Multiplexer ("Mux")
(here 2-to-1, n-bit-wide)



N instances of 1-bit-wide mux

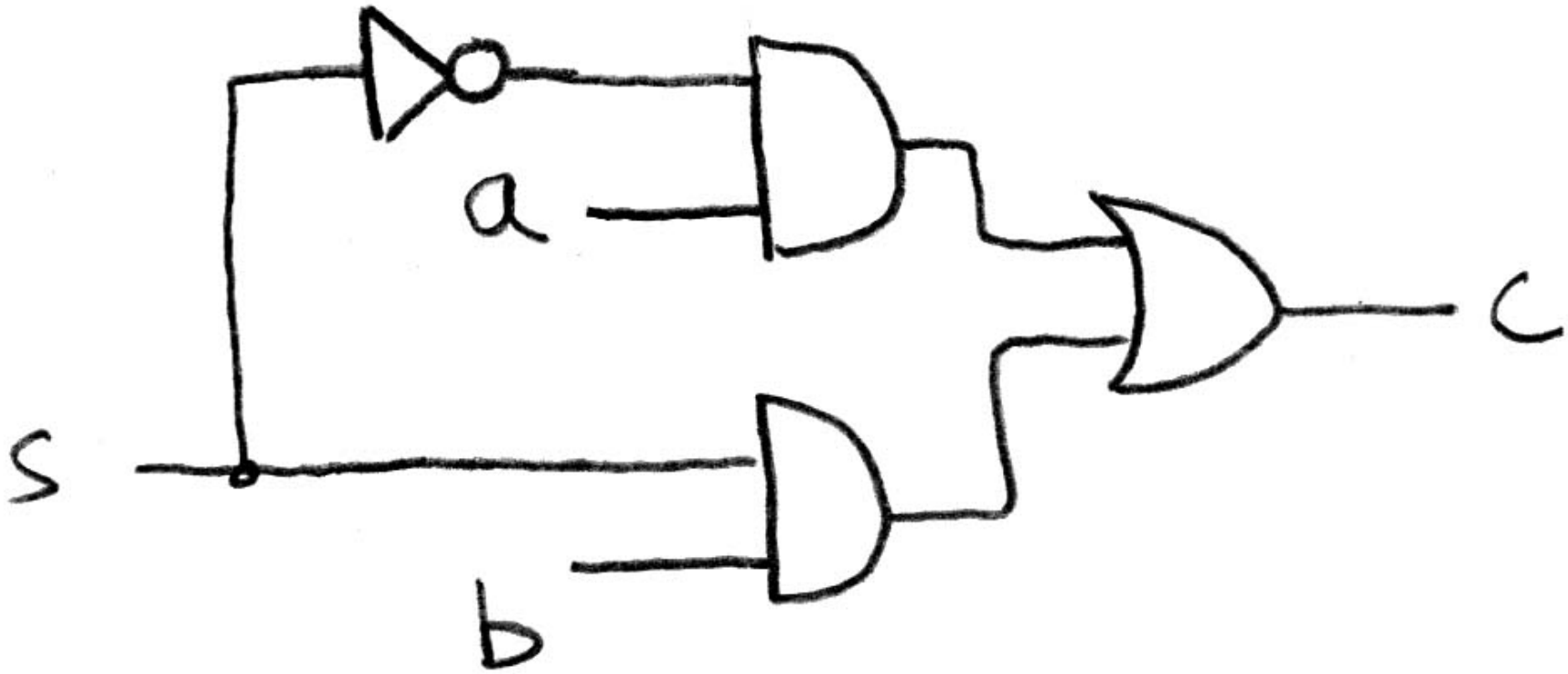
How many rows in TT?



$$
\begin{aligned}
c &= \overline{s}a\overline{b} + \overline{s}ab + s\overline{a}b + sab \\
&= \overline{s}(a\overline{b} + ab) + s(\overline{a}b + ab) \\
&= \overline{s}(a(\overline{b} + b)) + s((\overline{a} + a)b) \\
&= \overline{s}(a(1) + s((1)b) \\
&= \overline{s}a + sb
\end{aligned}
$$

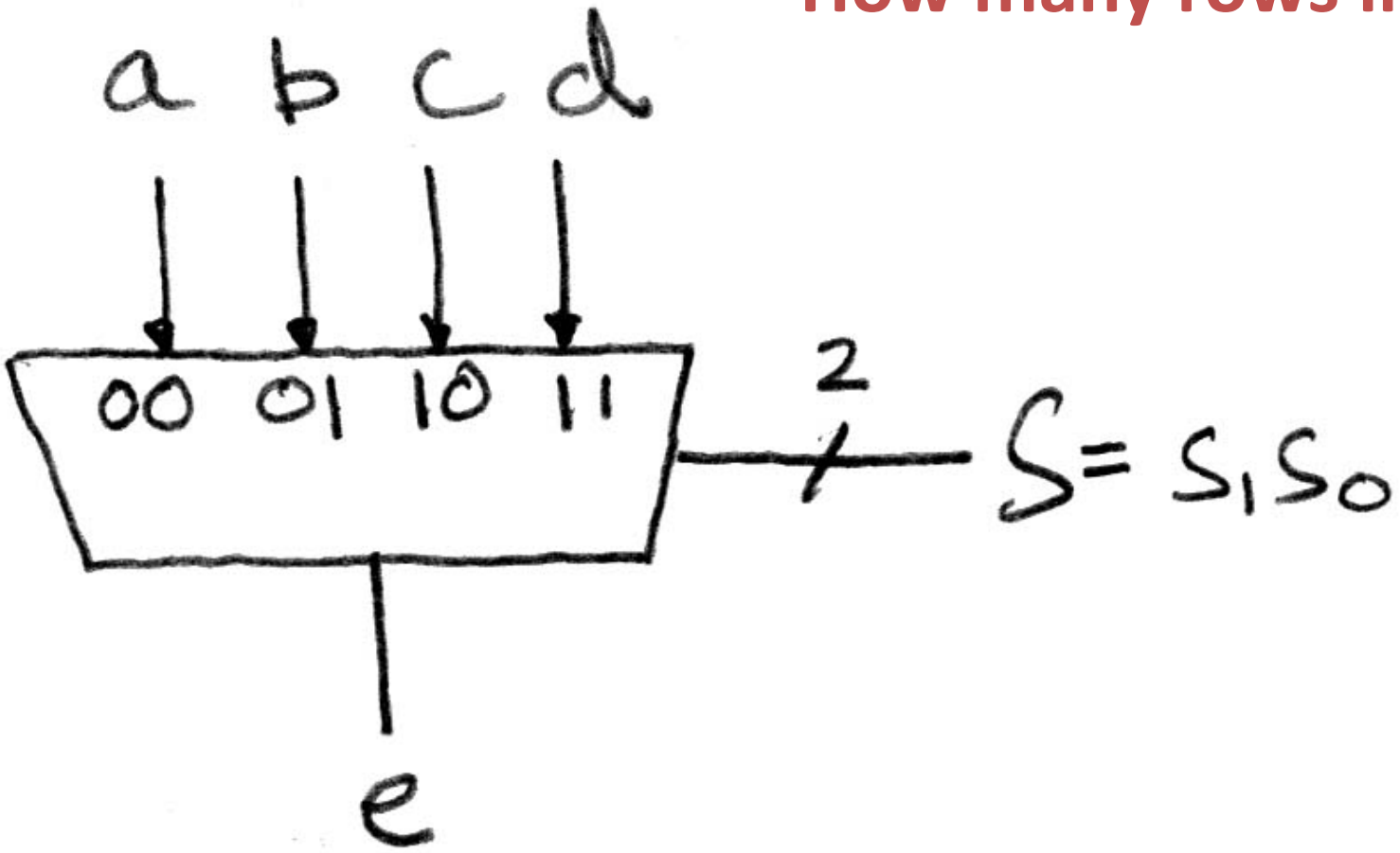
How do we build a 1-bit-wide mux?

$$\overline{s}a + sb$$



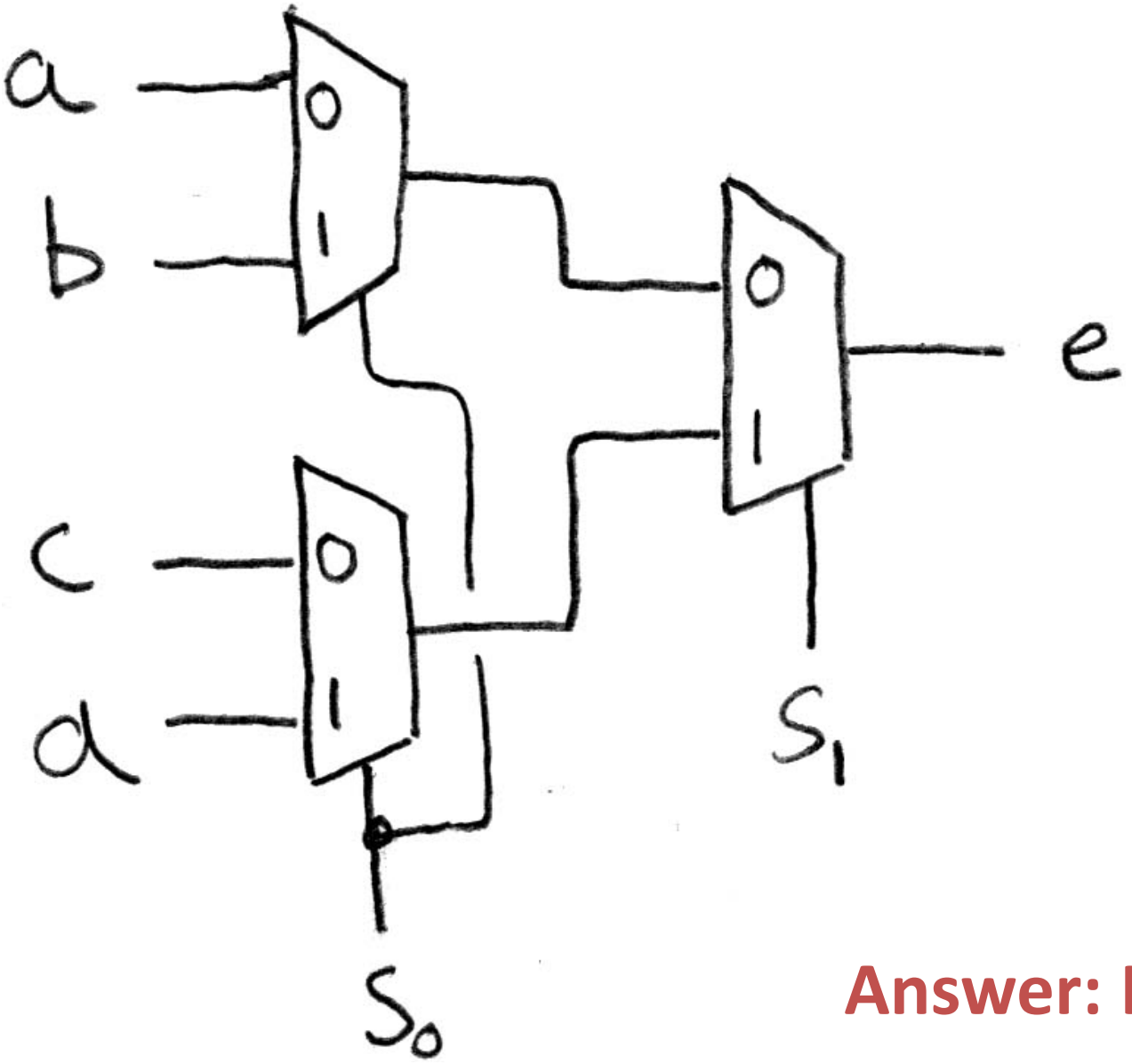
4-to-1 multiplexer?

How many rows in TT?



$$e = \overline{s_1}\,\overline{s_0}a + \overline{s_1}s_0b + s_1\overline{s_0}c + s_1s_0d$$

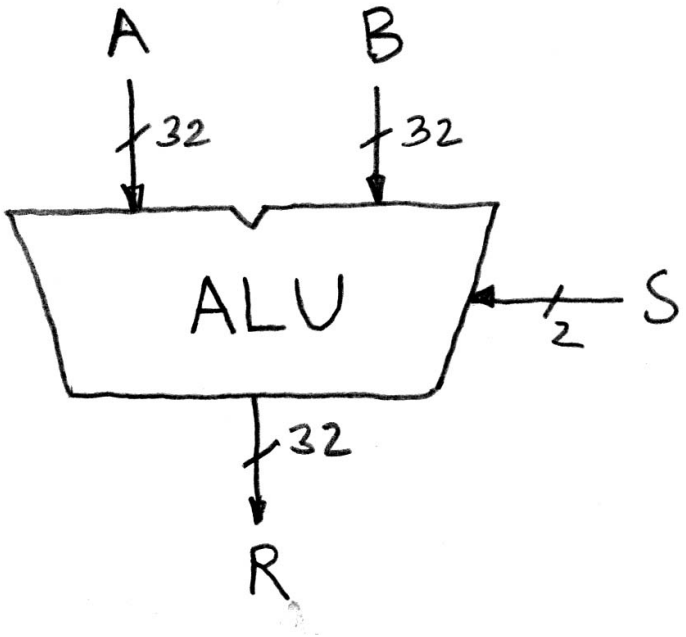
Another way to build 4-1 mux?



Answer: Hierarchically!

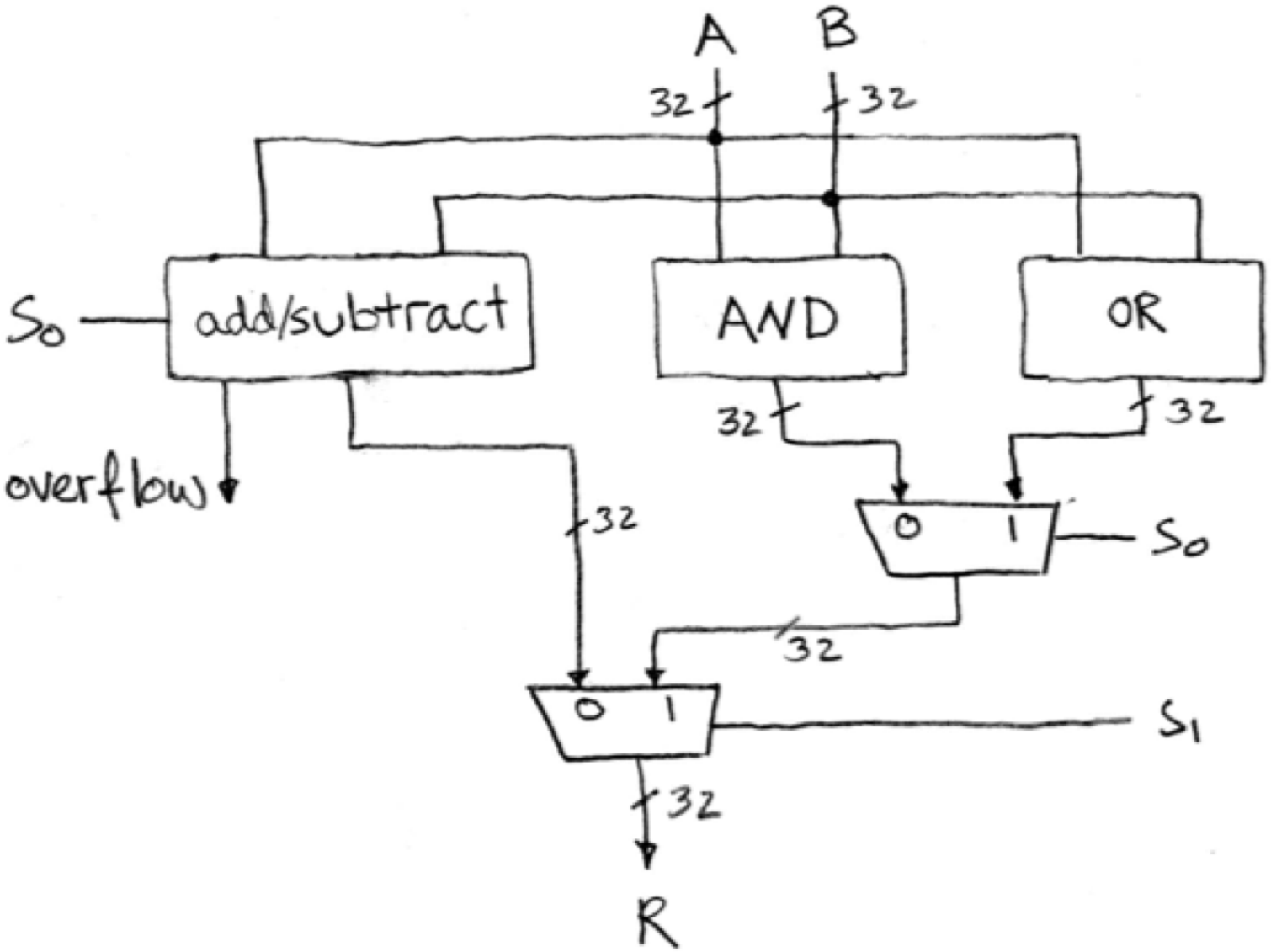
Arithmetic and Logic Unit

- Most processors contain a special logic block called the "Arithmetic and Logic Unit" (ALU)
- We'll show you an easy one that does ADD, SUB, bitwise AND, bitwise OR



when S=00, R=A+B
when S=01, R=A-B
when S=10, R=A AND B
when S=11, R=A OR B

Our simple ALU



Question

Convert the truth table to a boolean expression (no need to simplify):

A: F = xy + x(~y)

B: F = xy + (~x)y + (~x)(~y)

C: F = (~x)y + x(~y)

D: F = xy + (~x)y

E: F = (x+y)(~x+~y)

x	y	F(x,y)
0	0	0
0	1	1
1	0	0
1	1	1

How to design Adder/Subtractor?

- Truth-table, then determine canonical form, then minimize and implement as we've seen before
- Look at breaking the problem down into smaller pieces that we can cascade or hierarchically layer

Adder/Subtractor – One-bit adder LSB…

	a_3	a_2	a_1	a_0
+	b_3	b_2	b_1	b_0
	s_3	s_2	s_1	s_0

a_0	b_0	s_0	c_1
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s_0 =$$

$$c_1 =$$

Adder/Subtractor – One-bit adder (1/2)…

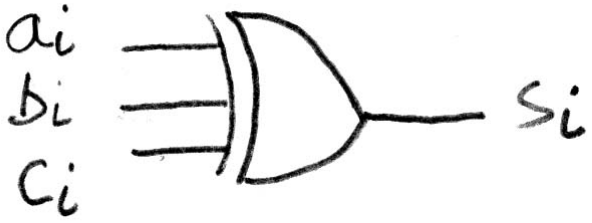
	c_3	c_2	c_1	
	a_3	a_2	a_1	a_0
+	b_3	b_2	b_1	b_0
	s_3	s_2	s_1	s_0

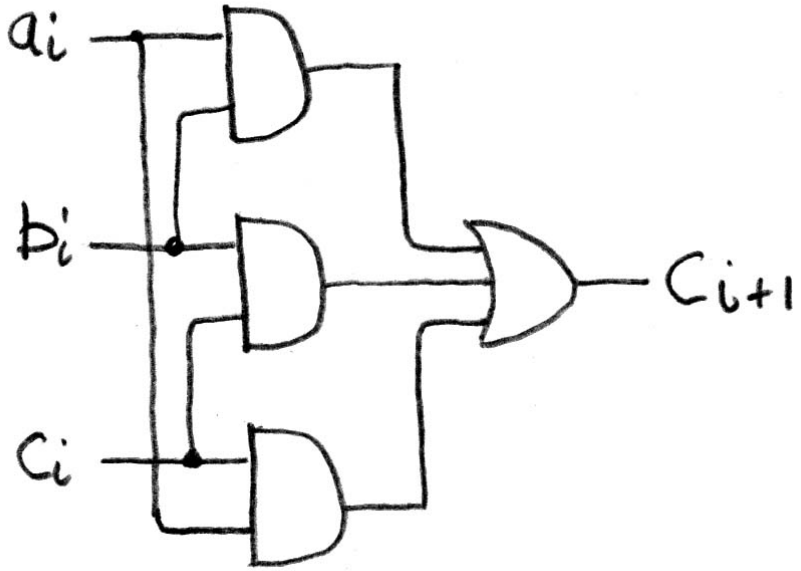
a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$s_i =$$

$$c_{i+1} =$$

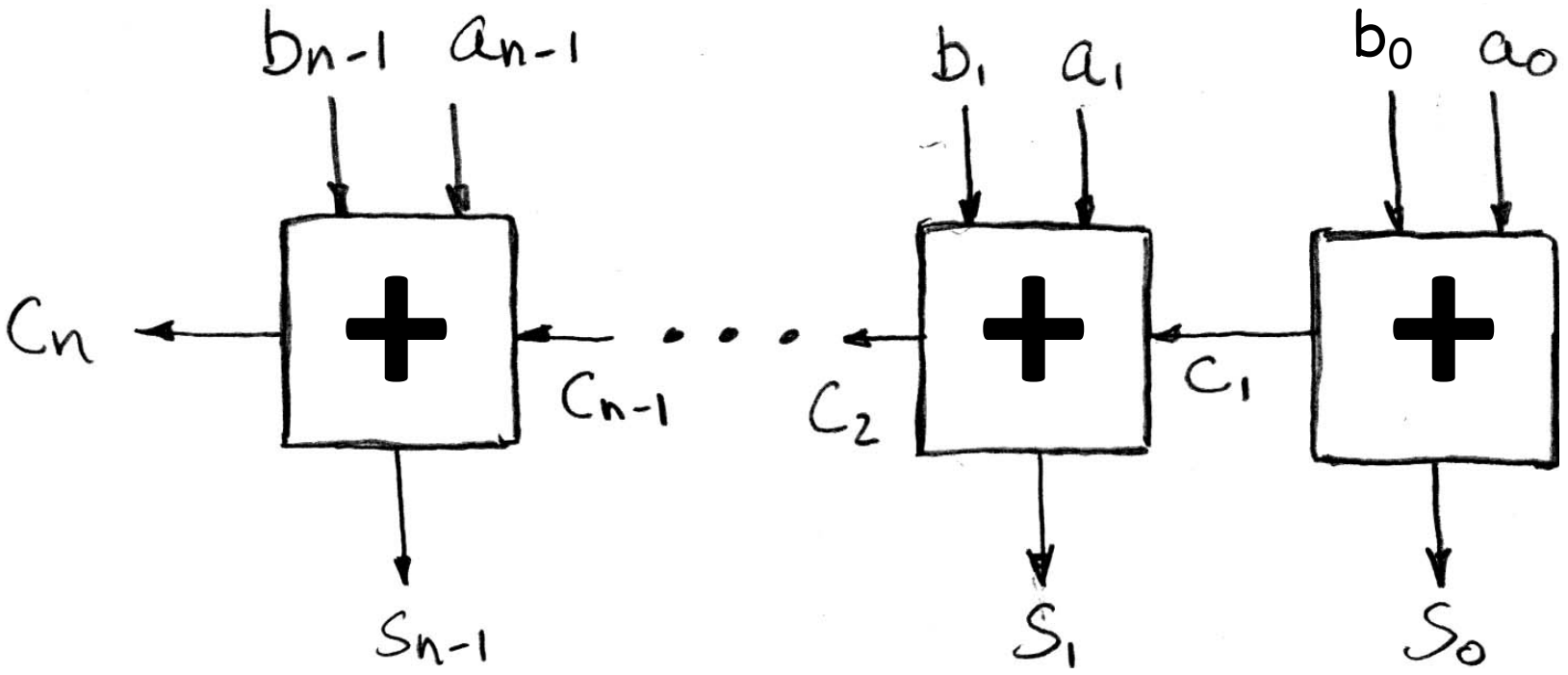
Adder/Subtractor – One-bit adder (2/2)





$$
\begin{aligned}
s_i &= \mathrm{XOR}(a_i, b_i, c_i) \\
c_{i+1} &= \mathrm{MAJ}(a_i, b_i, c_i) = a_i b_i + a_i c_i + b_i c_i
\end{aligned}
$$

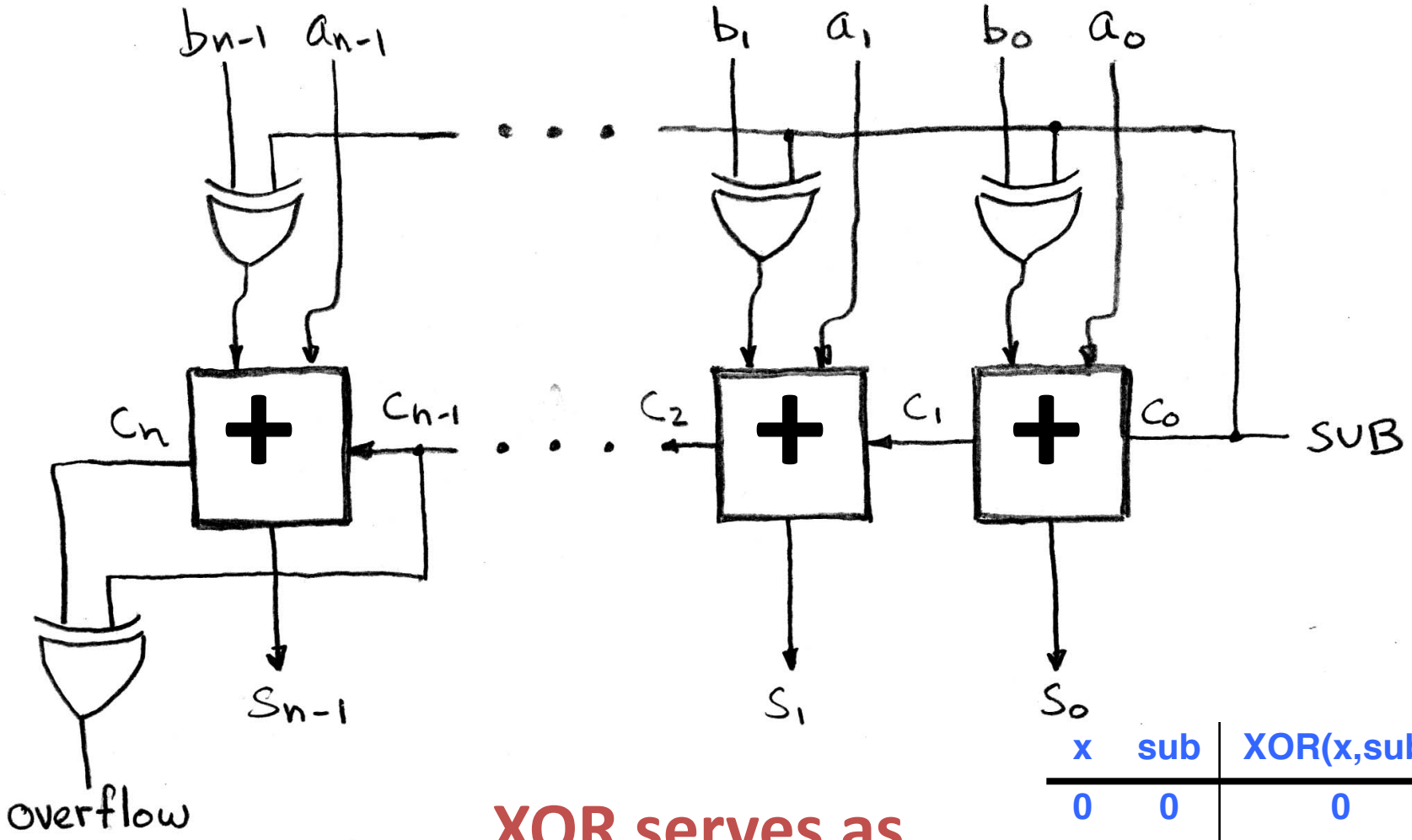
N 1-bit adders => 1 N-bit adder



What about overflow?

Overflow = c_n?

Extremely Clever Subtractor:
s = a + (-b)



x	sub	XOR(x,sub)
0	0	0
0	1	1
1	0	1
1	1	0

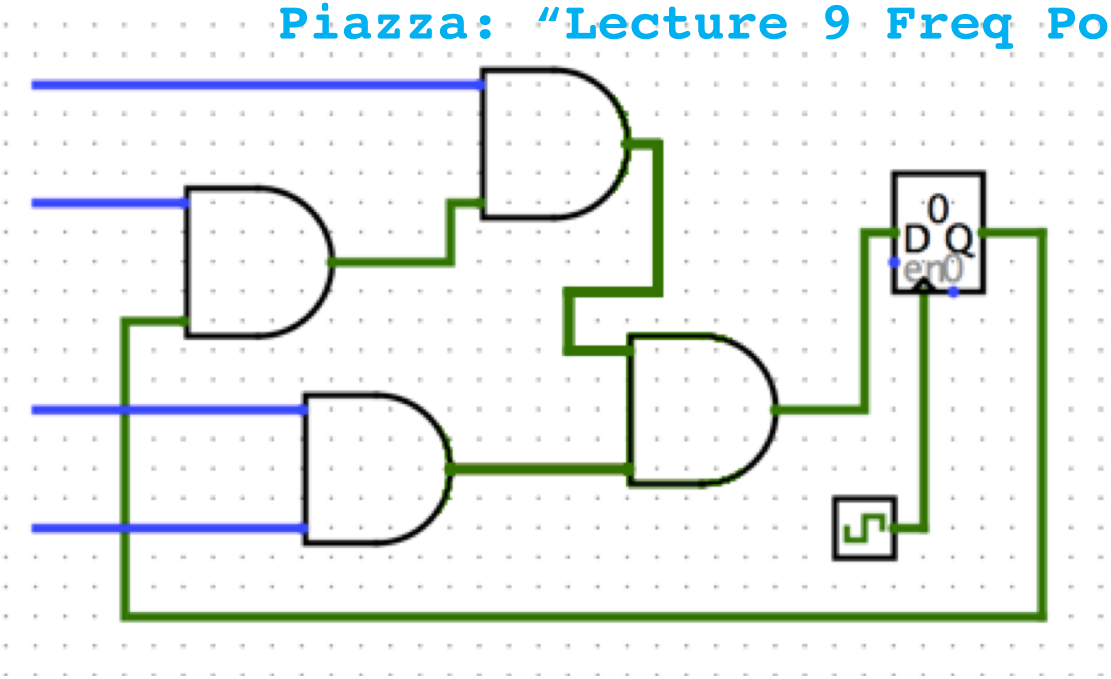
XOR serves as conditional inverter!

In Conclusion

- Finite State Machines have clocked state elements plus combinational logic to describe transition between states
  - Clocks synchronize D-FF change (Setup and Hold times important!)
- Standard combinational functional unit blocks built hierarchically from subcomponents

Question

Piazza: "Lecture 9 Freq Poll"



Clock->Q 1ns
Setup 1ns
Hold 1ns
AND delay 1ns

What is maximum clock frequency?

- A: 5 GHz
- B: 500 MHz
- C: 200 MHz
- D: 250 MHz
- E: 1/6 GHz