上海科技大学
ShanghaiTech University

CS283: **Robotics Spring 2024: Planning**

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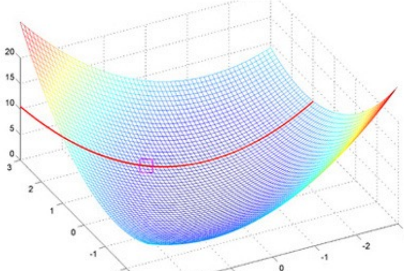
ShanghaiTech University

Path Planning: Overview of Algorithms

1. Optimal Control

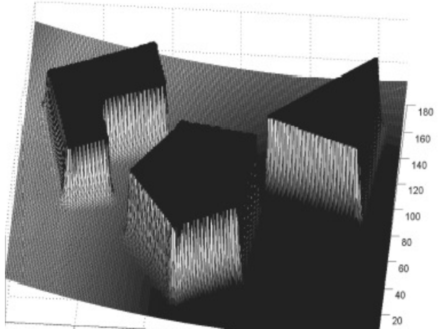
- Solves truly optimal solution
- Becomes intractable for even moderately complex as well as nonconvex problems



Source:
http://mitocw.udsm.ac.tz

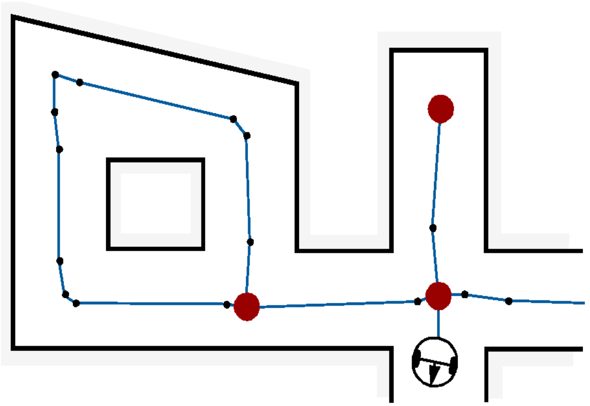
2. Potential Field

- Imposes a mathematical function over the state/configuration space
- Many physical metaphors exist
- Often employed due to its simplicity and similarity to optimal control solutions

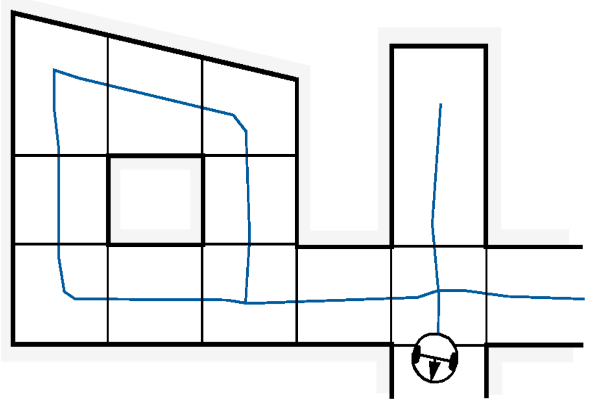


3. Graph Search

- Identify a set edges between nodes within the free space



- Where to put the nodes?



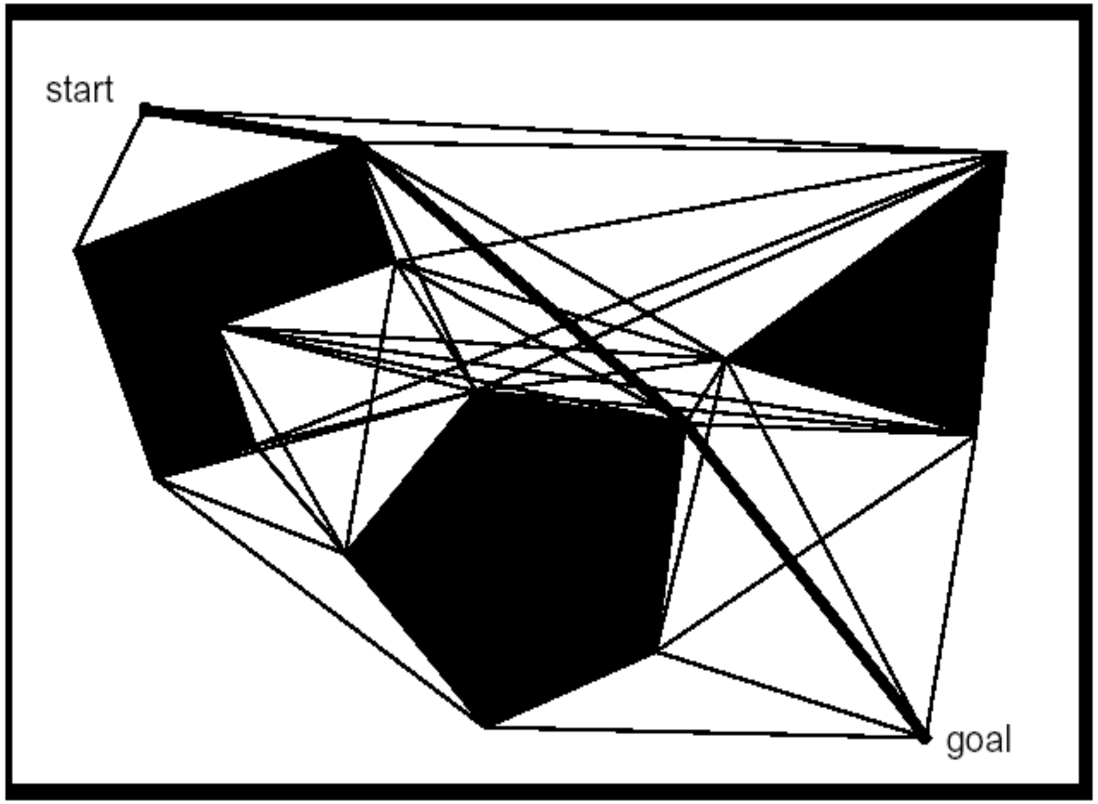
Graph Search

- Overview
  - Solves a least cost problem between two states on a (directed) graph
  - Graph structure is a discrete representation

- Limitations
  - State space is discretized → completeness is at stake
  - Feasibility of paths is often not inherently encoded

- Algorithms
  - (Preprocessing steps)
  - Breath first
  - Depth first
  - Dijkstra
  - A* and variants
  - D* and variants

Graph Construction: Visibility Graph



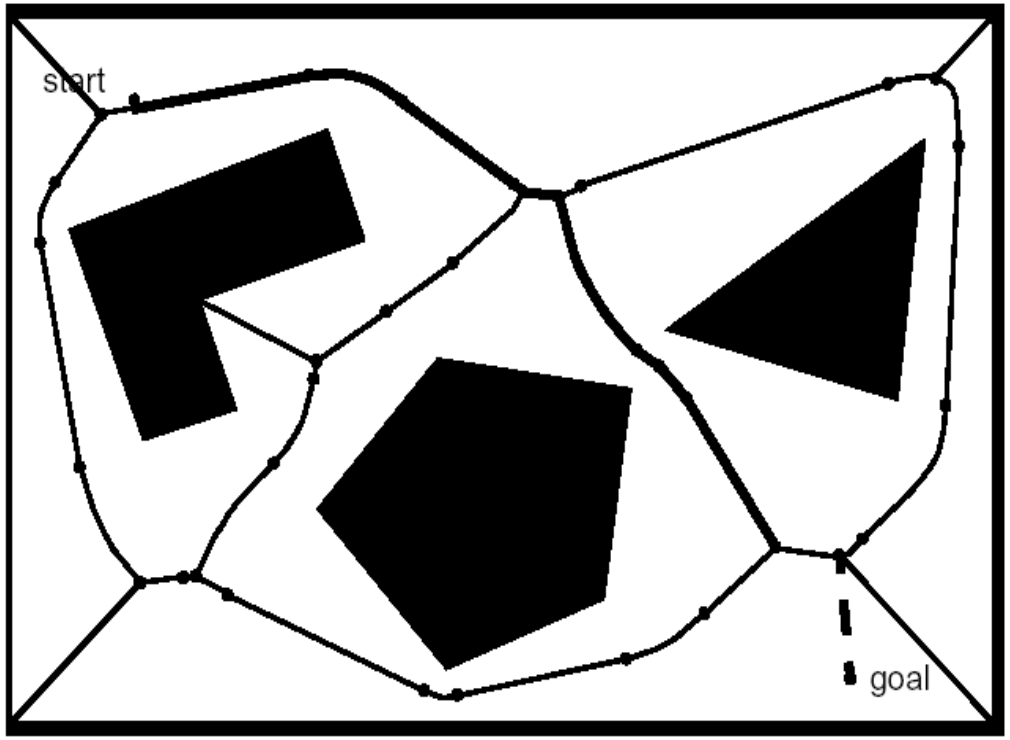
- Particularly suitable for polygon-like obstacles
- Shortest path length
- Grow obstacles to avoid collisions

Graph Construction: Visibility Graph

- Pros
  - The found path is optimal because it is the shortest length path
  - Implementation simple when obstacles are polygons

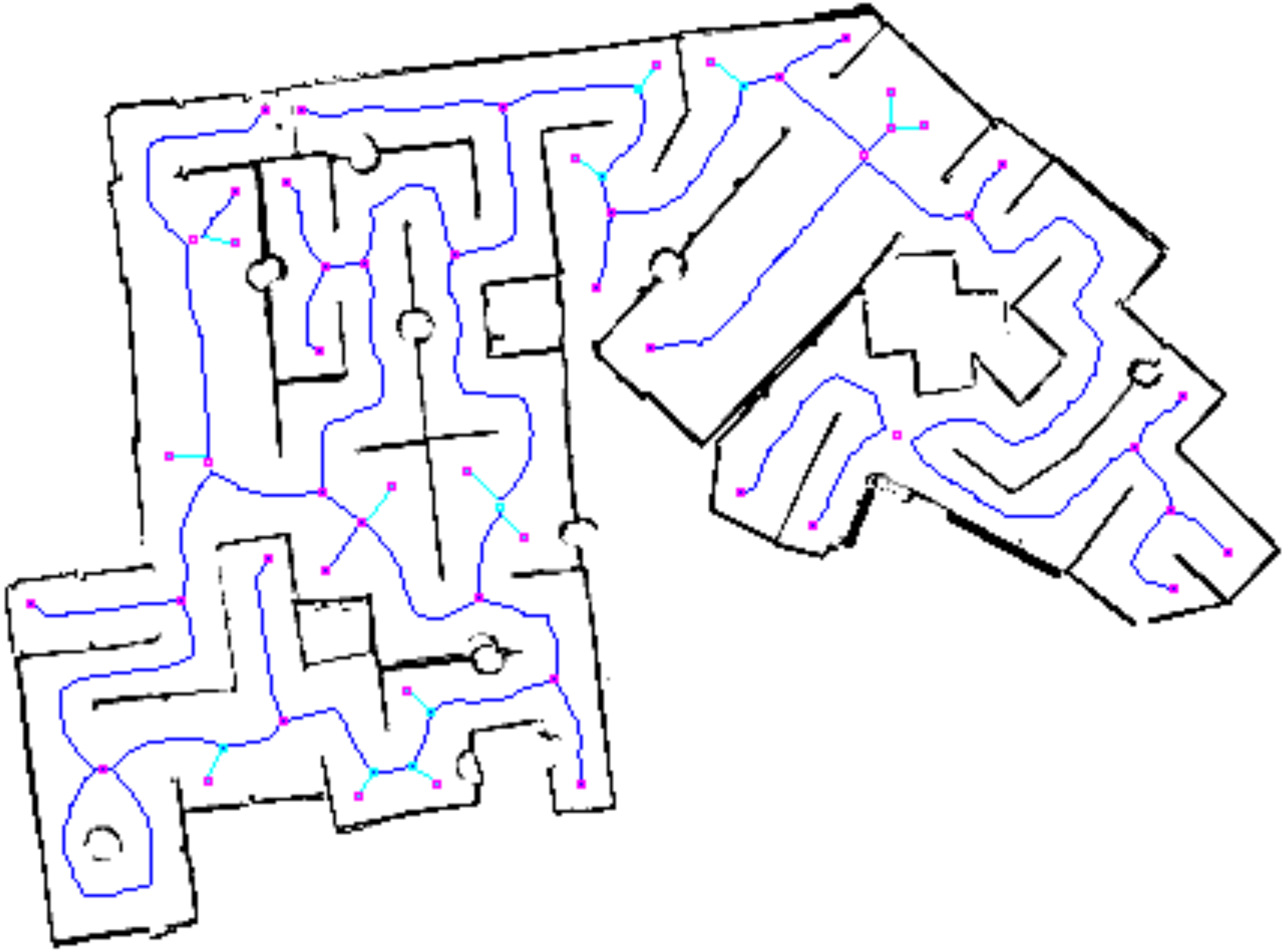
- Cons
  - The solution path found by the visibility graph tend to take the robot as close as possible to the obstacles: the common solution is to grow obstacles by more than robot's radius
  - Number of edges and nodes increases with the number of polygons
  - Thus it can be inefficient in densely populated environments

Graph Construction: Voronoi Diagram



- Tends to maximize the distance between robot and obstacles

Topology Graph

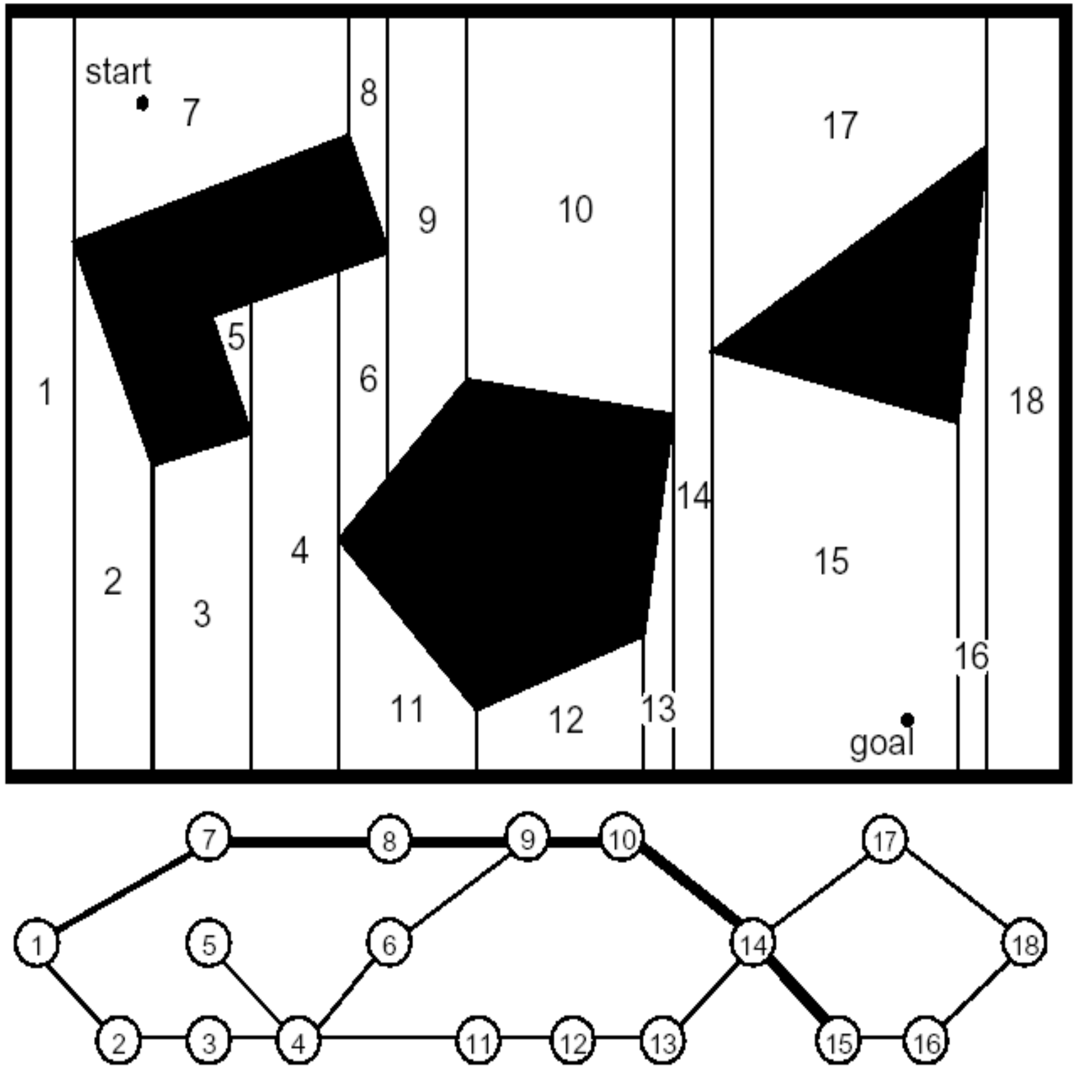


Graph Construction: Voronoi Diagram

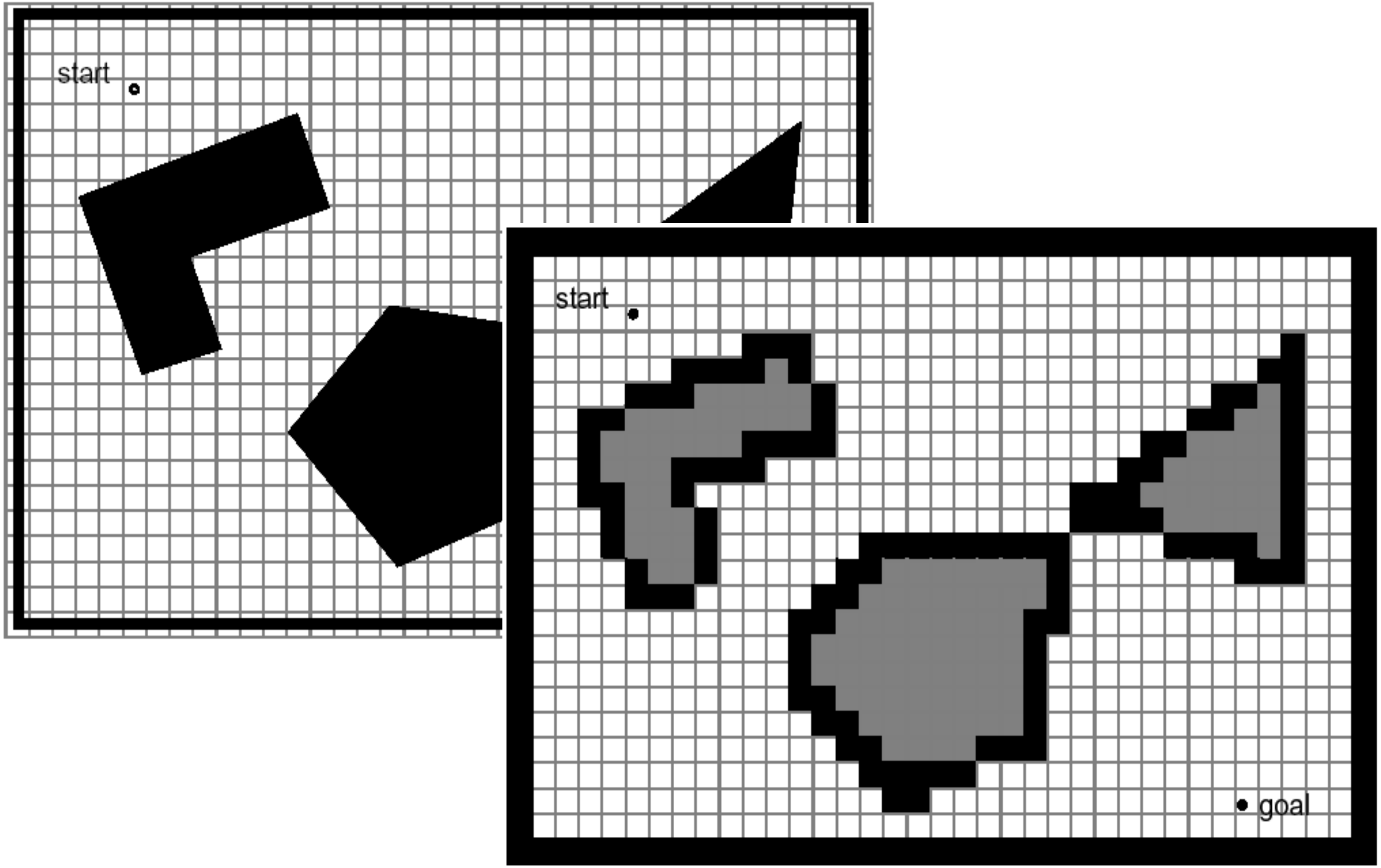
- Pros
  - Using range sensors like laser or sonar, a robot can navigate along the Voronoi diagram using simple control rules

- Cons
  - Because the Voronoi diagram tends to keep the robot as far as possible from obstacles, any short range sensor will be in danger of failing
  - Voronoi diagram can change drastically in open areas

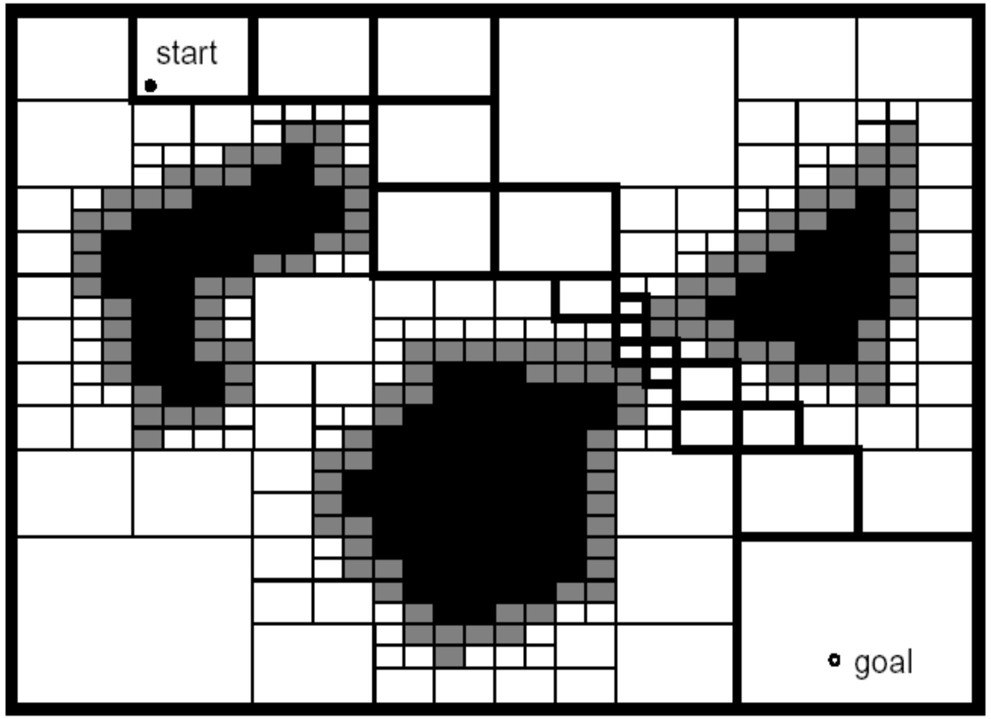
Graph Construction: Exact Cell Decomposition (2/4)



Graph Construction: Approximate Cell Decomposition (3/4)



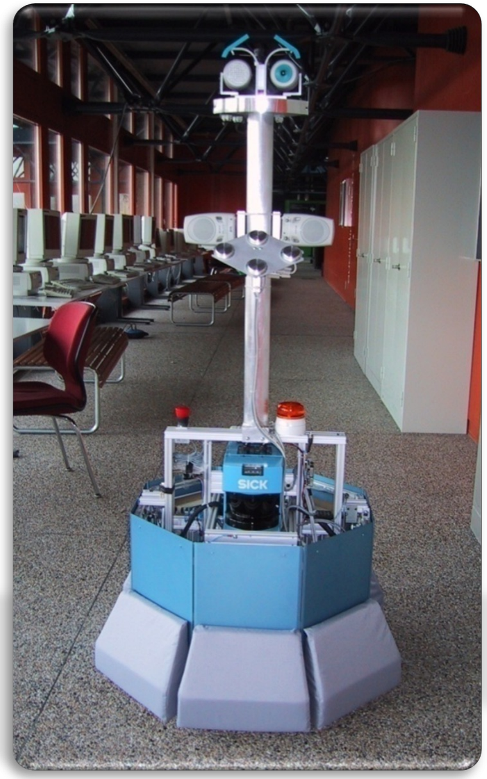
Graph Construction: Adaptive Cell Decomposition (4/4)



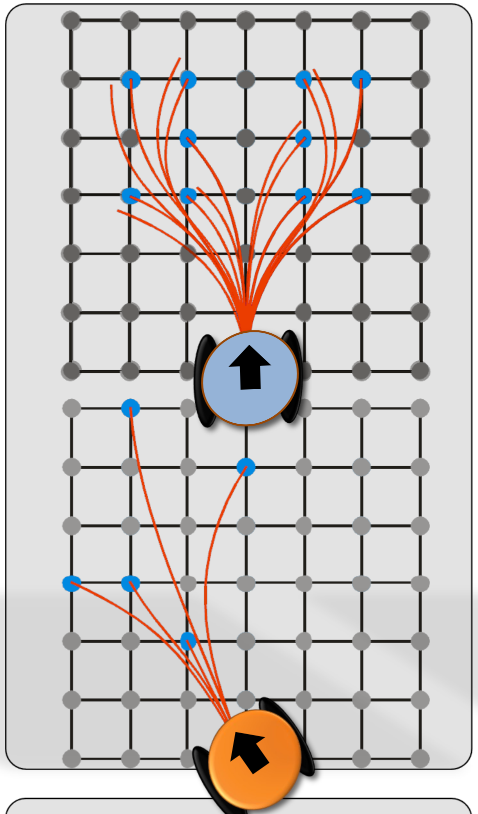
- Close relationship with map representation (Quadtree)!

Graph Construction: State Lattice Design (1/2)

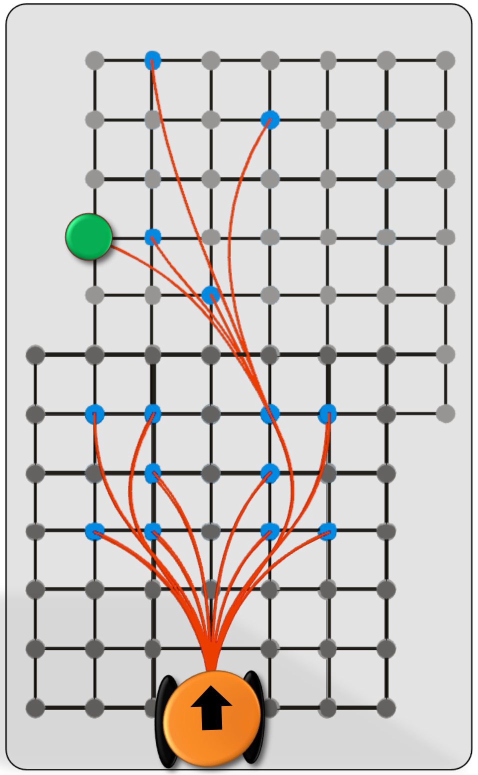
- Enforces edge feasibility



Offline:
Motion Model



Offline:
Lattice Gen.

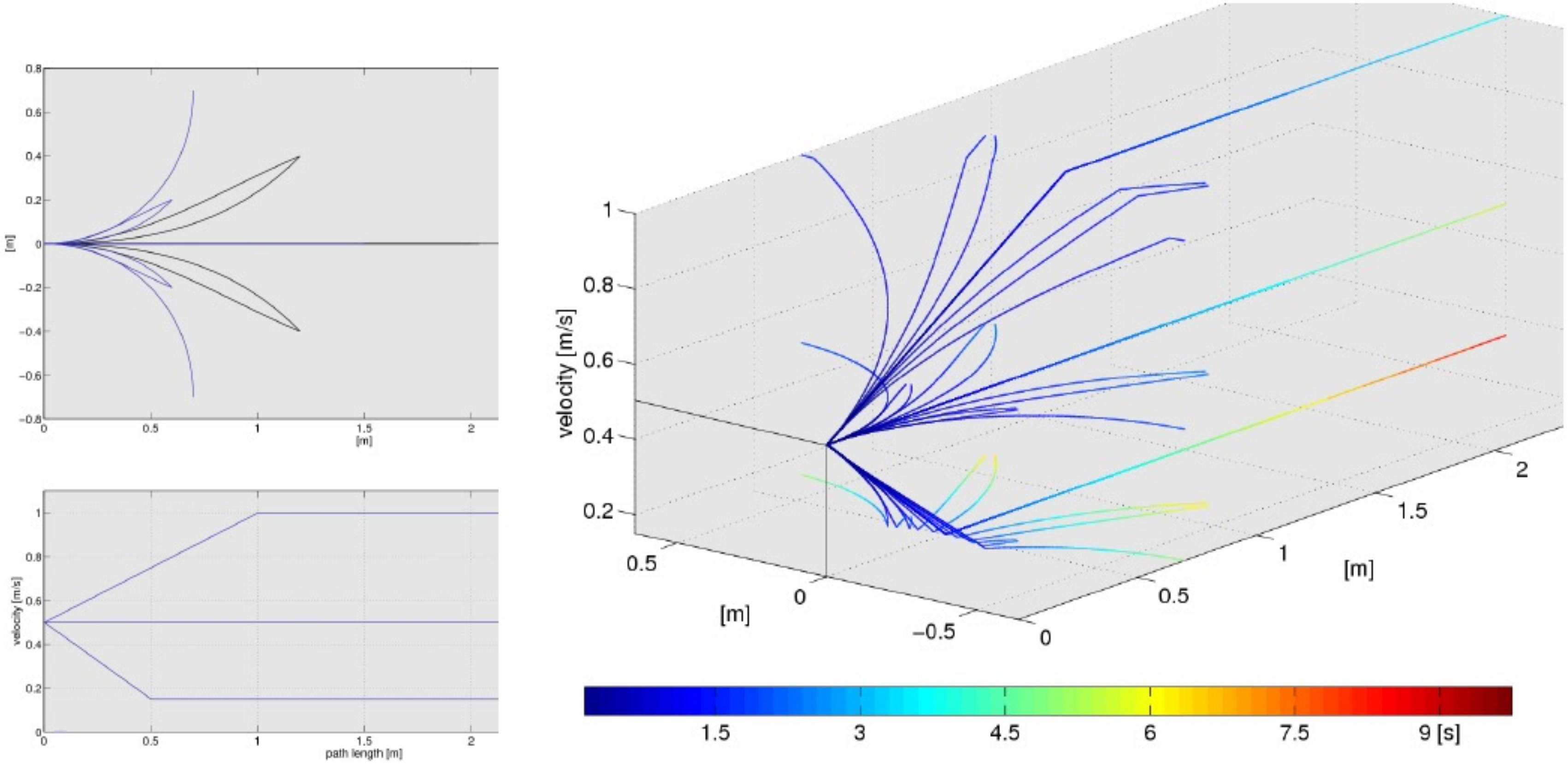


Online:
Incremental Graph Constr.

Graph Construction: State Lattice Design (2/2)

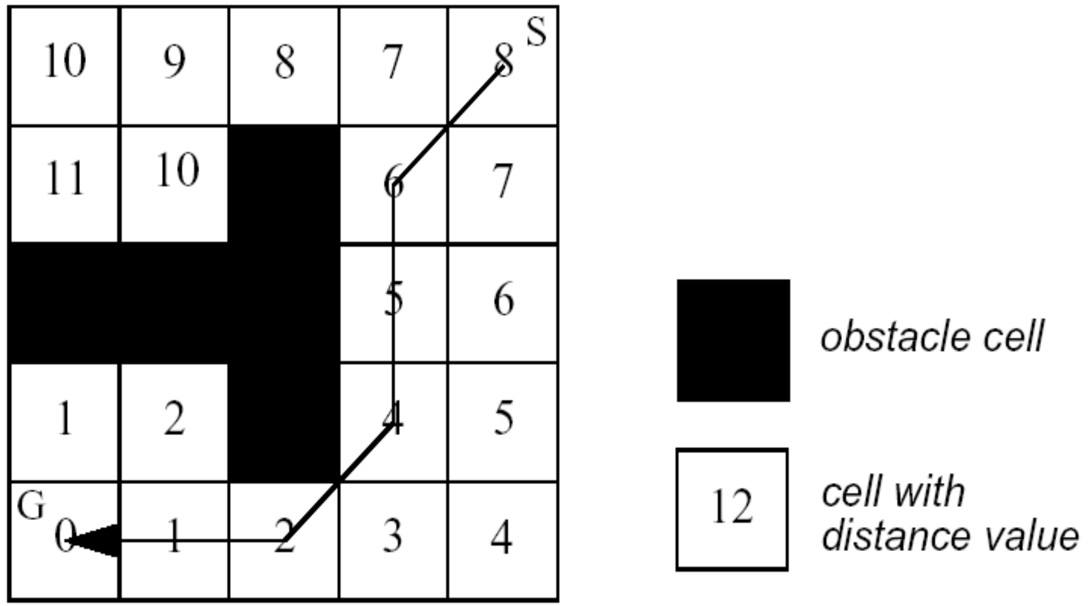
Martin Rufli

- State lattice encodes only kinematically feasible edges



Deterministic Graph Search

- Methods
  - Breath First
  - Depth First
  - **Dijkstra**
  - A* and variants
  - D* and variants
  - ...



ADMIN

HW4 is out

- Start early …
- First 4 tasks are config only – build a Gazebo simulation and use ROS2 Navigation 2 to perform simple autonomy
- Task 5: Implement frontier exploration yourself ☺

Project

- May 26th, 22:00: due date for the intermediate project report
- June 25th, 22:00: due date for the final demo. (You are welcome to do it earlier!)
- June 28th, 22:00: due date for the final report. (You are welcome to finish it earlier!)
- June 28th, 22:00: due date for the webpage. (You are welcome to finish it earlier!)

- Schedule Project meeting with me for tomorrow, Monday or Tuesday (April 26, April 29 or April 30)!
  - You contact me!
  - Counts towards your “attend weekly project meeting” score!

Project Safety!

- You work with real robotic hardware => can be dangerous!

- Don’t stick your fingers where they don’t belong!
- Be aware of the moving robot!
- Be careful when handling/ charging the batteries! If you are unsure, ask one of my students for help!
- Be careful with electric components.

- Be nice to the robot!

DIJKSTRA‘S ALGORITHM

EDSGER WYBE DIJKSTRA

1930 - 2002

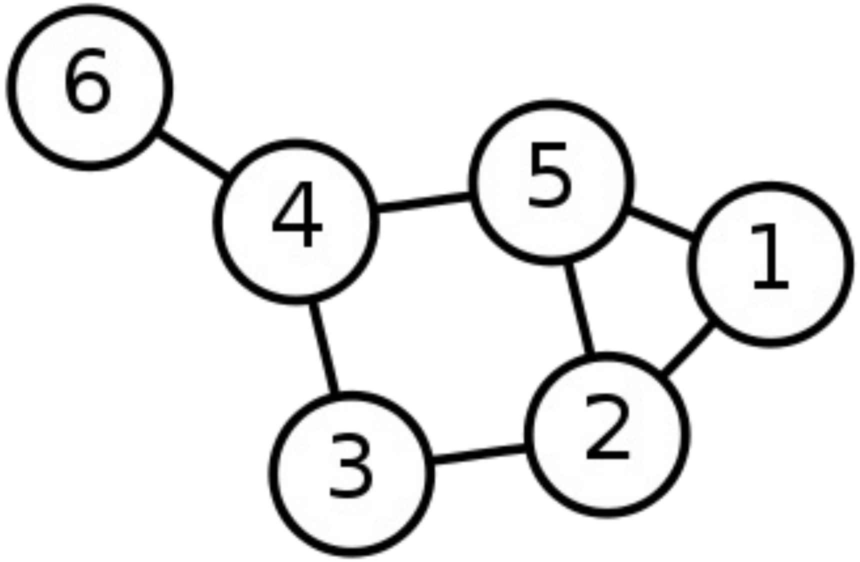
"Computer Science is no more about computers than astronomy is about telescopes."

http://www.cs.utexas.edu/~EWD/

SINGLE-SOURCE SHORTEST PATH PROBLEM

- **Single-Source Shortest Path Problem** - The problem of finding shortest paths from a source vertex *v* to all other vertices in the graph.

- **Graph**
- Set of vertices and edges
- Vertex:
  - Place in the graph; connected by:
- Edge: connecting two vertices
  - Directed or undirected (undirected in Dijkstra's Algorithm)
  - Edges can have weight/ distance assigned



Dijkstra material from http://www.cs.utexas.edu/~tandy/barrera.ppt

Dijkstra's Algorithm

- Assign all vertices infinite distance to goal
- Assign 0 to distance from start
- Add all vertices to the queue

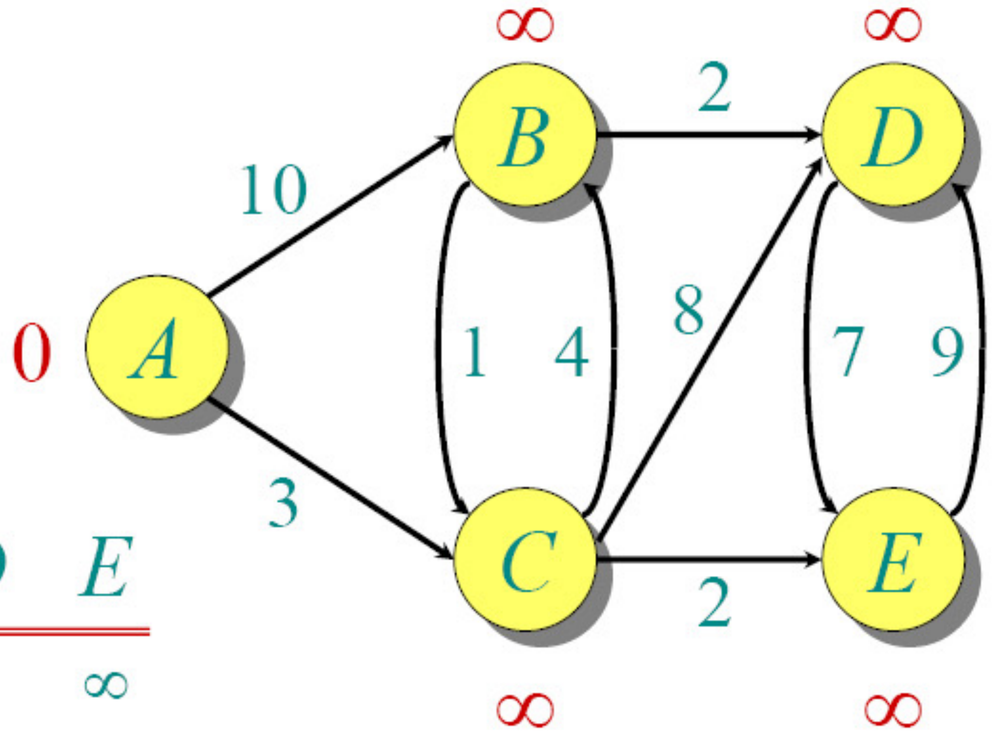
- While the queue is not empty:
  - Select vertex with smallest distance and remove it from the queue
  - Visit all neighbor vertices of that vertex,
  - calculate their distance and
  - update their (the neighbors) distance if the new distance is smaller

Dijkstra's Algorithm - Pseudocode

```
dist[s] ← 0                                  (distance to source vertex is zero)
for  all v ∈ V–{s}
        do  dist[v] ← ∞                      (set all other distances to infinity)
S ← Ø                                        (S, the set of visited vertices is initially empty)
Q← V                                         (Q, the queue initially contains all vertices)
while Q ≠Ø                                   (while the queue is not empty)
do   u ← mindistance(Q, dist)                (select the element of Q with the min. distance)
      S←S∪{u}                                (add u to list of visited vertices)
       for all v ∈ neighbors[u]
              do  if   dist[v] > dist[u] + w(u, v)          (if new shortest path found)
                          then      d[v] ←d[u] + w(u, v)    (set new value of shortest path)
                 (if desired, add traceback code)
return dist
```

Dijkstra Example

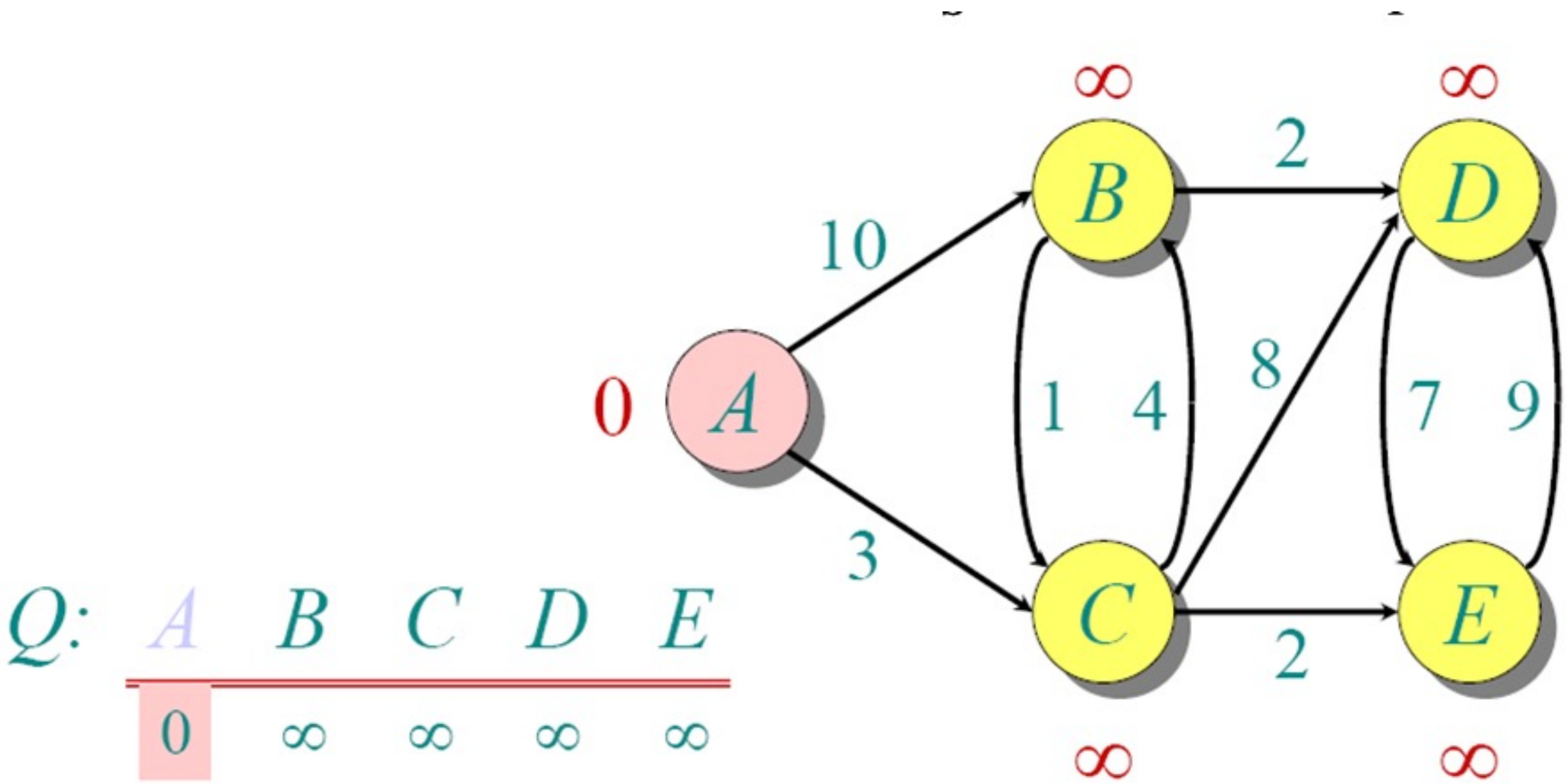
Initialize:



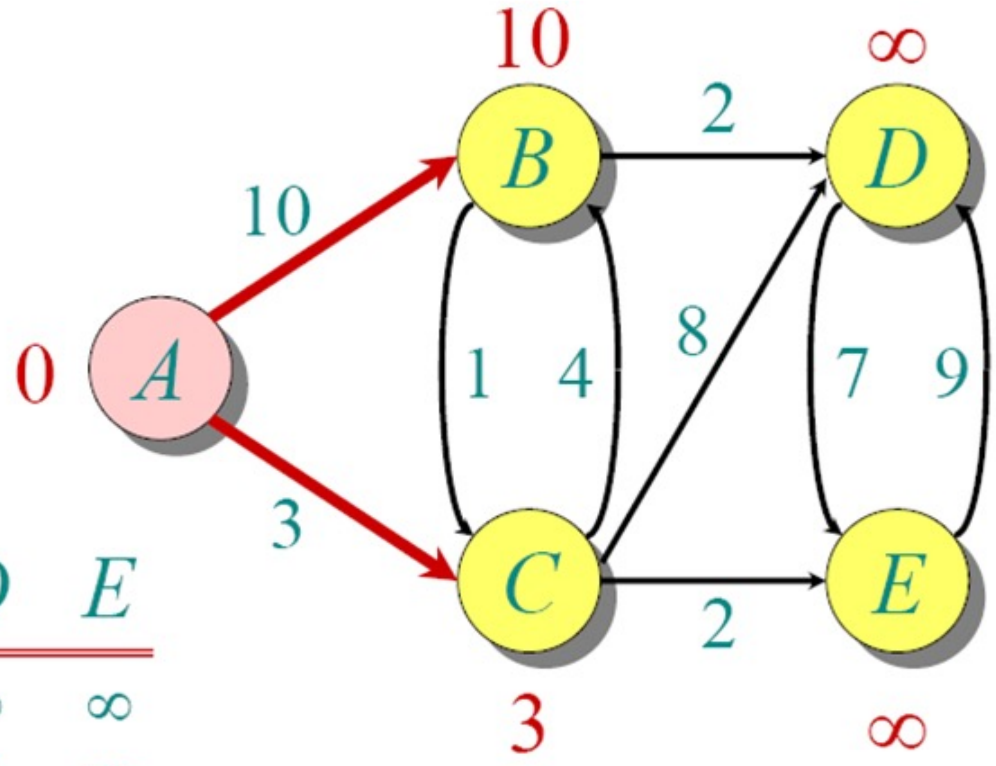
Q:	A	B	C	D	E
	0	∞	∞	∞	∞

S: {}

Dijkstra Example



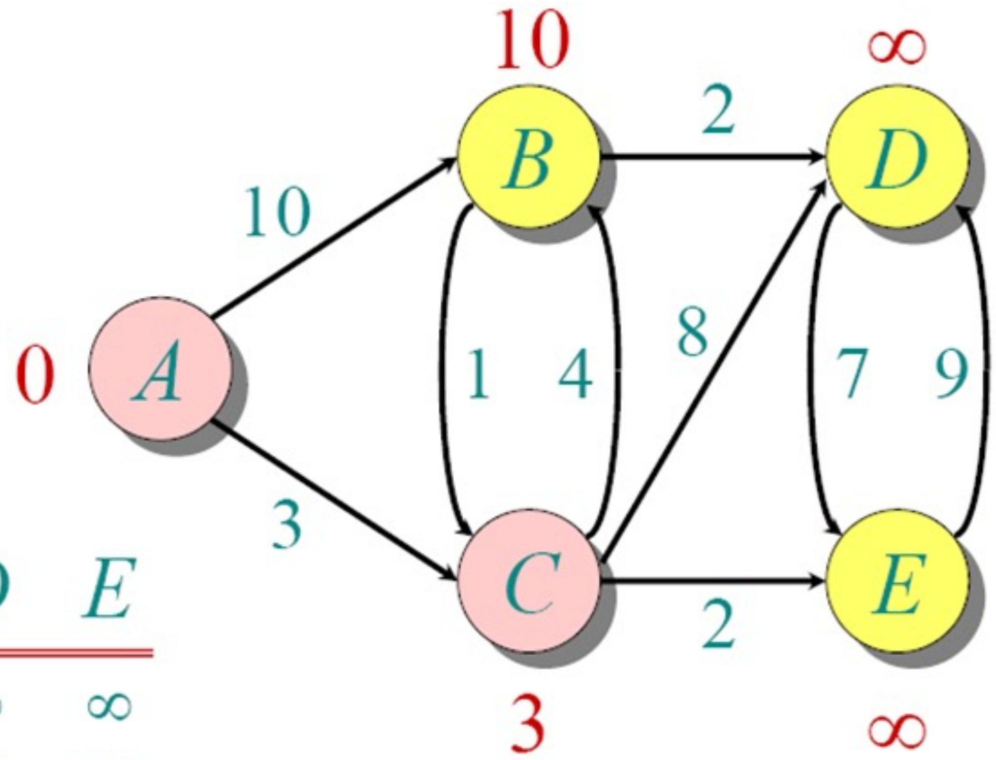
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { A }

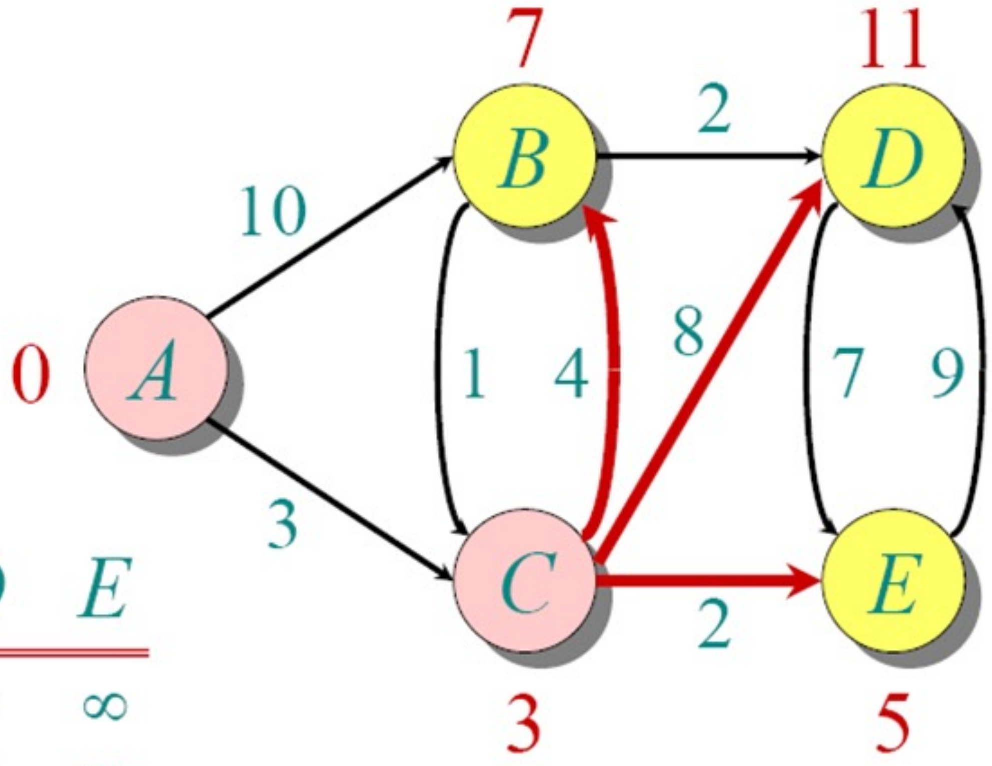
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { A, C }

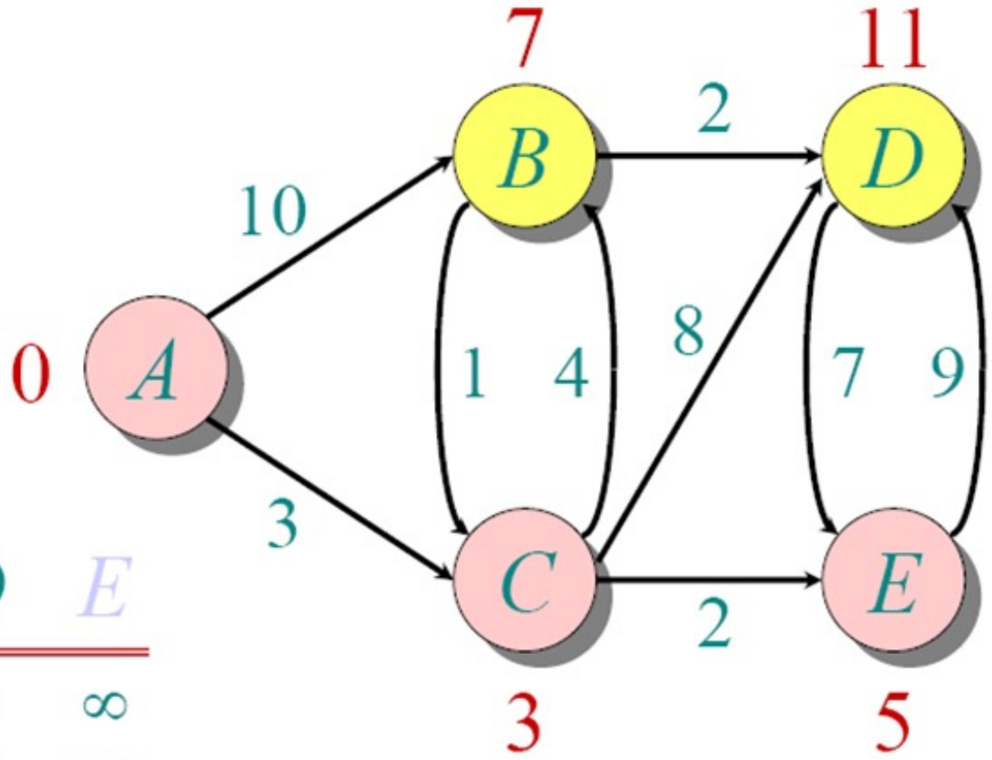
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C }

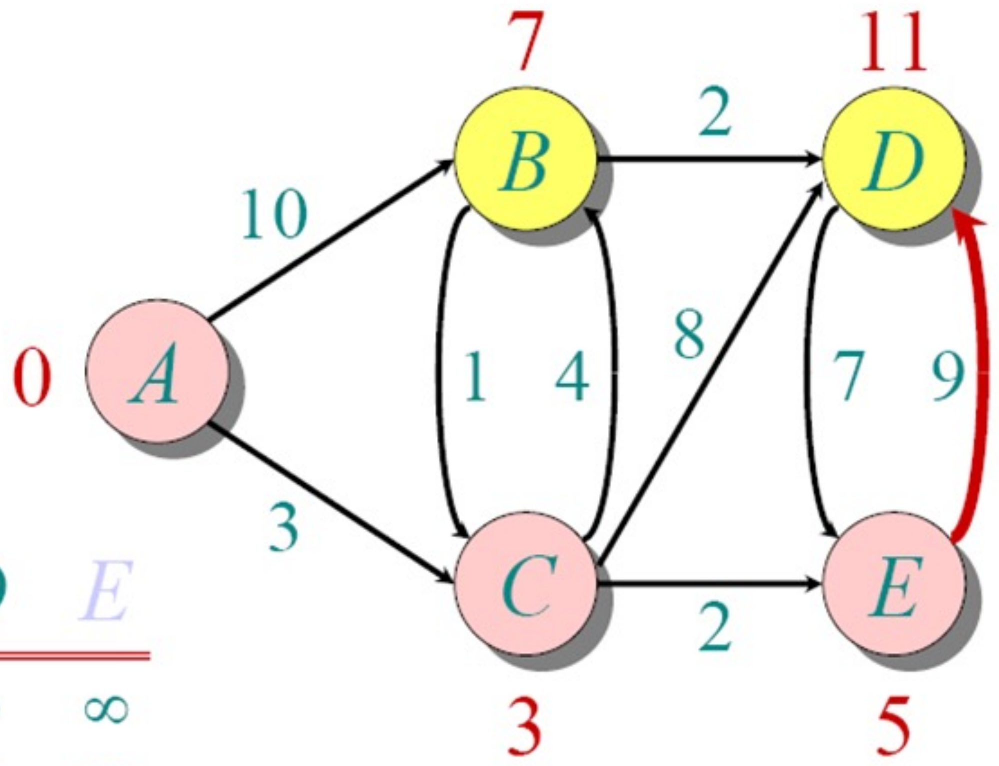
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C, E }

Dijkstra Example

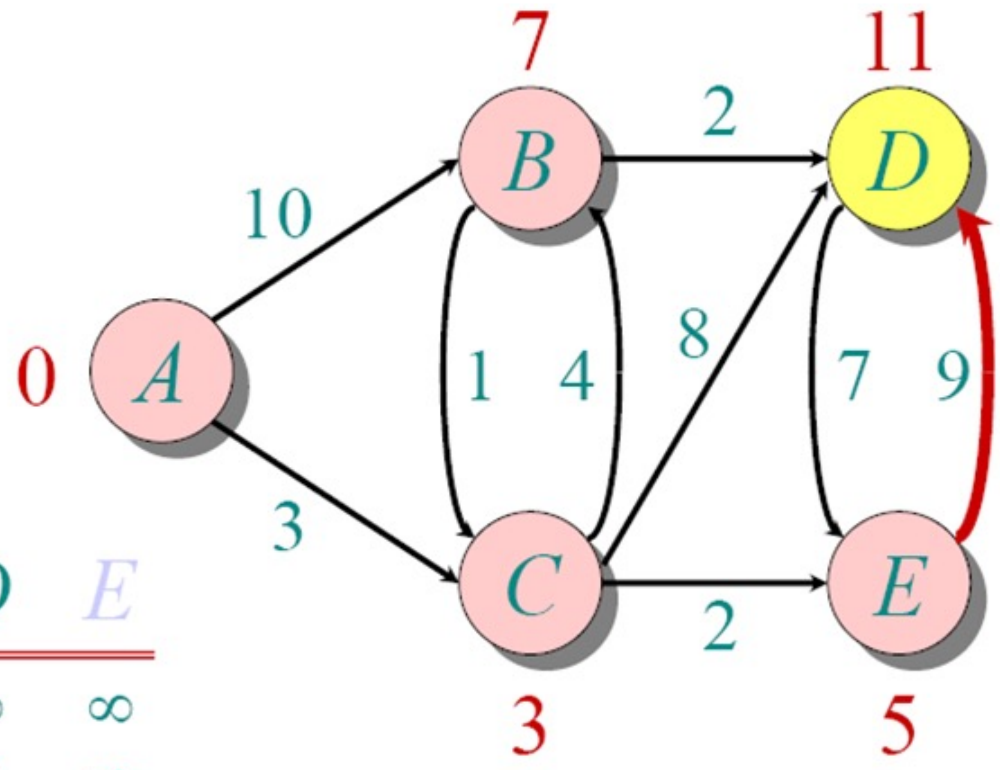


Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞
	7		11	5
	7		11	

S: { A, C, E }

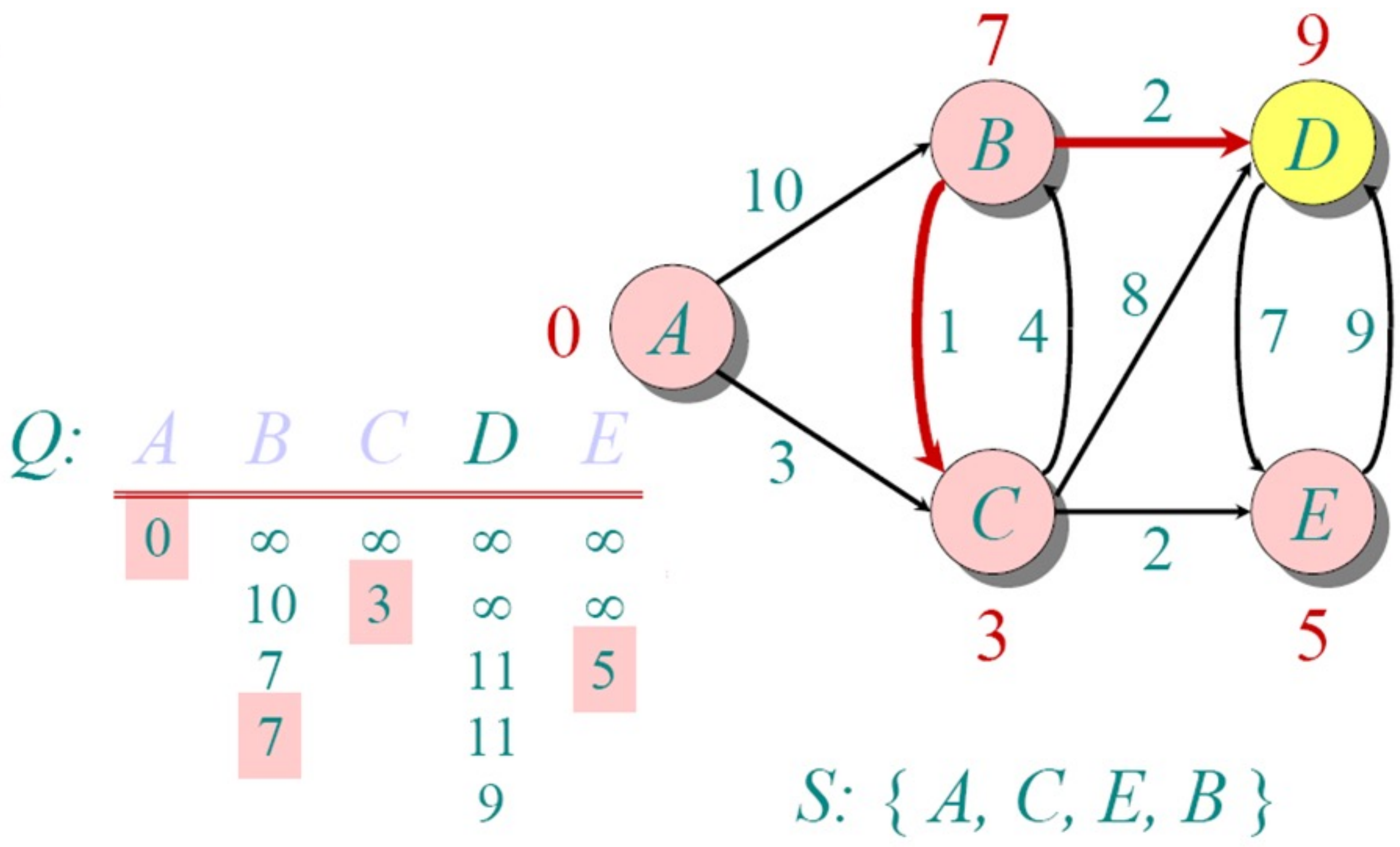
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

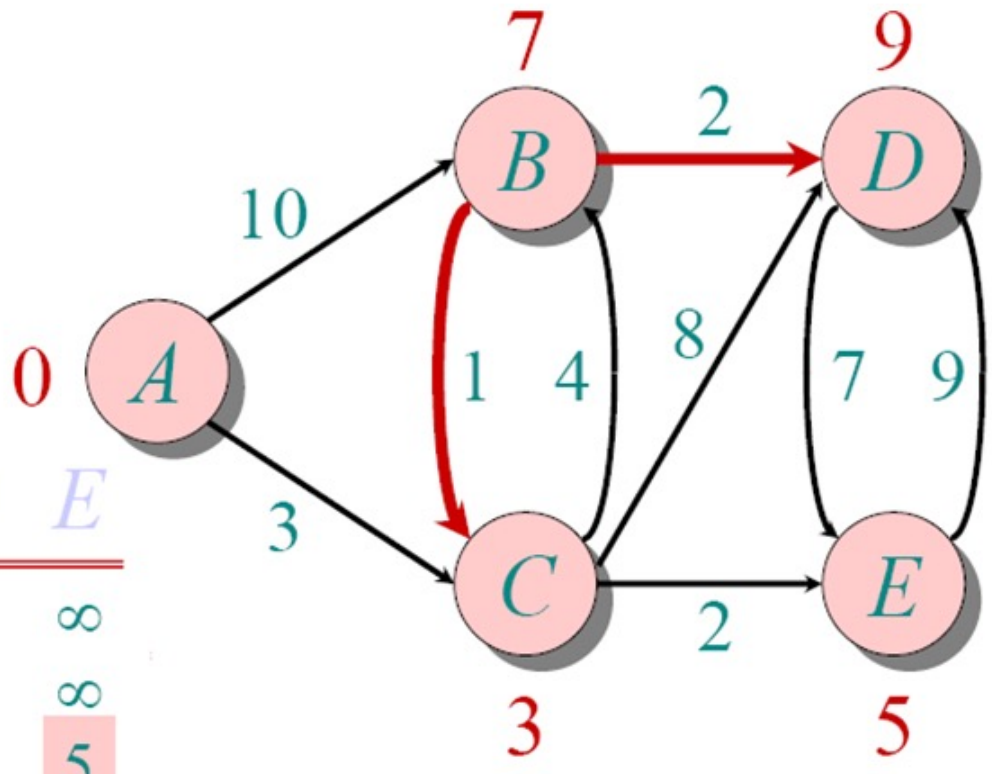
S: { A, C, E, B }

Dijkstra Example



Dijkstra Example

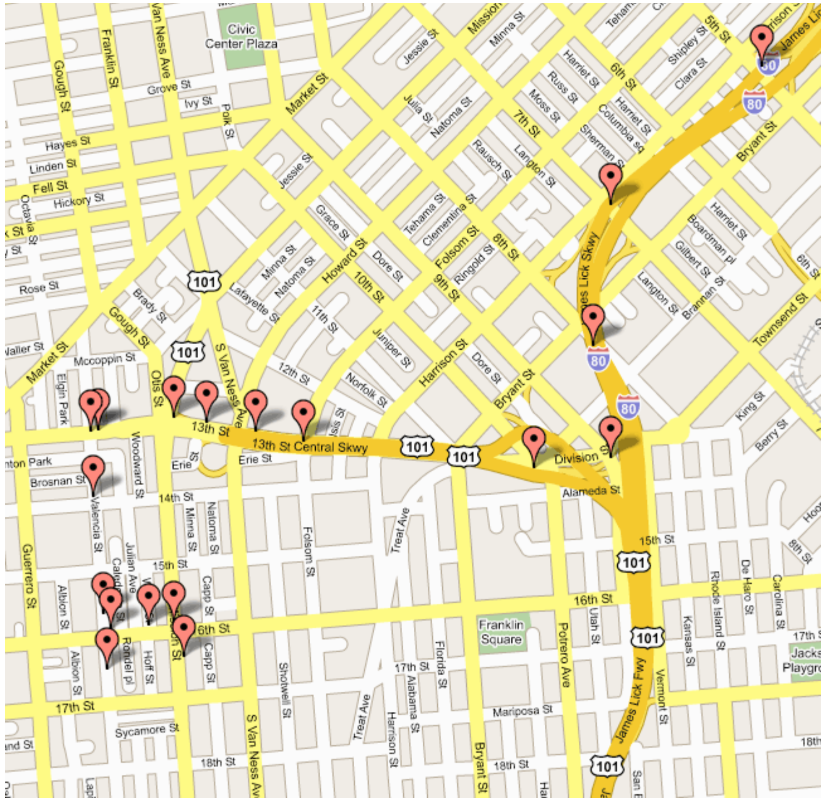
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

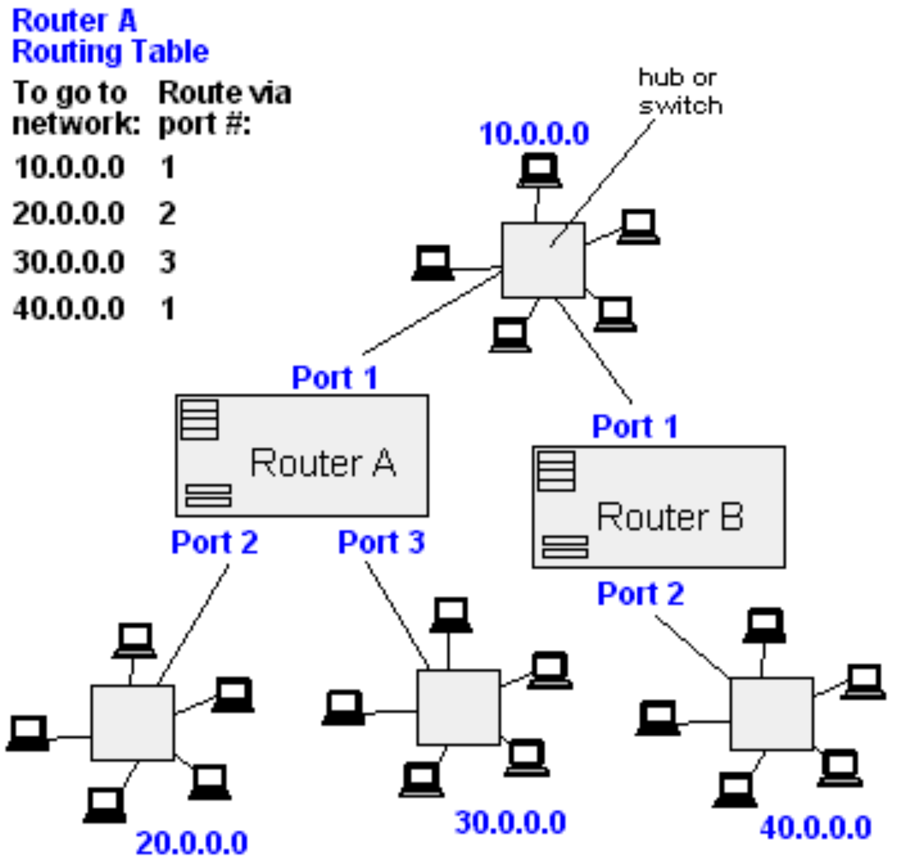


$S: \{ A, C, E, B, D \}$

APPLICATIONS OF DIJKSTRA'S ALGORITHM

- Navigation Systems
- Internet Routing

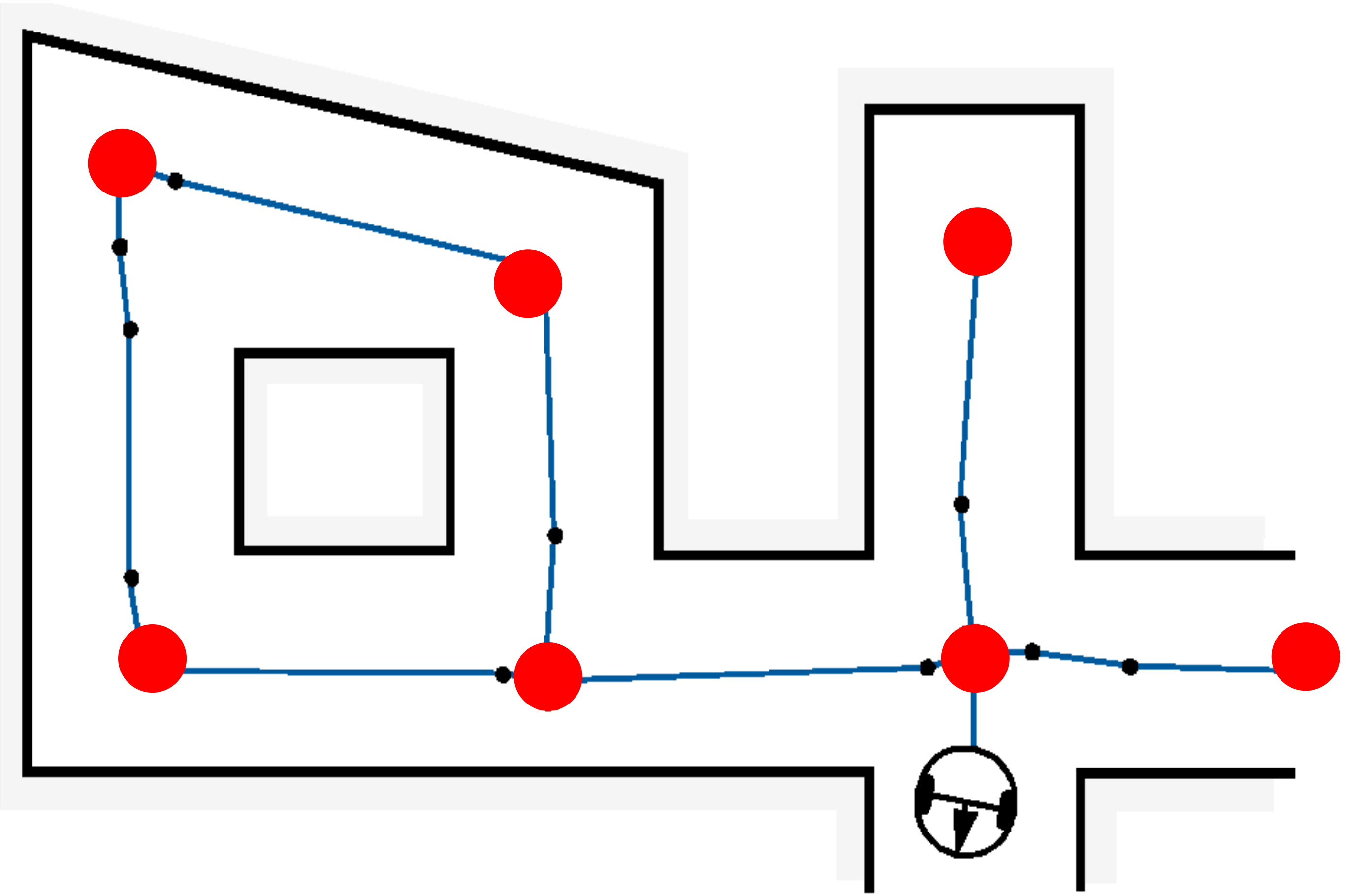




Dijkstra’s Algorithm for Path Planning: Topological Maps

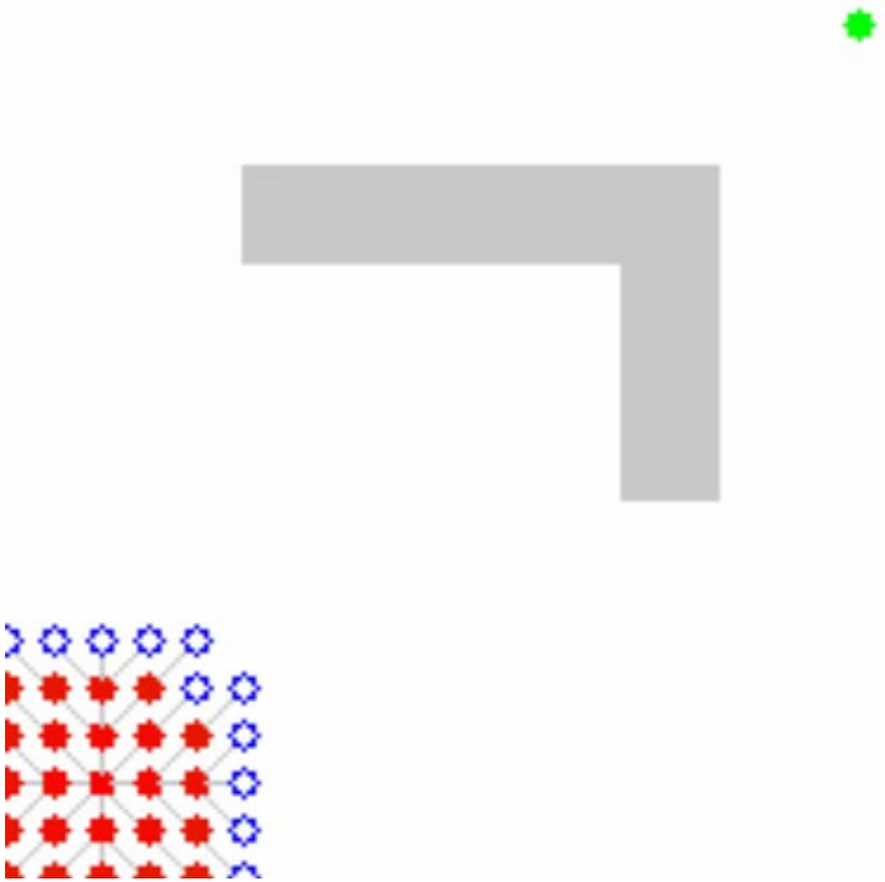
- Topological Map:
  - Places (vertices) in the environment (red dots)
  - Paths (edges) between them (blue lines)
  - Length of path = weight of edge

- => Apply Dijkstra’s Algorithm to find path from start vertex to goal vertex



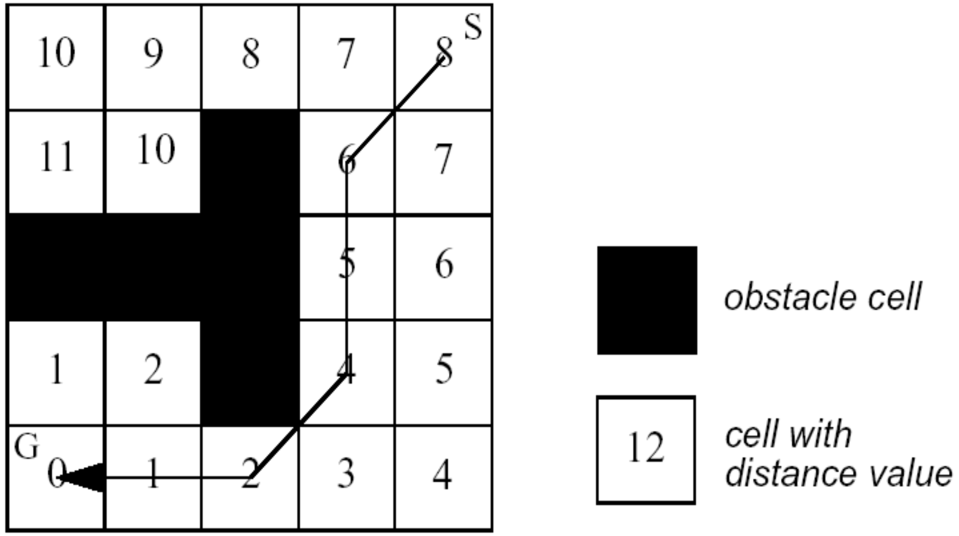
Dijkstra’s Algorithm for Path Planning: Grid Maps

- Graph:
  - Neighboring free cells are connected:
    - 4-neighborhood: up/ down/ left right
    - **8-neighborhood**: also diagonals
  - All edges have weight 1
- Stop once goal vertex is reached
- Per vertex: save edge over which the shortest distance from start was reached => Path

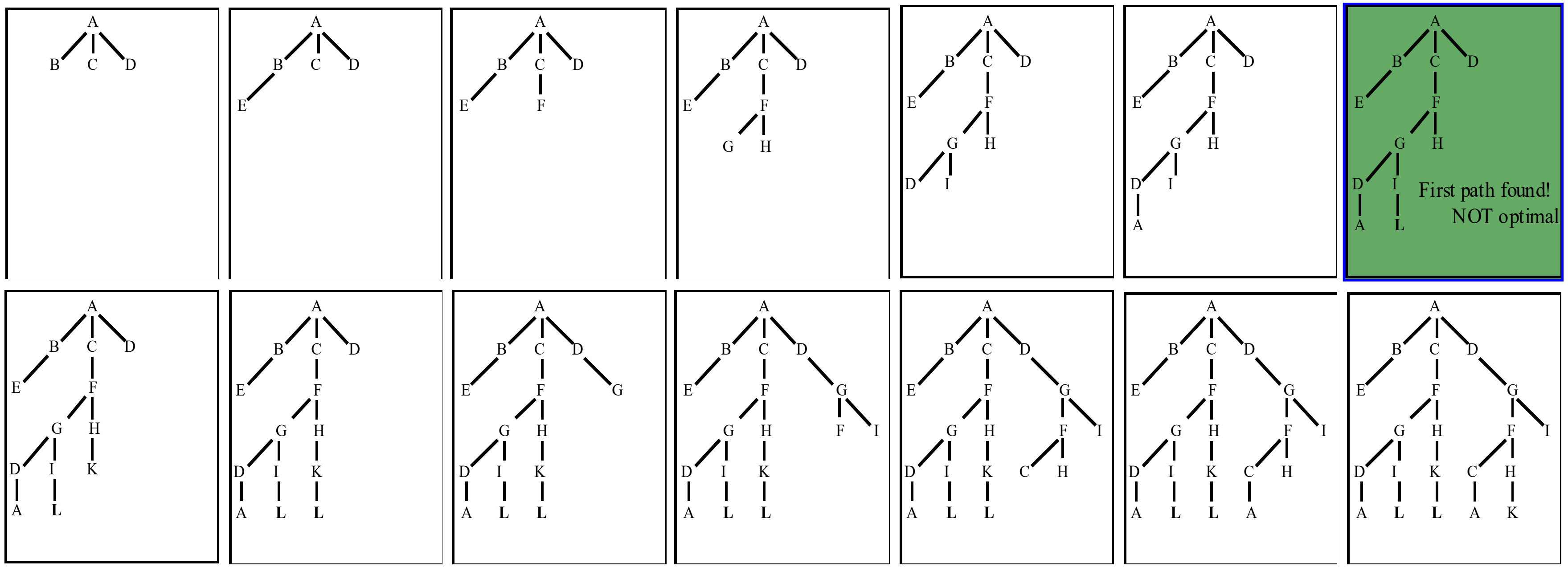


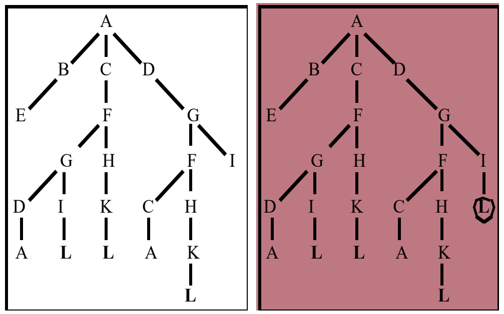
Graph Search Strategies: Breath-First Search

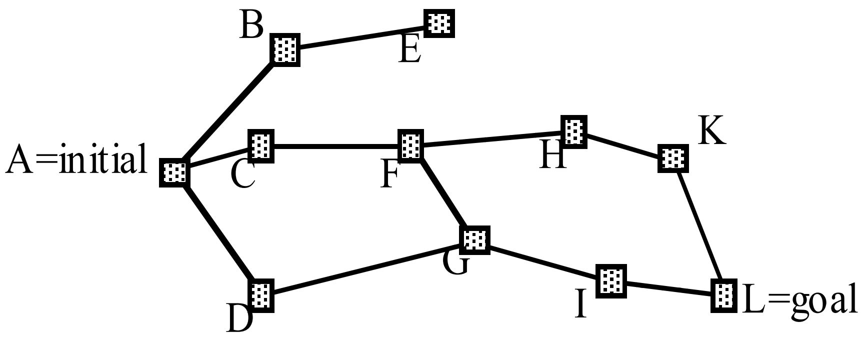
- Corresponds to a wavefront expansion on a 2D grid
- Breath-First: Dijkstra‘s search where all edges have weight 1



Graph Search Strategies: Depth-First Search

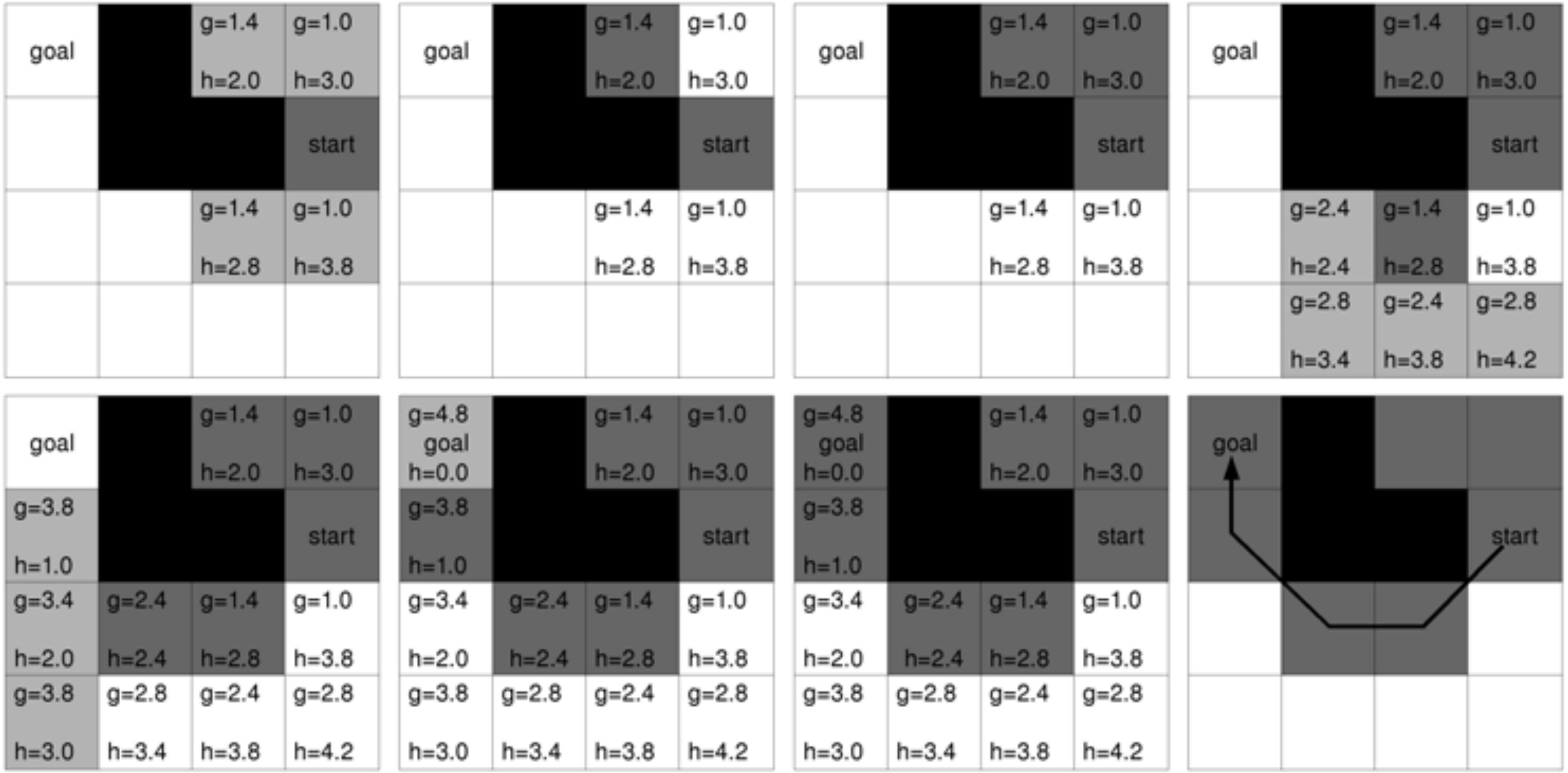






Graph Search Strategies: A* Search

- Similar to Dijkstra's algorithm, except that it uses a heuristic function h(n)
- f(n) = g(n) + h(n)



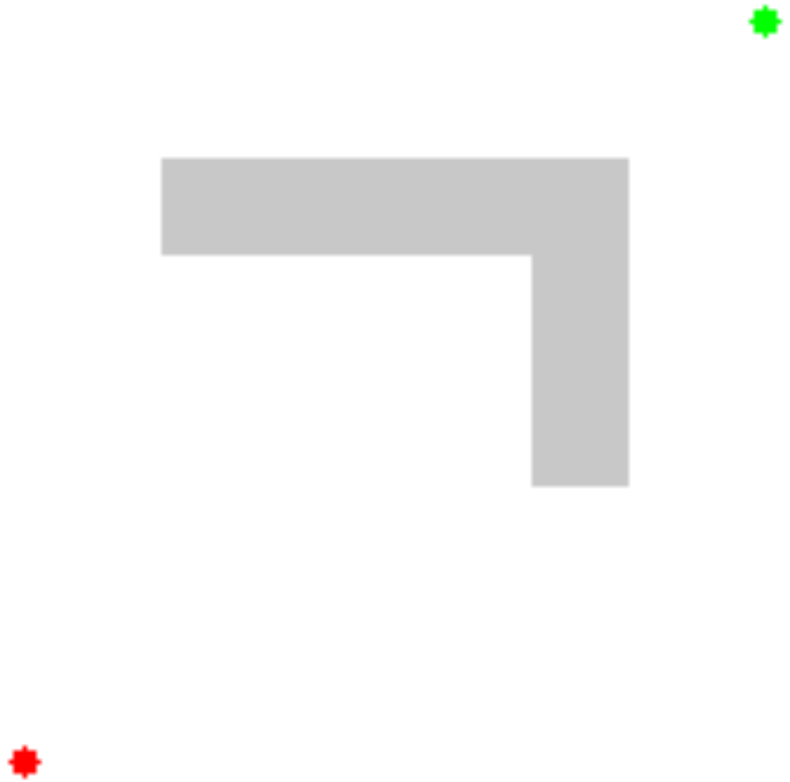
A*

- Developed 1986 as part of the Shakey project!
- Complexity:

Worst-case performance	$O(\|E\|) = O(b^d)$
Worst-case space complexity	$O(\|V\|) = O(b^d)$

b: branching factor
d: depth

- Good heuristic => small branching factor

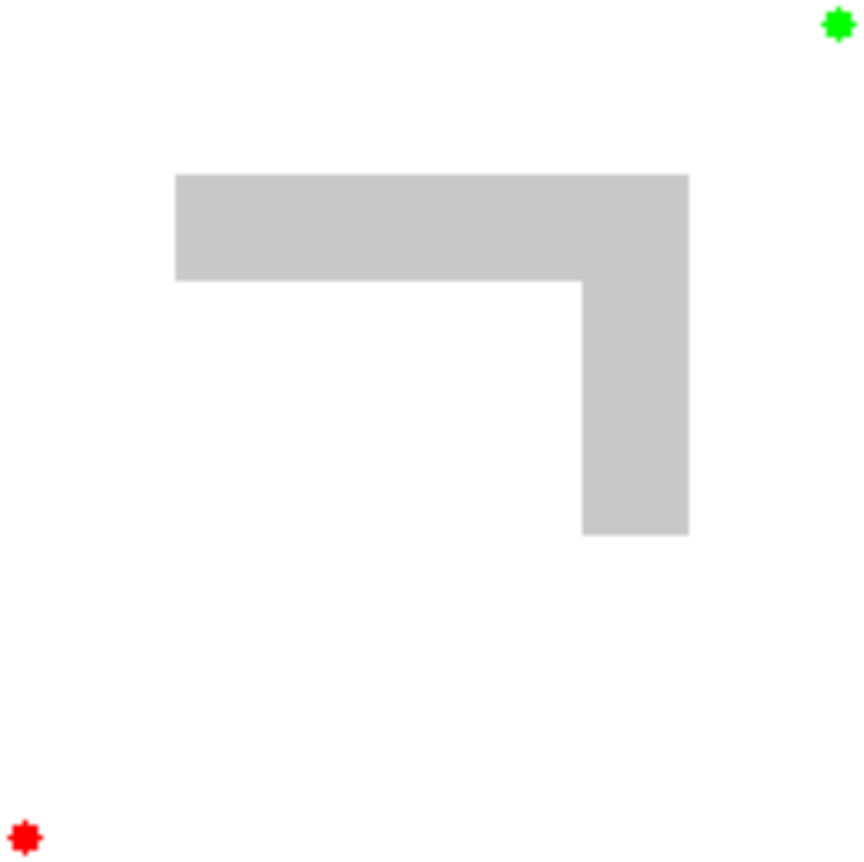


Optimal Planning

- Dijkstra finds the optimal path
- What about A*?
  - Find admissible heuristic h(n)
  - Admissible: do not overestimate the true cost-to-go
  - A* is optimal (finds optimal/ shortest path) if h(n) is admissible for all n
  - Admissible example: use geometric distance for h(n):

$$h(n) = \sqrt{(x_{goal} - x)^2 + (y_{goal} - y)^2}\ ).$$

  - Example: heuristic 5x geometric distance
  - h(n) = 0 => Dijkstra's Algorithm



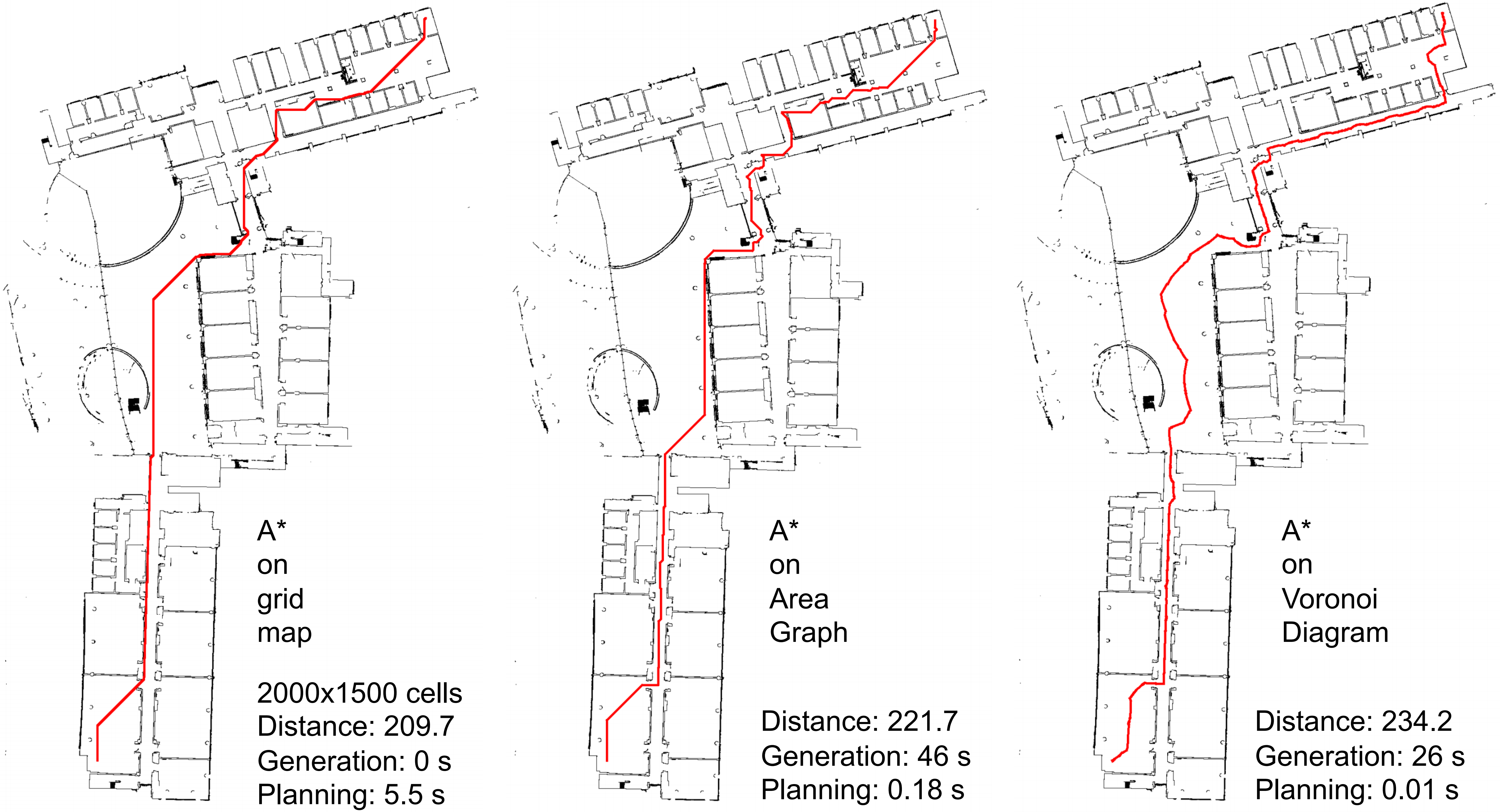
A*

- Hierarchical planning possible e.g.: Go to the library:
  - First plan how to get from SIST building to library
  - Then plan how to get from entrance of library to goal room (campus level vs. library level)

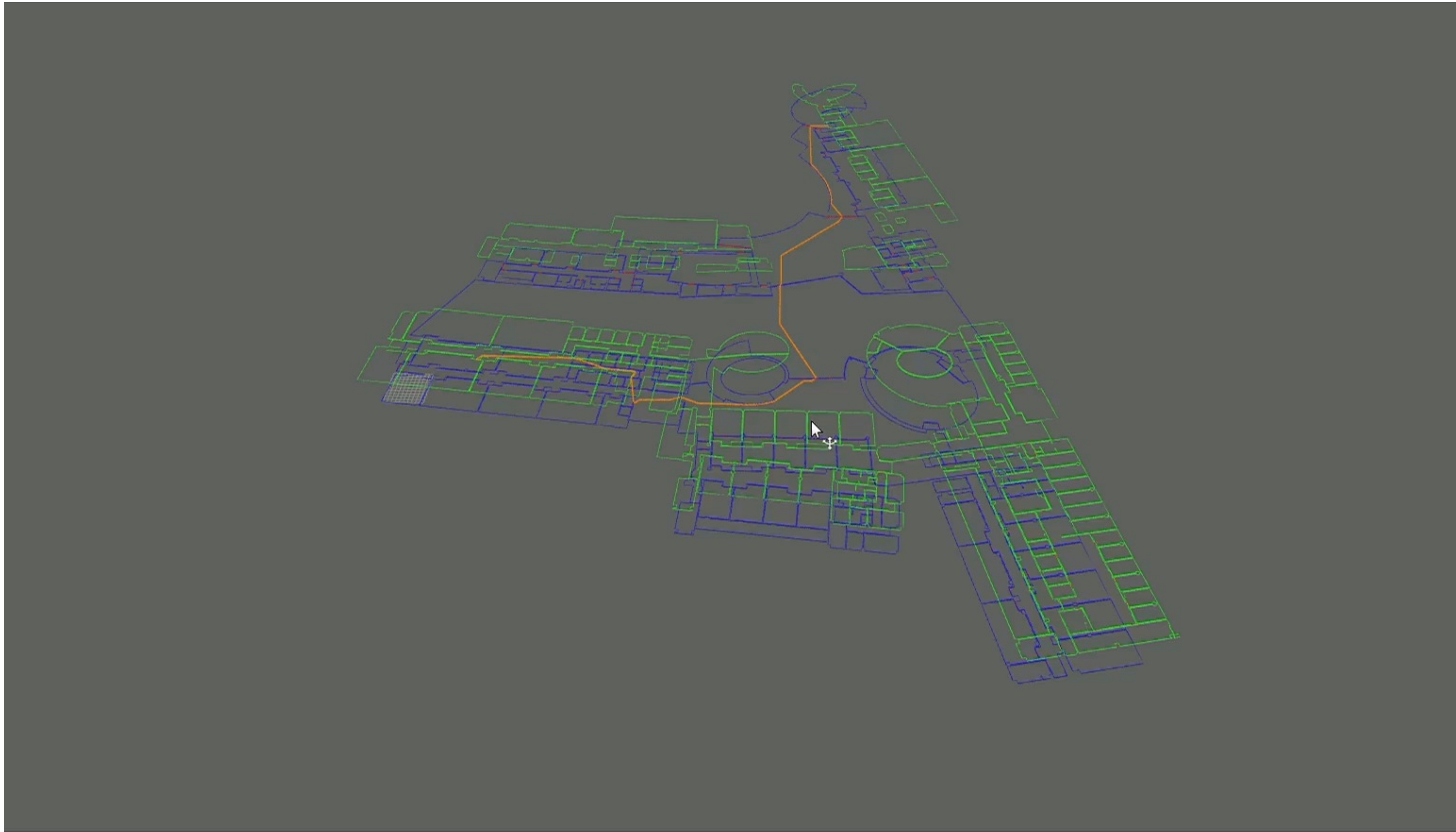
- Many variants of A* algorithms exist – with different properties

- A* as graph search: applications outside of robotics/ path planning
  - Video games
  - Parsing with stochastic grammars in natural language processing

- Graph on which planning is done matters!
  - E.g.: Grid map; Pose Graph; Topological Graph; Open Street Map; Lattice Graphs; …

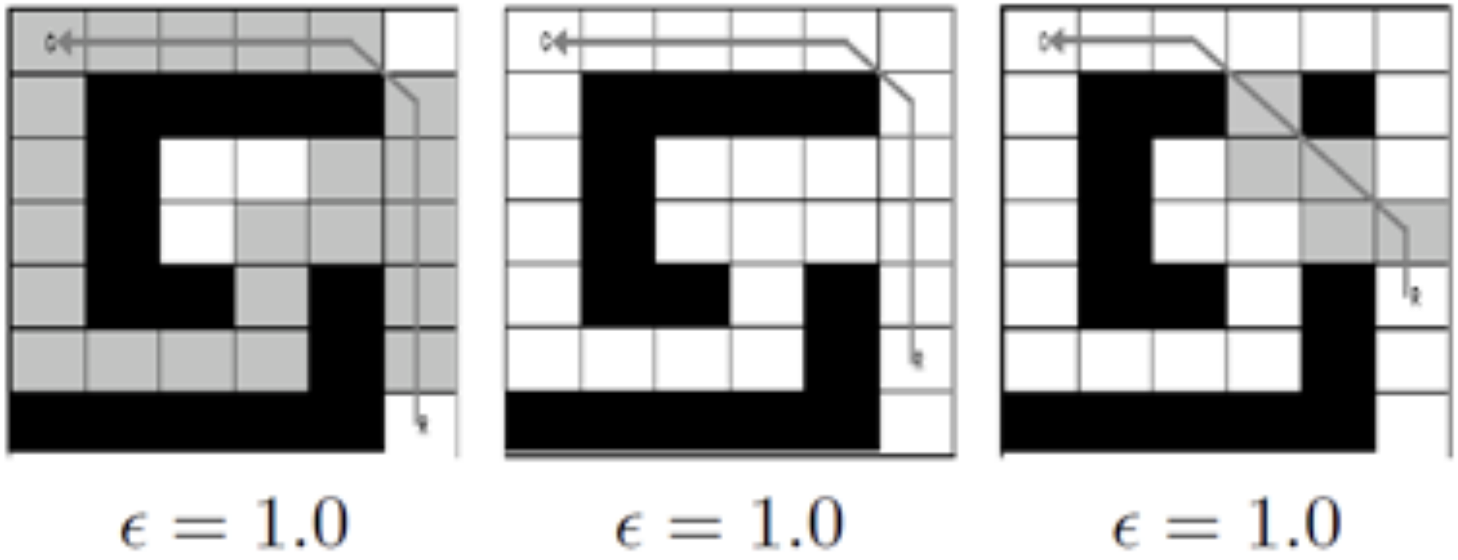

A*
on
grid
map
2000x1500 cells
Distance: 209.7
Generation: 0 s
Planning: 5.5 s
A*
on
Area
Graph
Distance: 221.7
Generation: 46 s
Planning: 0.18 s
A*
on
Voronoi
Diagram
Distance: 234.2
Generation: 26 s
Planning: 0.01 s

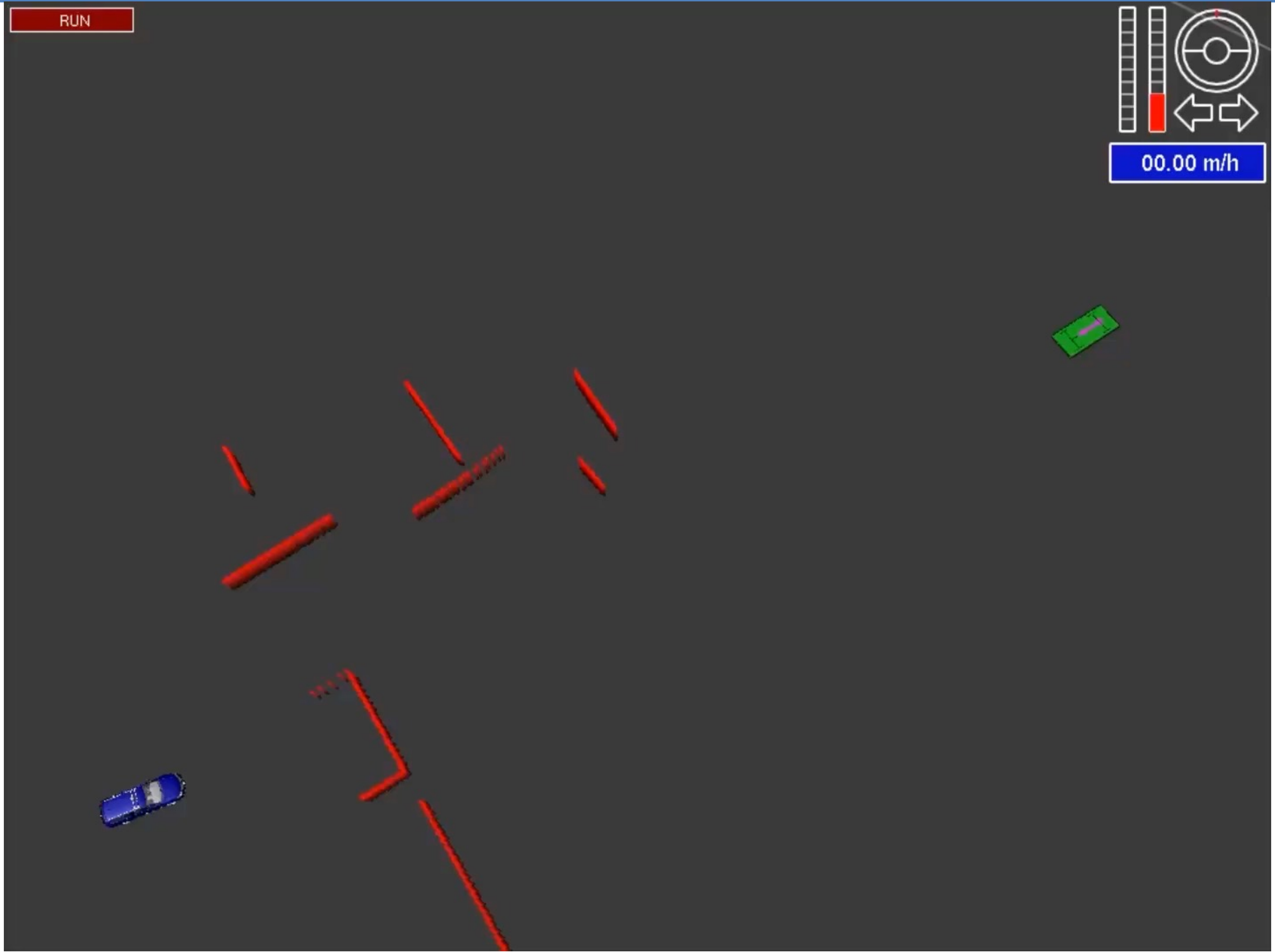
3D Path planned with A* in Area Graph of 2 stories of SIST & SEM buildings



Graph Search Strategies: D* Search

- Similar to A* search, except that the search starts from the goal outward
- f(n) = g(n) + ε h(n)
- First pass is identical to A*
- Subsequent passes reuse information from previous searches

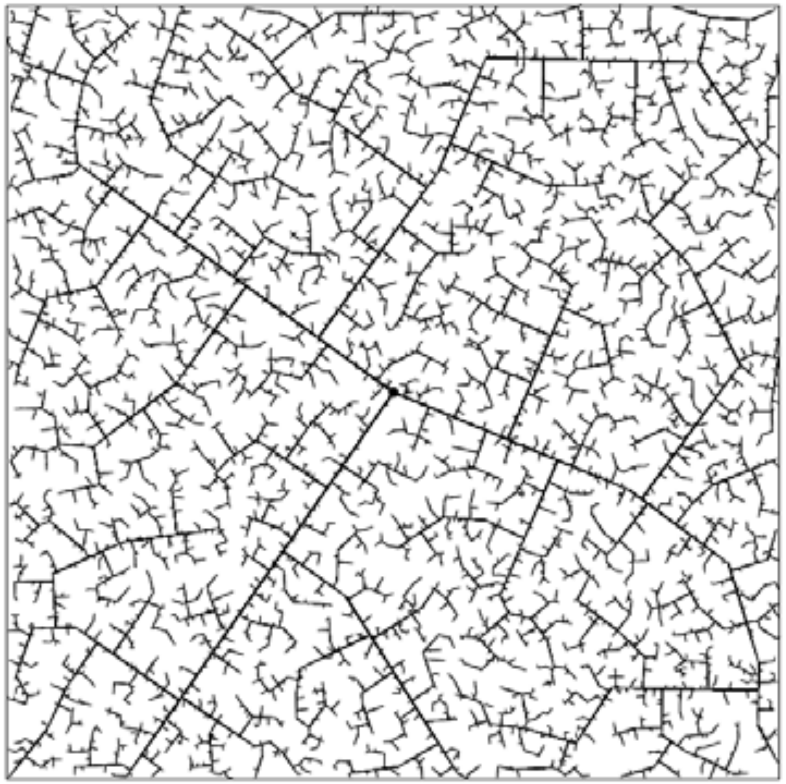



RUN
00.00 m/h

Graph Search Strategies: Randomized Search

- Most popular version is the rapidly exploring random tree (RRT)
  - Well suited for high-dimensional search spaces
  - Often produces highly suboptimal solutions

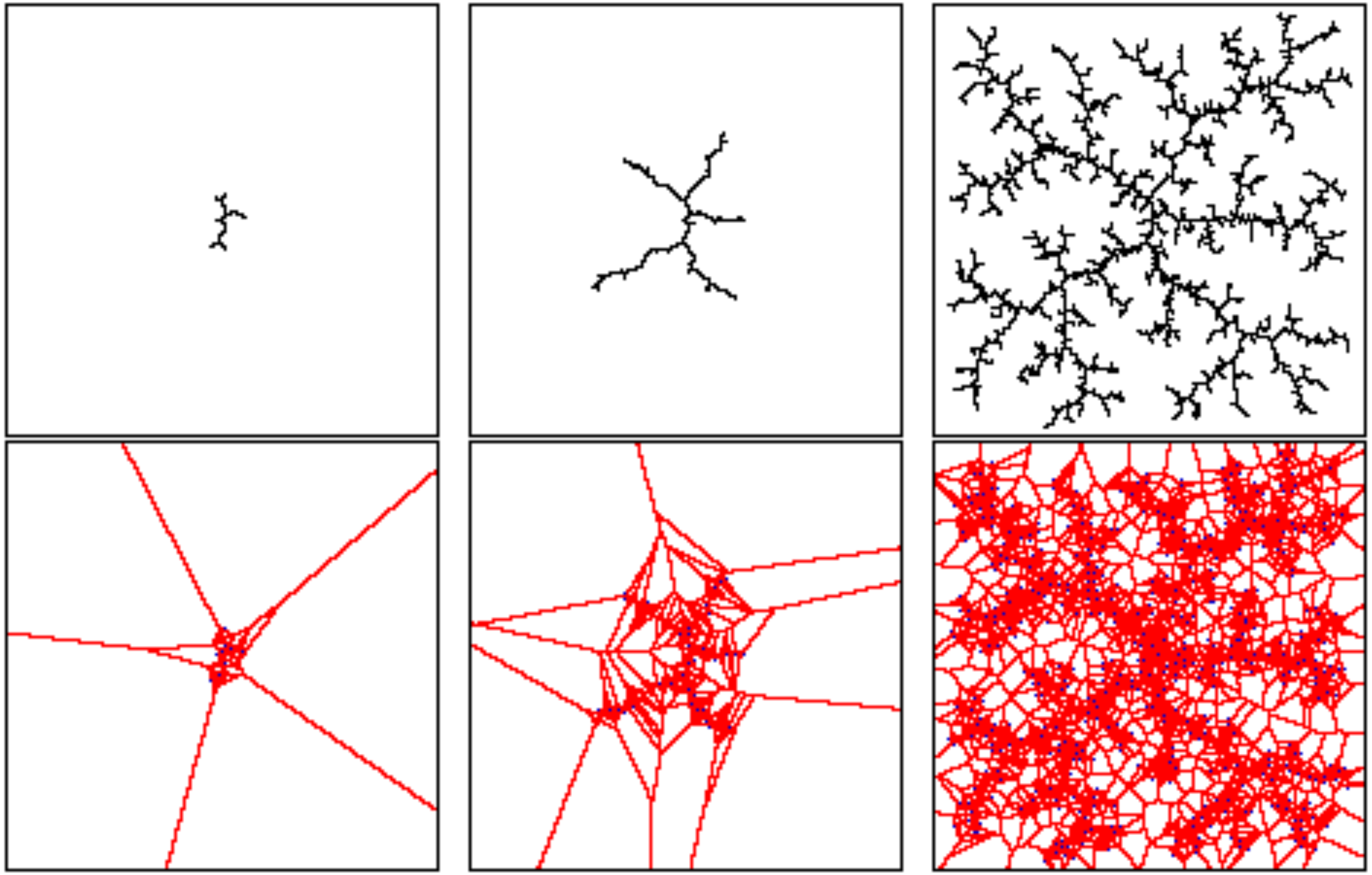
45 iterations

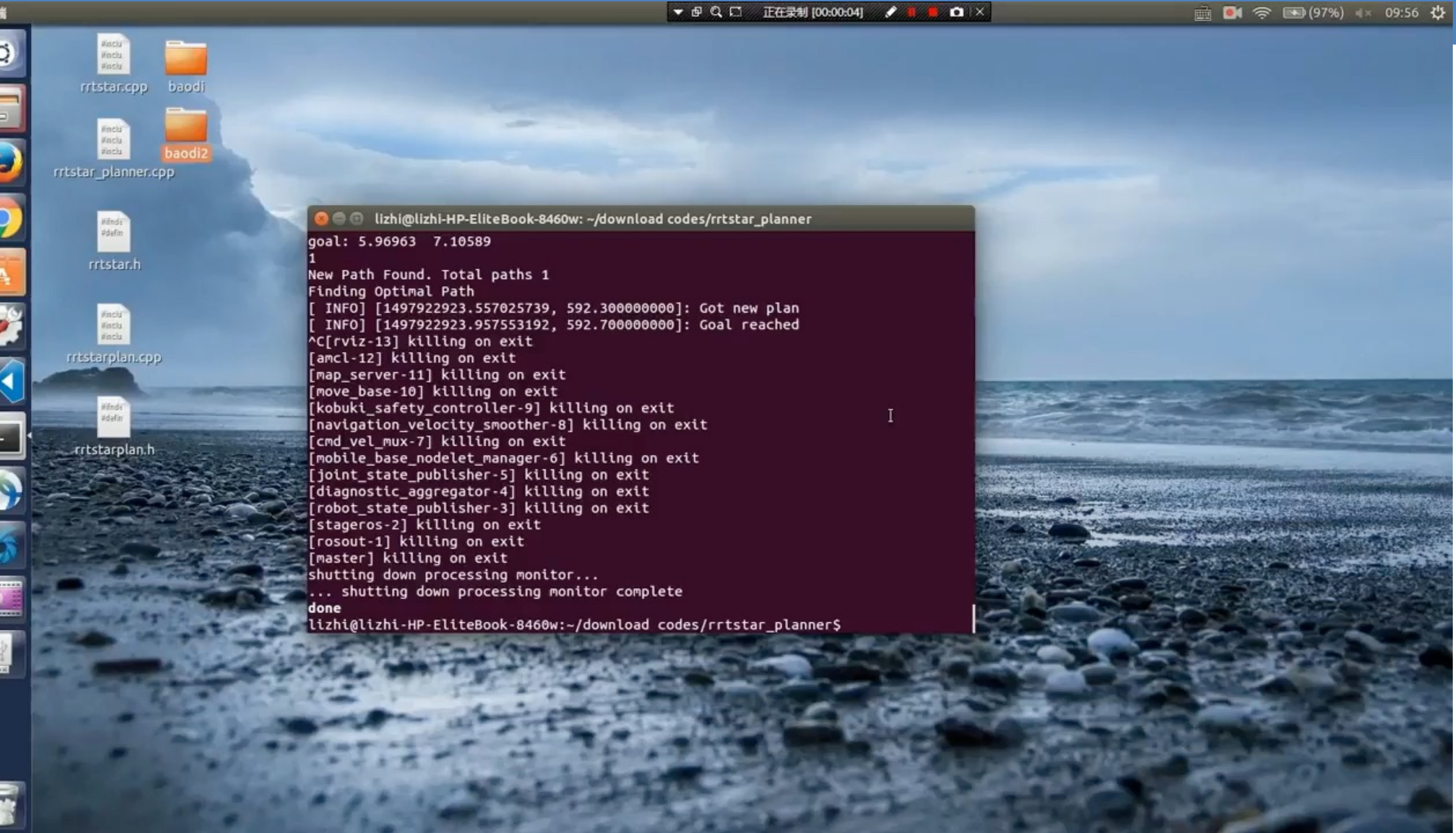


2345 iterations

Why are RRT's rapidly exploring?

The probability of a node to be selected for expansion is proportional to the area of its Voronoi region



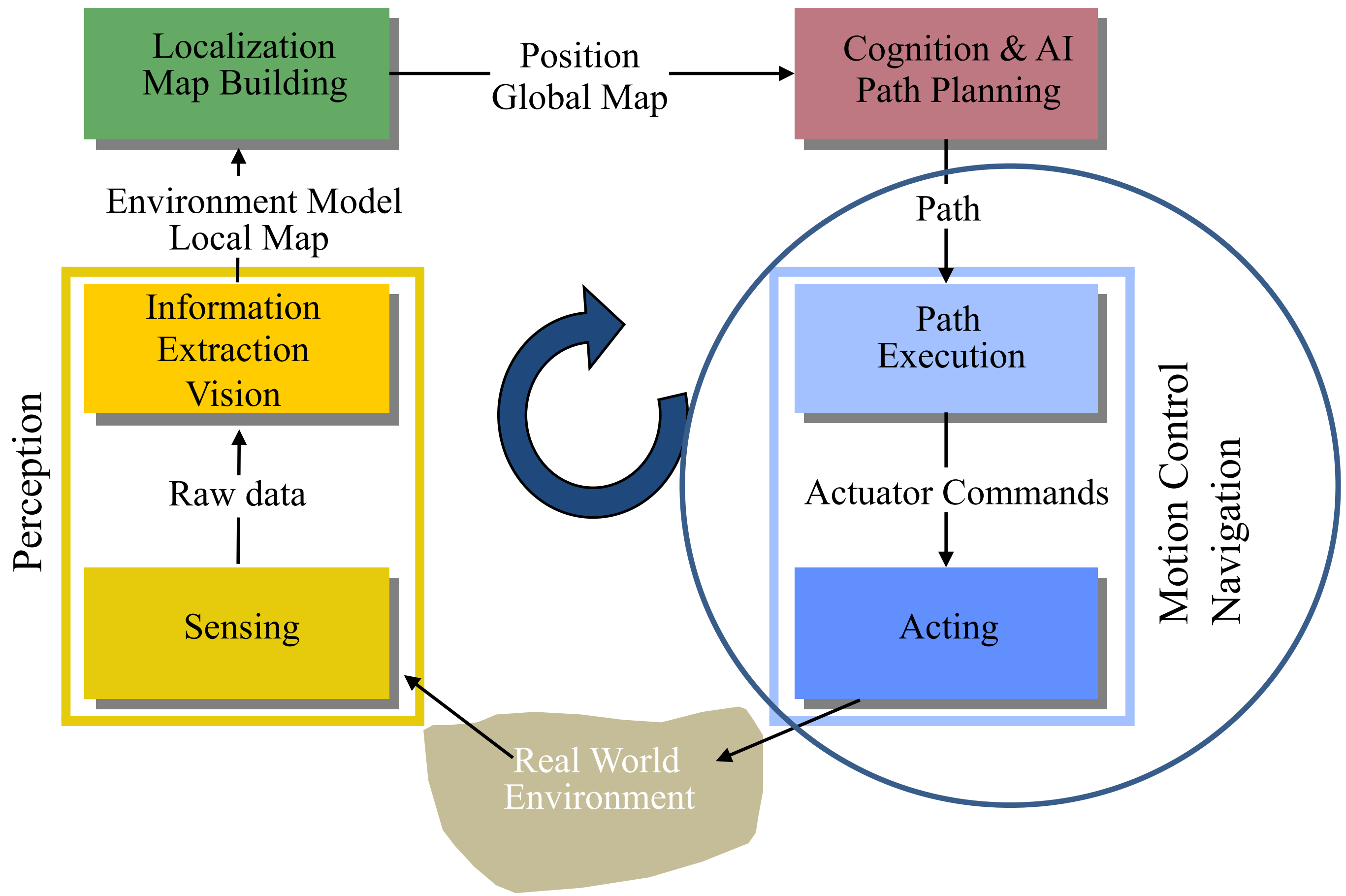

正在录制 [00:00:04]
(97%)
09:56
rrtstar.cpp
baodi
rrtstar_planner.cpp
baodi2
rrtstar.h
rrtstarplan.cpp
rrtstarplan.h
lizhi@lizhi-HP-EliteBook-8460w: ~/download codes/rrtstar_planner
goal: 5.96963 7.10589
1
New Path Found. Total paths 1
Finding Optimal Path
[INFO] [1497922923.557025739, 592.300000000]: Got new plan
[INFO] [1497922923.957553192, 592.700000000]: Goal reached
^C[rviz-13] killing on exit
[amcl-12] killing on exit
[map_server-11] killing on exit
[move_base-10] killing on exit
[kobuki_safety_controller-9] killing on exit
[navigation_velocity_smoother-8] killing on exit
[cmd_vel_mux-7] killing on exit
[mobile_base_nodelet_manager-6] killing on exit
[joint_state_publisher-5] killing on exit
[diagnostic_aggregator-4] killing on exit
[robot_state_publisher-3] killing on exit
[stageros-2] killing on exit
[rosout-1] killing on exit
[master] killing on exit
shutting down processing monitor...
... shutting down processing monitor complete
done
lizhi@lizhi-HP-EliteBook-8460w:~/download codes/rrtstar_planner$

teb_local_planner

An optimal trajectory planner for mobile robots based on Timed-Elastic-Bands

http://wiki.ros.org/teb_local_planner

General Control Scheme for Mobile Robot Systems



This lecture: Control

With material from Roland Siegwart and Davide Scaramuzza, ETH Zurich

- Autonomous mobile robots move around in the environment. Therefore **ALL** of them:
  - They need to know **where** they **are**.
  - They need to know **where** their **goal** is.
  - <u>They need to know **how** to get there.</u>

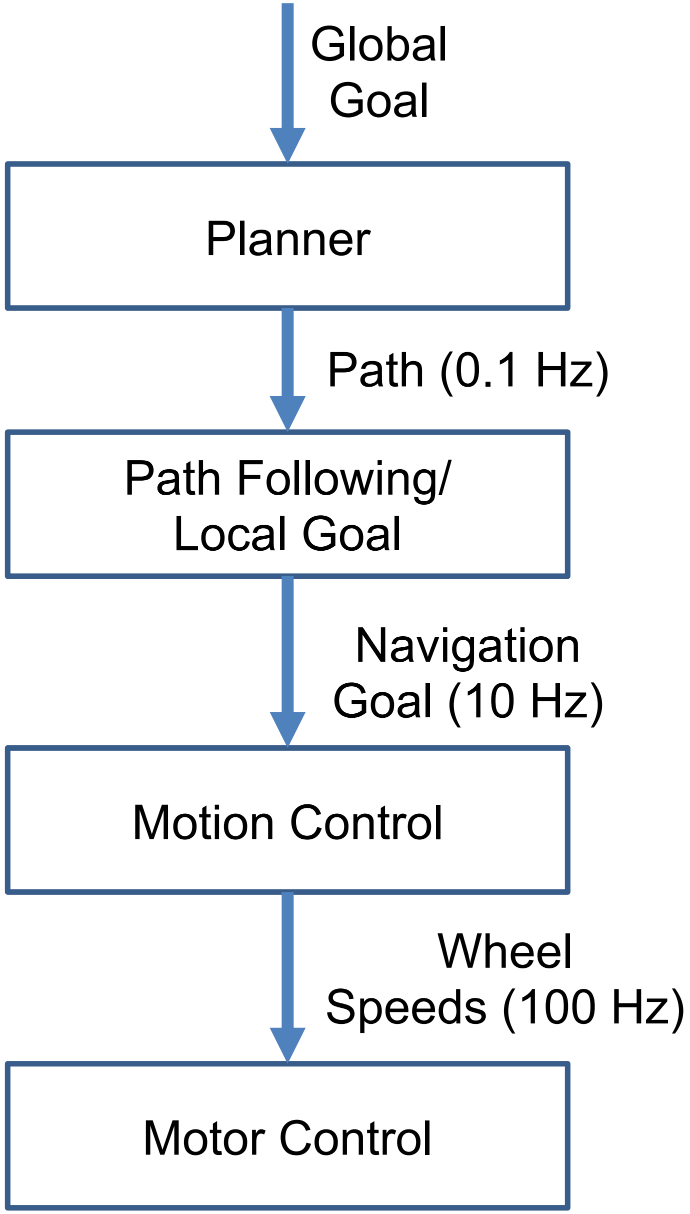
- Different levels:
  - Control:
    - How much power to the motors to move in that direction, reach desired speed
  - Navigation:
    - Avoid obstacles
    - Classify the terrain in front of you
    - Predict the behavior (motion) of other agents (humans, robots, animals, machines)
  - Planning:
    - Long distance path planning
    - What is the way, optimize for certain parameters

Navigation, Motion & Motor Control

- Navigation/ Motion Control:
  - Where to drive to **next** in order to reach goal
  - Output: motion vector (direction) and speed
  - For example:
    - follow path (Big Model)
    - go to unexplored area (Big Model)
    - drive forward (Small Model)
    - be attracted to goal area (Small Model)

- Motion Control:
  - How use propulsion to achieve motion vector

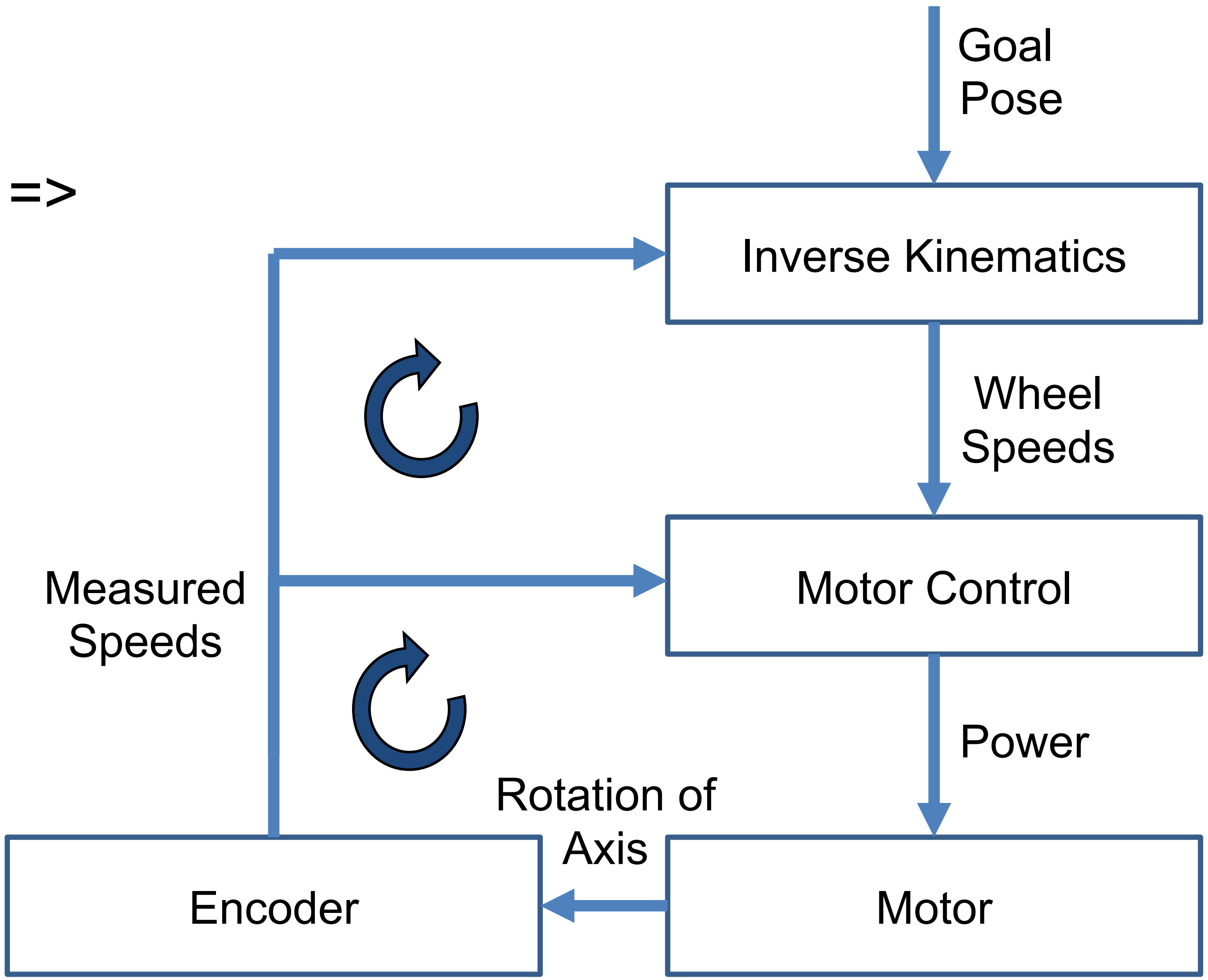
- Motor Control:
  - How much power to achieve propulsion (wheel speed)



Control Hierarchy

- Assume we have a goal pose (close by)
- Calculate Inverse Kinematics =>
- Desired wheel speeds
  - Typically not just one wheel =>
  - Many motor controllers, motors, encoders

- Motor control loop
- Pose control loop



ROS2 Navigation 2

navigation.ros.org/

More infos next class!