CS 110 Computer Architecture Lecture 10:

Finite State Machines, Functional Units

Instructors:

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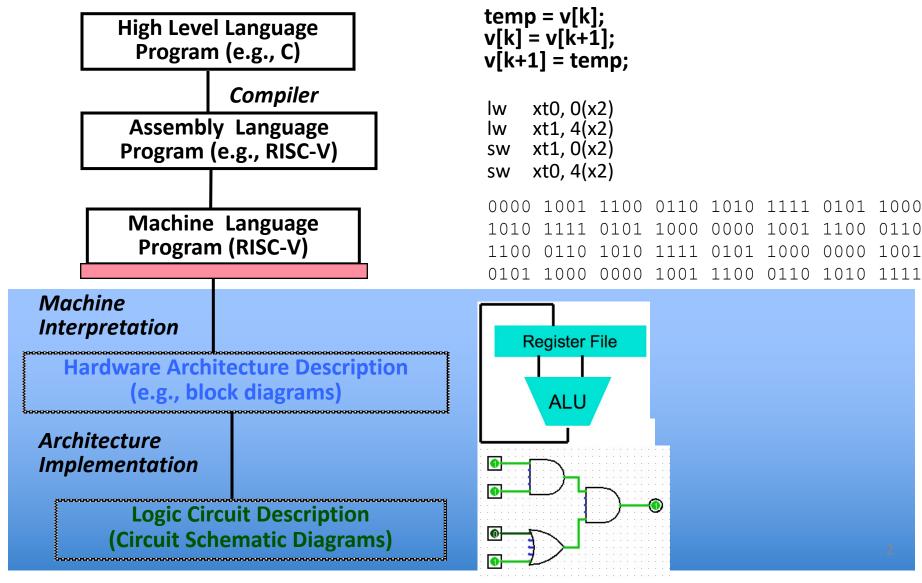
https://robotics.shanghaitech.edu.cn/courses/ca/20s/

School of Information Science and Technology SIST

ShanghaiTech University

Slides based on UC Berkley's CS61C

Levels of Representation/Interpretation



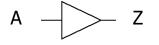
Outline

- Timing and Critical Path
- Finite State Machine
- Admin
- Functional Units
 - Multiplexer
 - ALU
 - Adder/ Subtractor

Combinational Logic Symbols

 Common combinational logic systems have standard symbols called logic gates



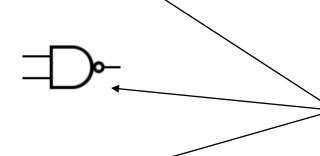


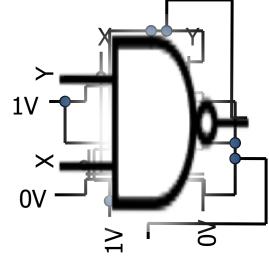
AND, NAND

$$\frac{A}{B}$$



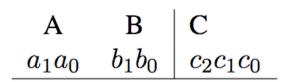
$$A \longrightarrow Z$$

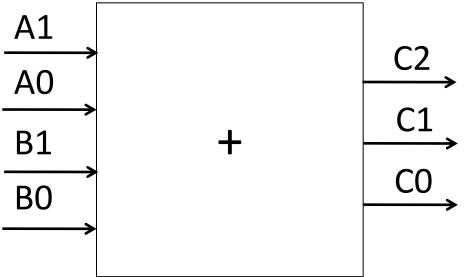




Inverting versions (NOT, NAND, NOR) easiest to implement with CMOS transistors (the switches we have available and use most)

Truth Table Example #2: 2-bit Adder



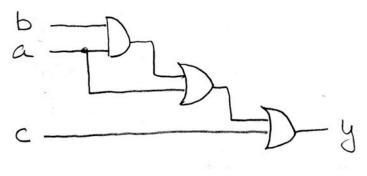


How Many Rows?

Truth Table Example #3: 32-bit Unsigned Adder

A	В	C	_
000 0	000 0	000 00	-
000 0	000 1	000 01	
•	•	•	How
•	•	•	Many Rows?
•	•	•	
111 1	111 1	111 10	

Boolean Algebra: Circuit & Algebraic Simplification



$$y = ((ab) + a) + c$$

$$\downarrow = ab + a + c$$

$$= a(b+1) + c$$

$$= a(1) + c$$

$$= a + c$$

$$\downarrow$$

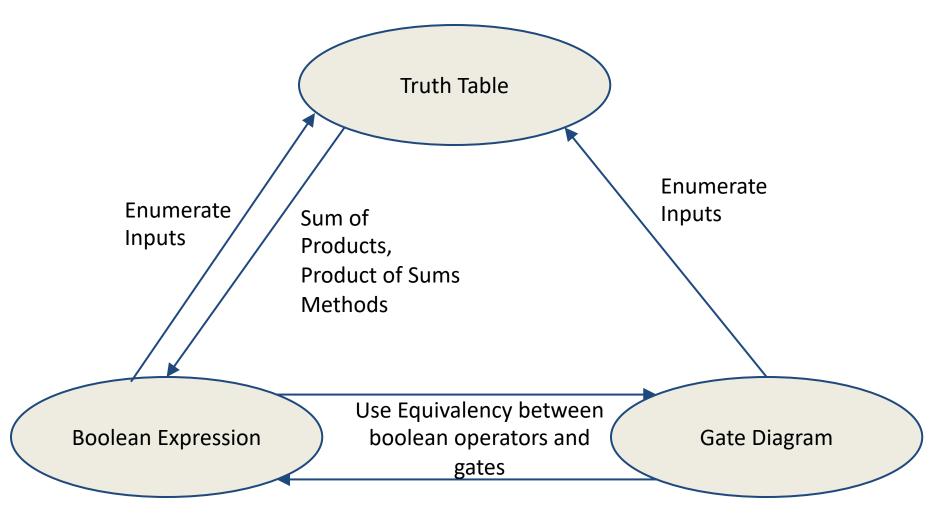
original circuit

equation derived from original circuit

algebraic simplification

simplified circuit

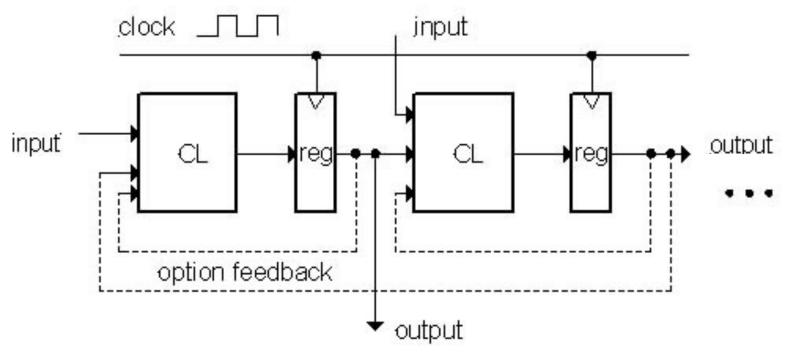
Representations of Combinational Logic (groups of logic gates)



Type of Circuits

- Synchronous Digital Systems consist of two basic types of circuits:
 - Combinational Logic (CL) circuits
 - Output is a function of the inputs only, not the history of its execution
 - E.g., circuits to add A, B (ALUs)
 - Sequential Logic (SL)
 - Circuits that "remember" or store information
 - aka "State Elements"
 - E.g., memories and registers (Registers)

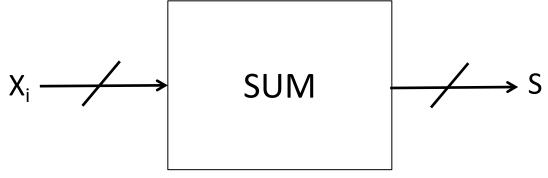
Model for Synchronous Systems



- Collection of Combinational Logic blocks separated by registers
- Feedback is optional
- Clock signal(s) connects only to clock input of registers
- Clock (CLK): steady square wave that synchronizes the system
- Register: several bits of state that samples on rising edge of CLK (positive edge-triggered) or falling edge (negative edge-triggered)

Accumulator Example

Why do we need to control the flow of information?



Want: S=0;

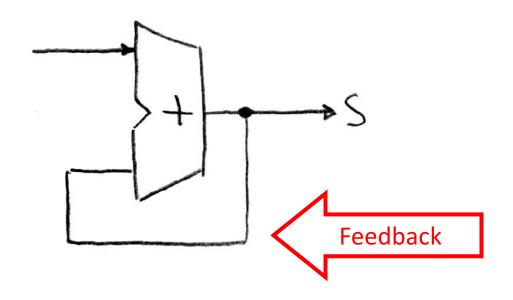
for
$$(i=0; i< n; i++)$$

 $S = S + X_i$

Assume:

- Each X value is applied in succession, one per cycle
- After n cycles the sum is present on S

First Try: Does this work?

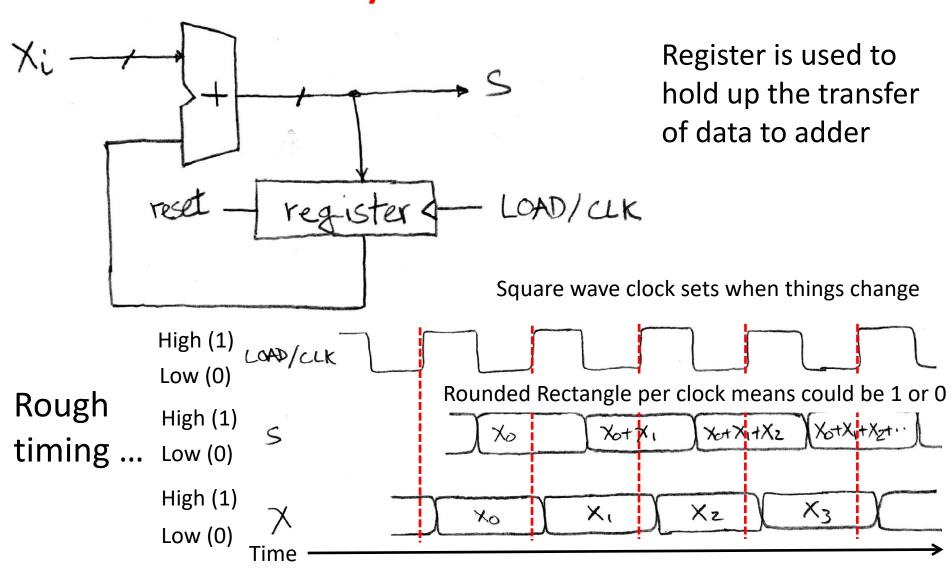


No!

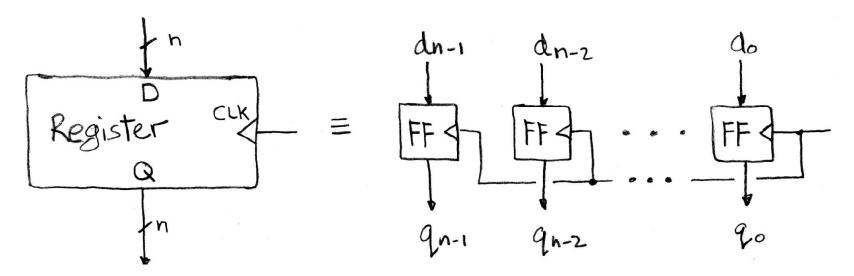
Reason #1: How to control the next iteration of the 'for' loop?

Reason #2: How do we say: 'S=0'?

Second Try: How About This?



Register Internals



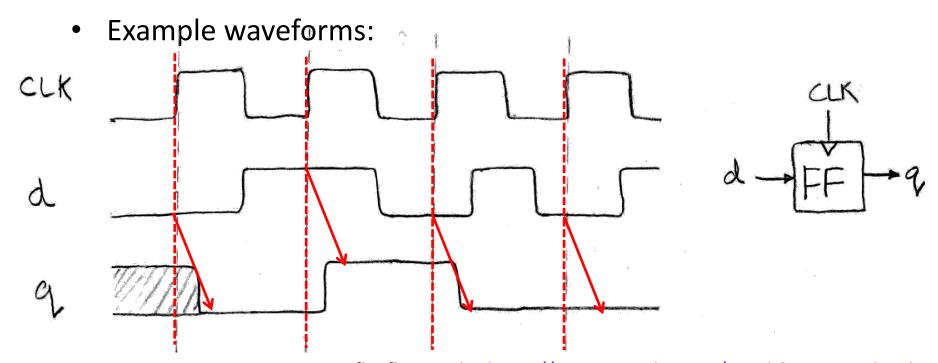
- n instances of a "Flip-Flop"
- Flip-flop name because the output flips and flops between 0 and 1
- D is "data input", Q is "data output"
- Also called "D-type Flip-Flop"

Flip-Flop Operation

- Edge-triggered d-type flip-flop
 - This one is "positive edge-triggered"



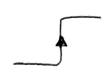
 "On the rising edge of the clock, the input d is sampled and transferred to the output. At all other times, the input d is ignored."



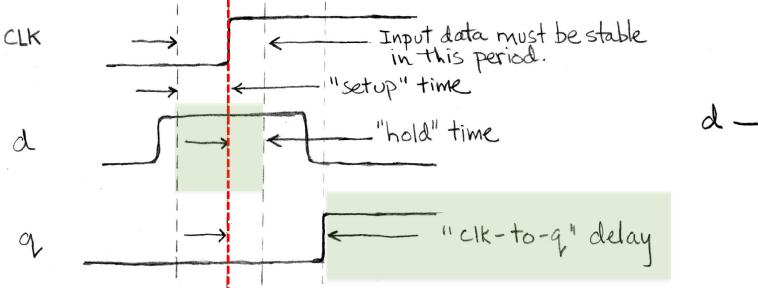
How a flip flop works: https://www.youtube.com/watch?v=-aQH0ybMd3U

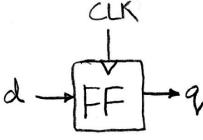
Flip-Flop Timing

- Edge-triggered d-type flip-flop
 - This one is "positive edge-triggered"



- "On the rising edge of the clock, the input d is sampled and transferred to the output. At all other times, the input d is ignored."
- Example waveforms (more detail):

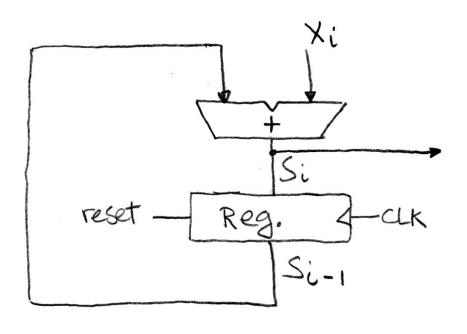




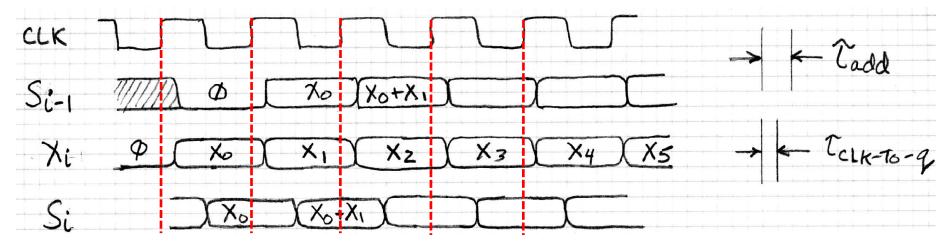
Hardware Timing Terms

- Setup Time: when the input must be stable before the edge of the CLK
- Hold Time: when the input must be stable after the edge of the CLK
- "CLK-to-Q" Delay: how long it takes the output to change, measured from the edge of the CLK

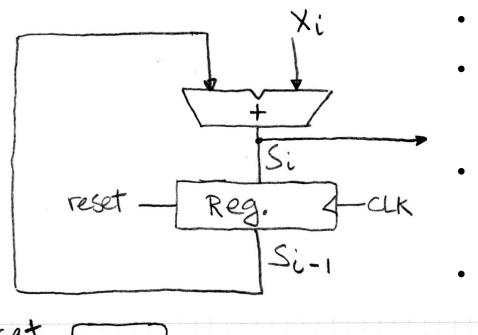
Accumulator Timing 1/2



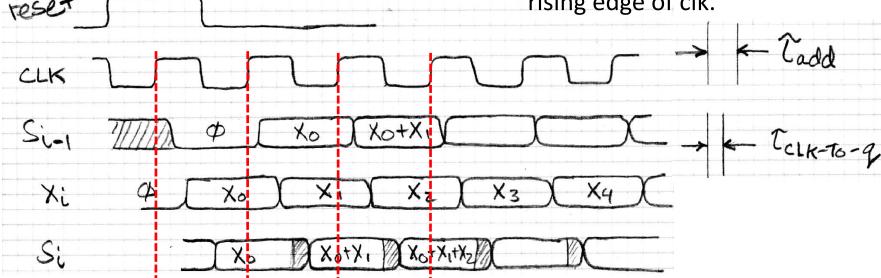
- Reset input to register is used to force it to all zeros (takes priority over D input).
- S_{i-1} holds the result of the ith-1 iteration.
- Analyze circuit timing starting at the output of the register.



Accumulator Timing 2/2

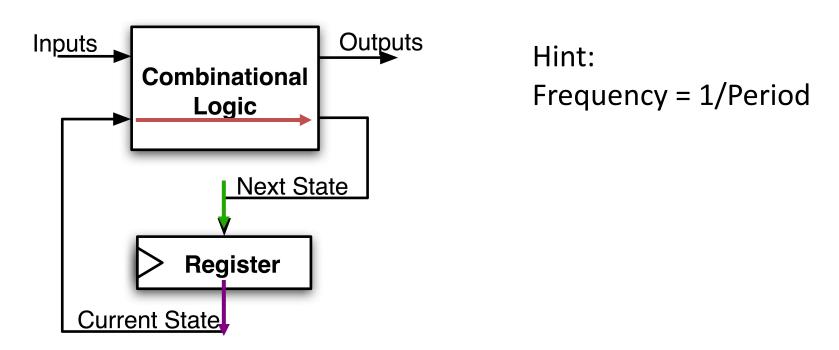


- reset signal shown.
- Also, in practice X might not arrive to the adder at the same time as S_{i-1}
- S_i temporarily is wrong, but register always captures correct value.
- In good circuits, instability never happens around rising edge of clk.



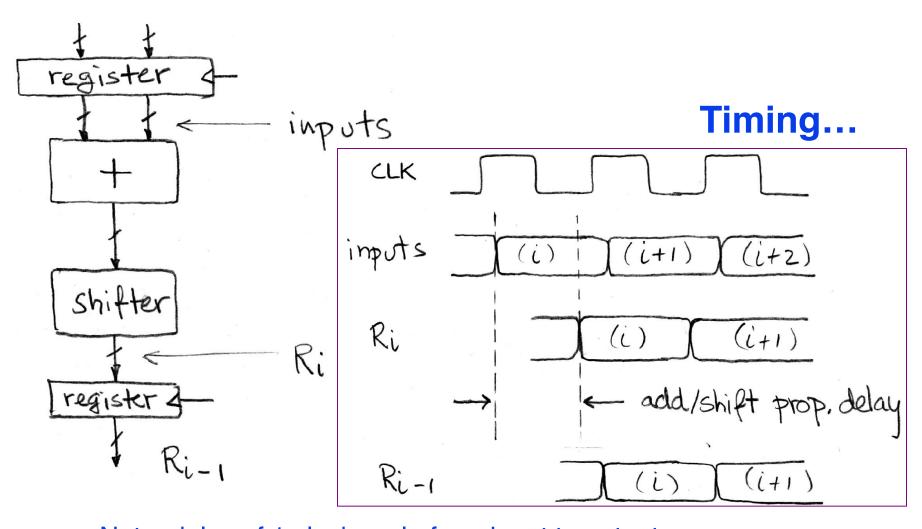
Maximum Clock Frequency

What is the maximum frequency of this circuit?

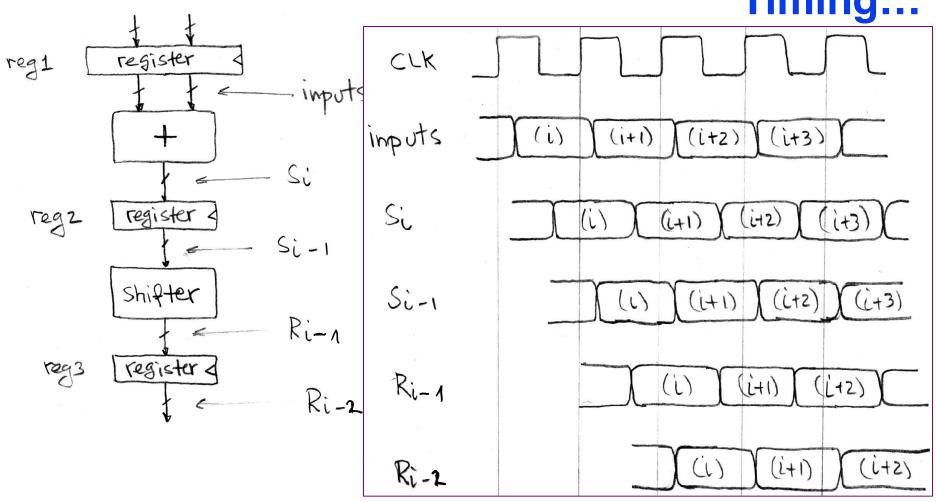


Max Delay = CLK-to-Q Delay + CL Delay + Setup Time

Critical Paths



Note: delay of 1 clock cycle from input to output. Clock period limited by propagation delay of adder/shifter. Pipelining to improve performance



- Insertion of register allows higher clock frequency.
- More outputs per second (higher bandwidth)
- But each individual result takes longer (greater latency)

Recap of Timing Terms

- Clock (CLK) steady square wave that synchronizes system
- Setup Time when the input must be stable <u>before</u> the rising edge of the CLK
- Hold Time when the input must be stable <u>after</u> the rising edge of the CLK
- "CLK-to-Q" Delay how long it takes the output to change, measured from the rising edge of the CLK
- Flip-flop one bit of state that samples every rising edge of the CLK (positive edge-triggered)
- Register several bits of state that samples on rising edge of CLK or on LOAD (positive edge-triggered)

Problems with Clocking

- The clock period must be longer than the critical path
 - Otherwise, you will get the wrong answers
 - But it can be even longer than that
- Critical path:
 - clk->q time
 - Necessary to get the output of the registers
 - worst case combinational logic delay
 - Setup time for the next register
- Must meet all of these to be correct

Hold-Time Violations...

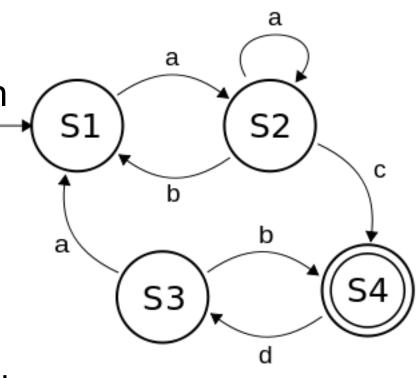
- An alternate problem can occur...
 - Clk->Q + best case combinational delay < Hold time...</p>
- What happens?
 - Clk->Q + data propagates...
 - And now you don't hold the input to the flip flop long enough
- Solution:
 - Add delay on the best-case path (e.g. two inverters)

Finite State Machines (FSM) Intro

 A convenient way to conceptualize computation over time

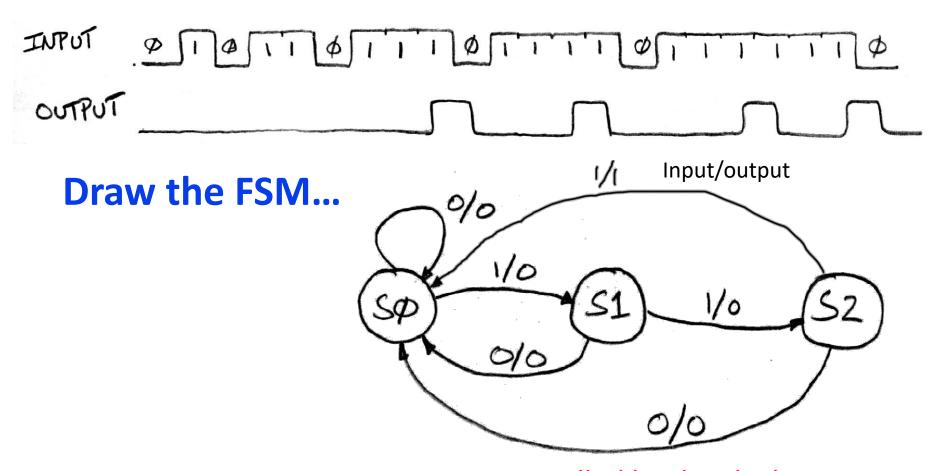
 We start at a state and given an input, we follow some edge to another (or the same) state

- The function can be represented with a "state transition diagram".
- With combinational logic and registers, any FSM can be implemented in hardware.



FSM Example: 3 ones...

FSM to detect the occurrence of 3 consecutive 1's in the input.

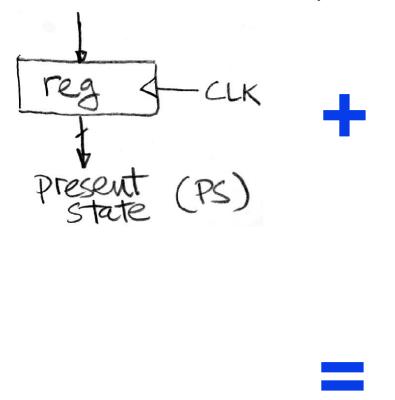


Assume state transitions are controlled by the clock: on each clock cycle the machine checks the inputs and moves to a new state and produces a new output...

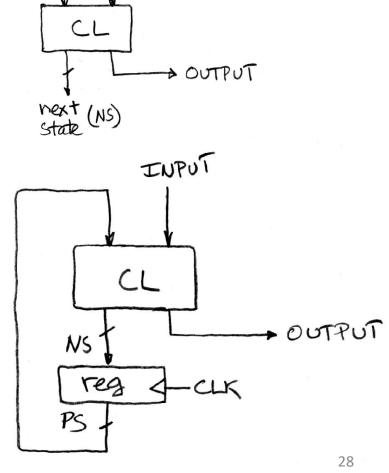
Hardware Implementation of FSM

INPUT

... Therefore a register is needed to hold the a representation of which state the machine is in. Use a unique bit pattern for each state.

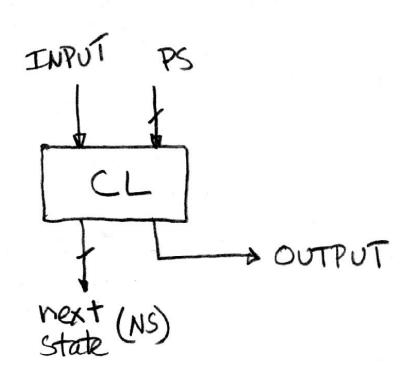


Combinational logic circuit is used to implement a function that maps from present state and input to next state and output.



FSM Combinational Logic

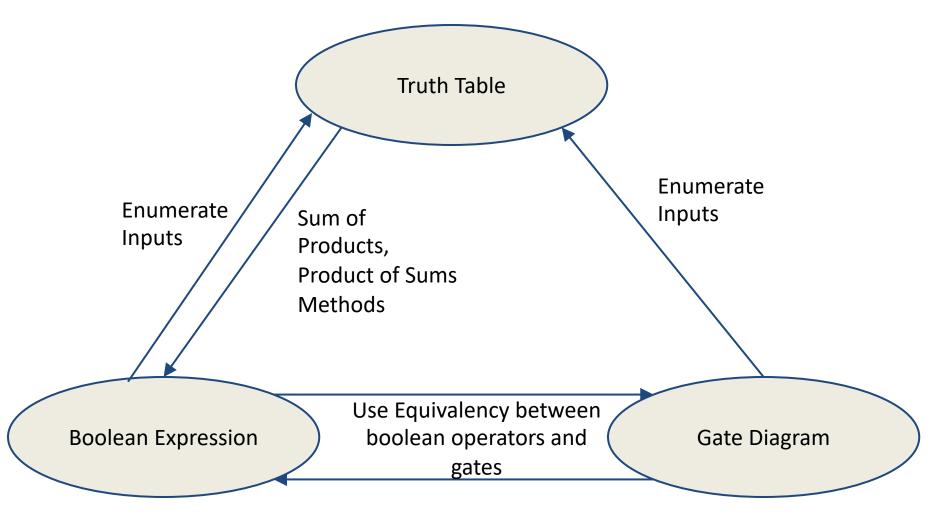
Specify CL using a truth table



Truth table...

PS	Input	NS	Output
00	0	00	0
00	1	01	0
01	0	00	0
01	1	10	0
10	0	00	0
10	1	00	1

Representations of Combinational Logic (groups of logic gates)



Admin

- P1.1 due tomorrow
- P1.2 will be published this week
- HW3 is out

- Midterm I
 - April 6 during lecture hours
- Contents:
 - Everything till (including) Datapath (next lecture)

Midterm I

- Switch cell phones off! (not silent mode – off!)
 - Put them in your bags.
- Bags in the front. On the table: nothing but: pen, 1 drink, 1 snack, your student ID card and your cheat sheet!
- The RISC V green card will be provided
- No other electronic devices are allowed!
 - No ear plugs, music, smartwatch...
- Anybody touching any electronic device will FAIL the course!
- Anybody found cheating (copy your neighbors answers, additional material, ...) will FAIL the course!







COMPUTER CREANIZATION AND DESIGN

THE HA. WARE/SOFTWA INTERFACE



EDITION



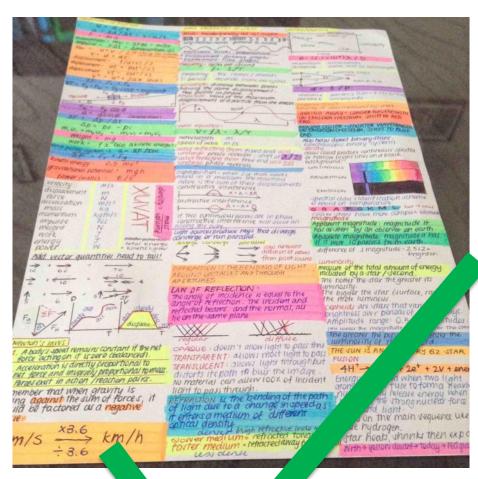


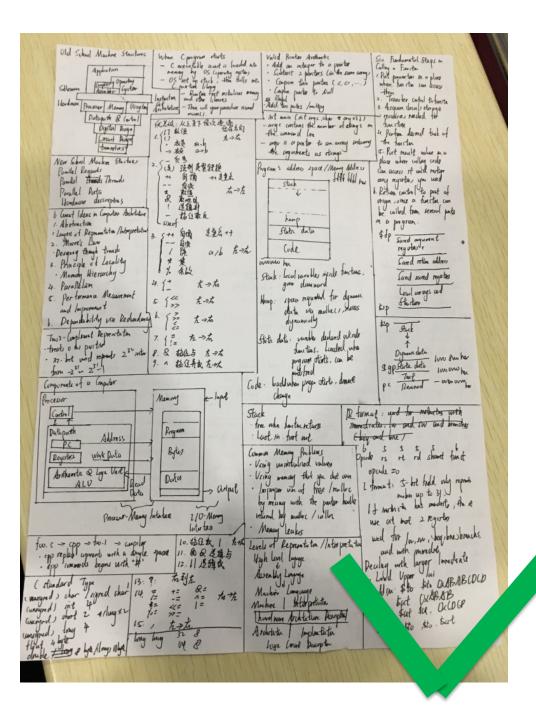
DAVID A. PATTERSON JOHN L. HENNESSY

Cheat Sheet

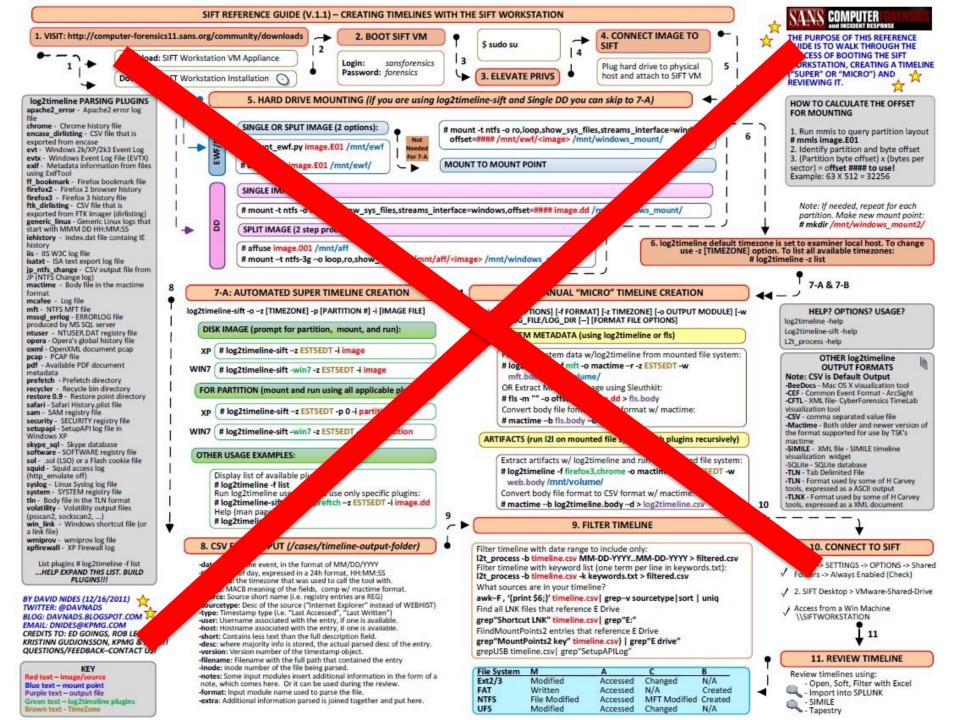
- 1 A4 Cheat Sheet allowed (double sided)
 - Midterm II: 2 pages
 - Final: 3 pages
- Rules:
 - Hand-written not printed!
 - Your <u>name</u> in pinyin on the top!
 - Cheat Sheets not complying to this rule will be confiscated!







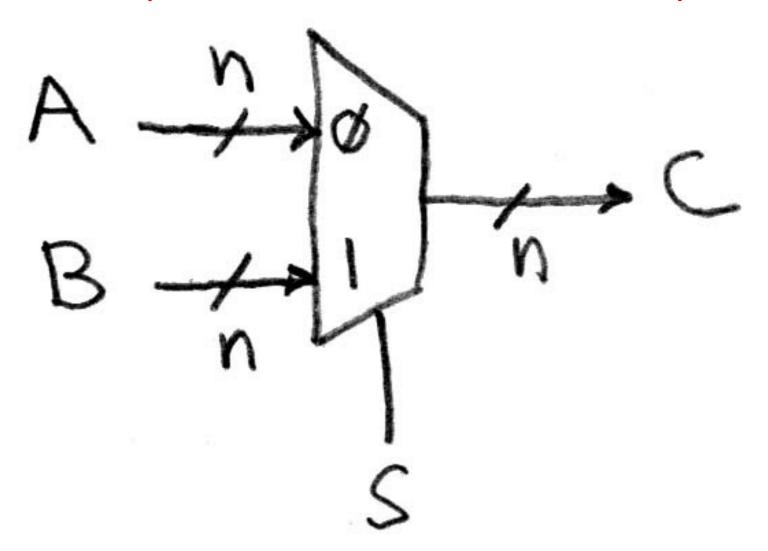
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Building Standard Functional Units

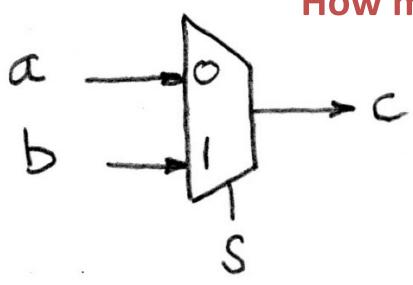
- Data multiplexers
- Arithmetic and Logic Unit
- Adder/ Subtractor

Data Multiplexer ("Mux") (here 2-to-1, n-bit-wide)



N instances of 1-bit-wide mux

How many rows in TT?



$$c = \overline{s}a\overline{b} + \overline{s}ab + s\overline{a}b + sab$$

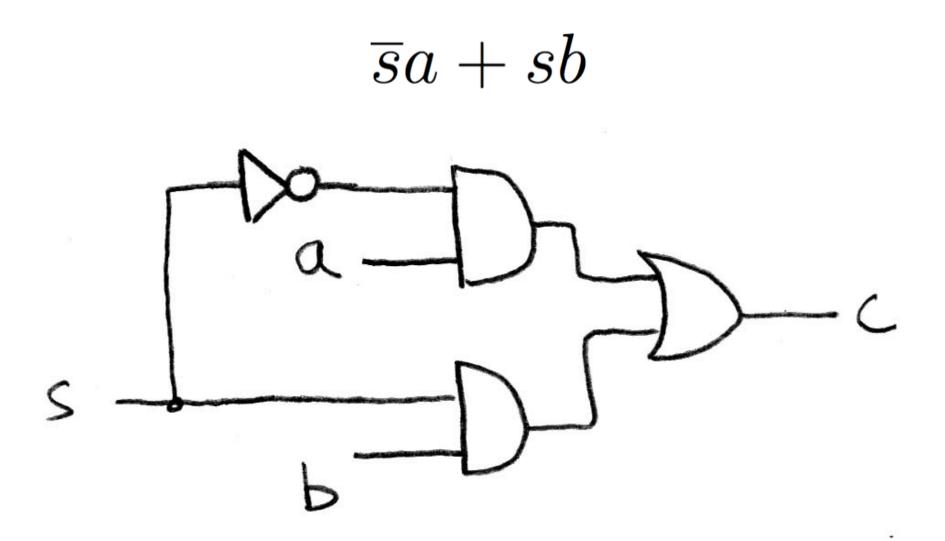
$$= \overline{s}(a\overline{b} + ab) + s(\overline{a}b + ab)$$

$$= \overline{s}(a(\overline{b} + b)) + s((\overline{a} + a)b)$$

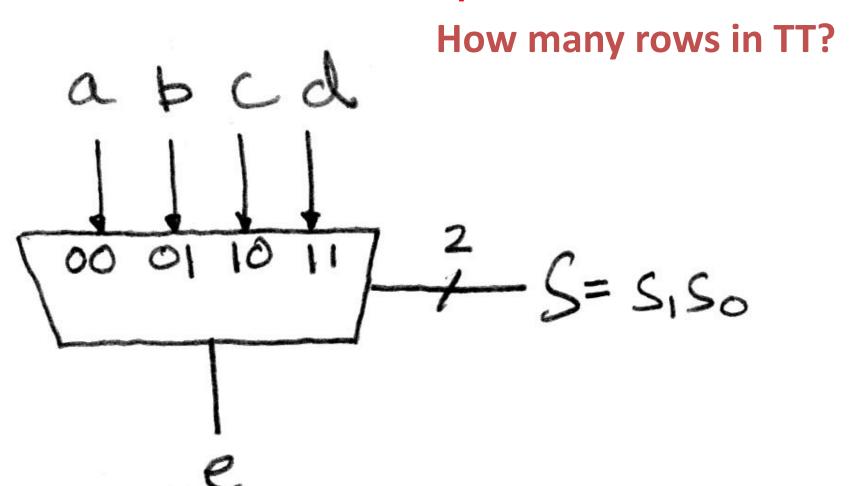
$$= \overline{s}(a(1) + s((1)b))$$

$$= \overline{s}a + sb$$

How do we build a 1-bit-wide mux?

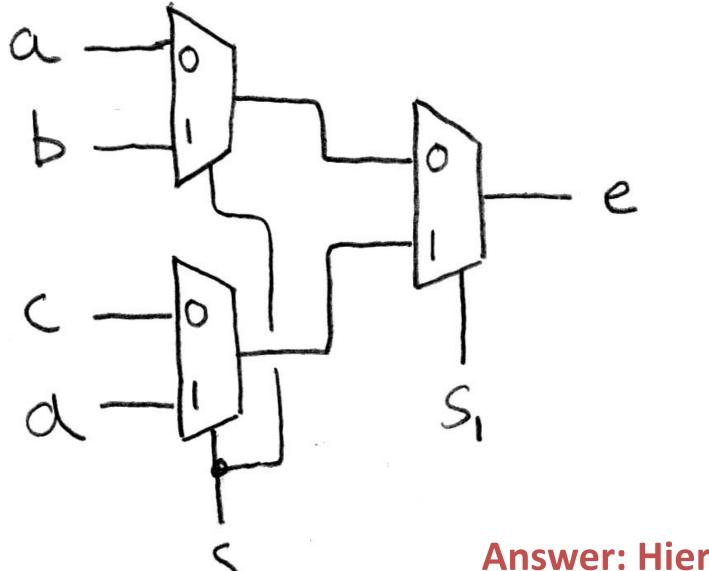


4-to-1 multiplexer?



$$e = \overline{s_1}\overline{s_0}a + \overline{s_1}s_0b + s_1\overline{s_0}c + s_1s_0d$$

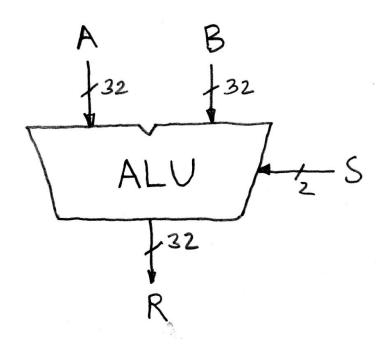
Another way to build 4-1 mux?



Answer: Hierarchically!

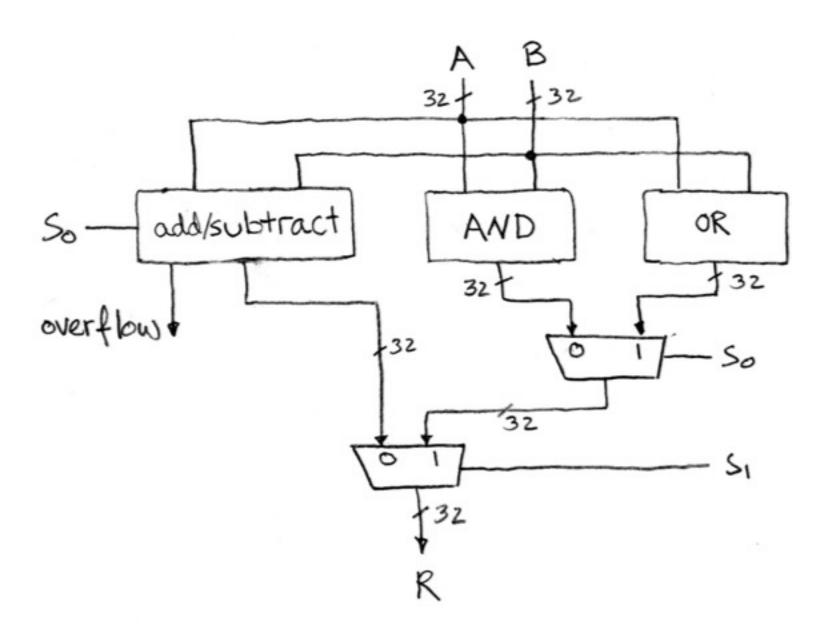
Arithmetic and Logic Unit

- Most processors contain a special logic block called the "Arithmetic and Logic Unit" (ALU)
- We'll show you an easy one that does ADD,
 SUB, bitwise AND, bitwise OR



when S=00, R=A+B when S=01, R=A-B when S=10, R=A AND B when S=11, R=A OR B

Our simple ALU



Question

Convert the truth table to a boolean expression (no need to simplify):

A:
$$F = xy + x(^{\sim}y)$$

B:
$$F = xy + (^x)y + (^x)(^y)$$

C:
$$F = (^x)y + x(^y)$$

D:
$$F = xy + (^x)y$$

E:
$$F = (x+y)(^x+^y)$$

X	y	F(x,y)
0	0	0
0	1	1
1	0	0
1	1	1

How to design Adder/Subtractor?

- Truth-table, then determine canonical form, then minimize and implement as we've seen before
- Look at breaking the problem down into smaller pieces that we can cascade or hierarchically layer

Adder/Subtractor – One-bit adder LSB...

-	\mathbf{a}_0	b_0	\mathbf{s}_0	c_1
_	0	0	0	0
	0	1	1	0
	1	0	1	0
	1	1	0	1

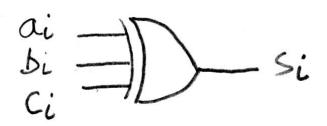
$$s_0 = c_1 = c_1 = c_1$$

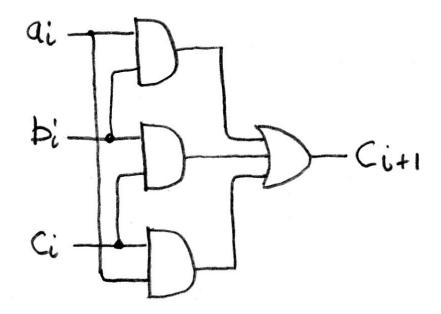
Adder/Subtractor – One-bit adder (1/2)...

						\mathbf{a}_i	b_i	c_i	s_i	c_{i+1}
	Ca	Ca	Ca			0	0	0	0	0
		\mathbf{c}_2				0	0	1	1	0
	\mathbf{a}_3	\mathbf{a}_2				0	1	0	l	
+	b_3	b_2	b_1	b_0		0	1	1	0	1
	S 3	\mathbf{s}_2	\mathbf{s}_1	s_0	•	1	0	0	ı	
	0	2	1	j		1	0	1	0	1
						1	1	0	0	1
						1	1	1	1	1

$$s_i = c_{i+1} =$$

Adder/Subtractor – One-bit adder (2/2)

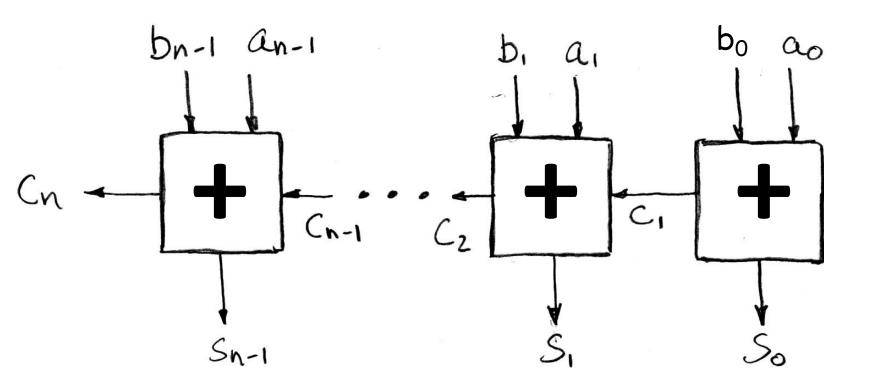




$$s_i = XOR(a_i, b_i, c_i)$$

 $c_{i+1} = MAJ(a_i, b_i, c_i) = a_i b_i + a_i c_i + b_i c_i$

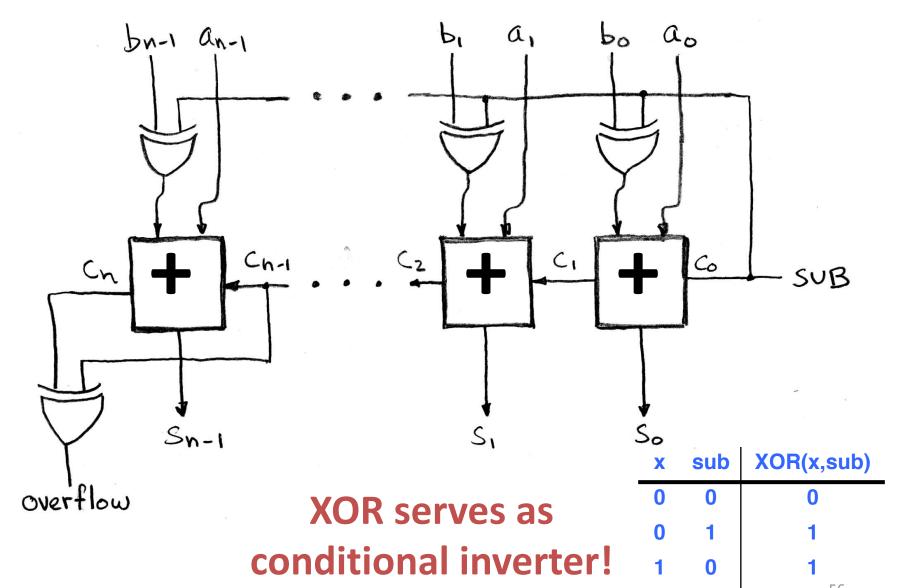
N 1-bit adders => 1 N-bit adder



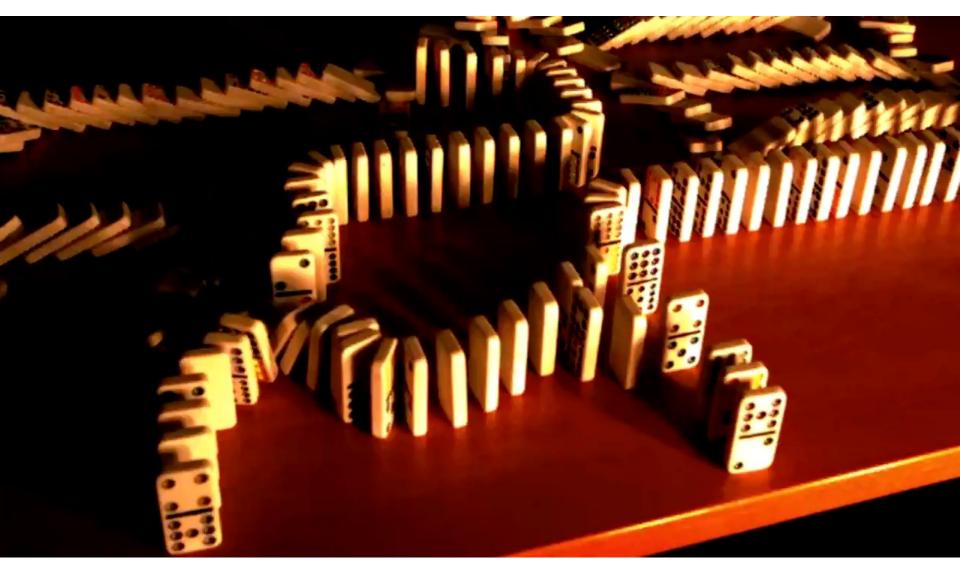
What about overflow? Overflow = c_n ?

Extremely Clever Subtractor:

$$s = a + (-b)$$



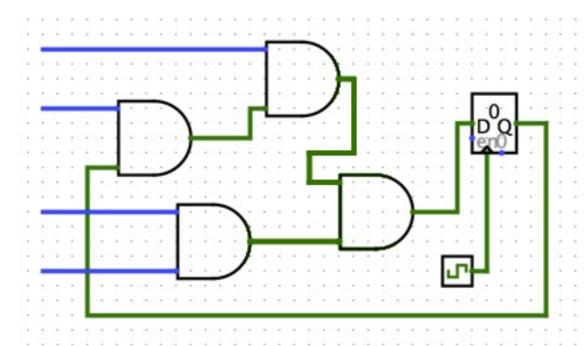
Domino Adder



Explanation: https://www.youtube.com/watch?v=INuPy-r1GuQ

4-bit adder: https://www.youtube.com/watch?v=OpLU bhu2w&t=0s

Question



Clock->Q 1ns
Setup 1ns
Hold 1ns
AND delay 1ns

What is maximum clock frequency?

• A: 5 GHz

• B: 500 MHz

• C: 200 MHz

• D: 250 MHz

• E: 1/6 GHz

In Conclusion

- Finite State Machines have clocked state elements plus combinational logic to describe transition between states
 - Clocks synchronize D-FF change (Setup and Hold times important!)
- Standard combinational functional unit blocks built hierarchically from subcomponents