上海科技大学
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CS283: **Robotics Spring 2024: Vision**

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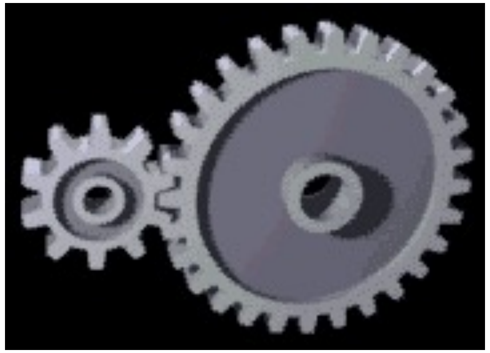
GEARS

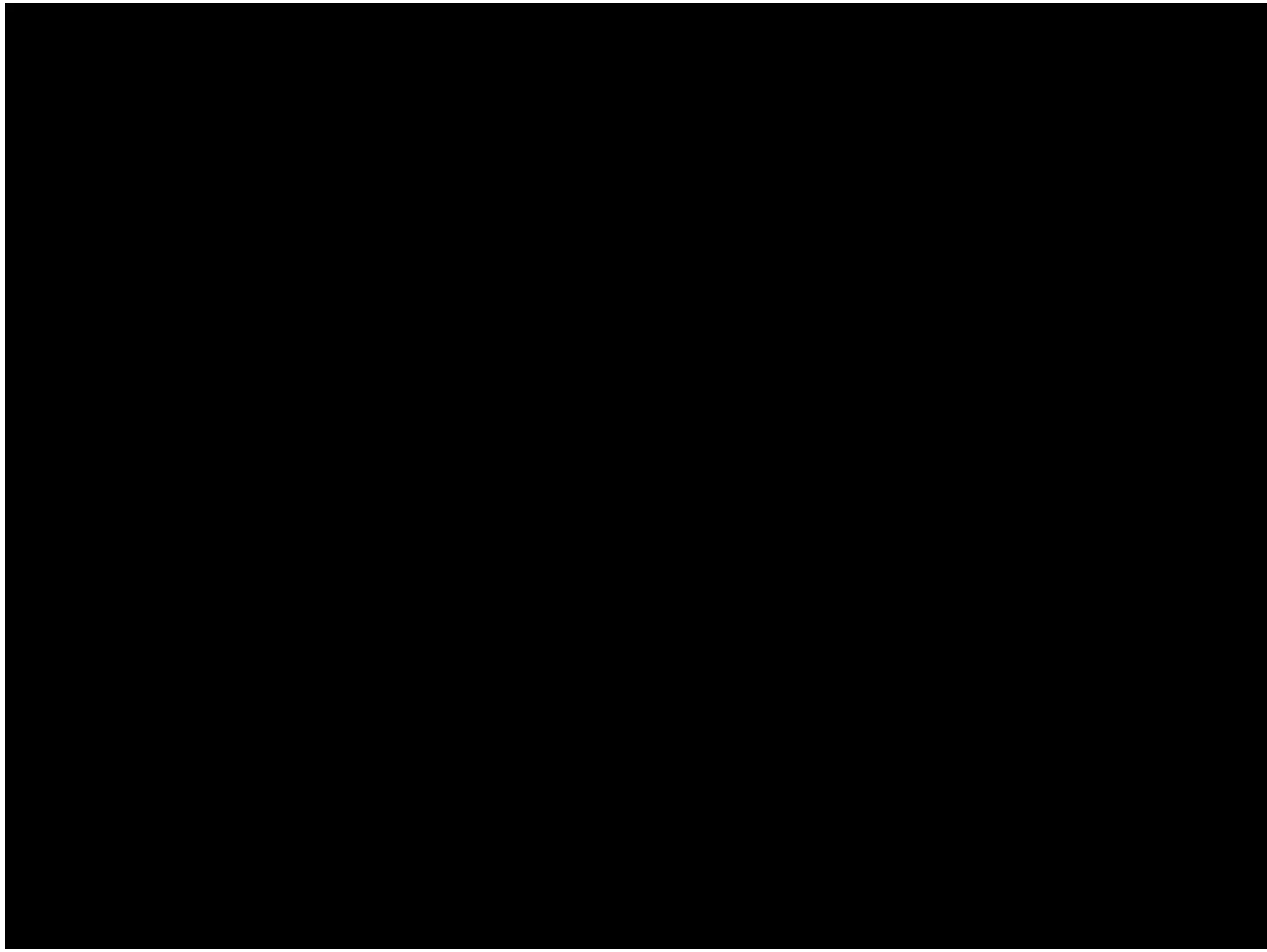
Gears

- Trade speed for torque
- See previous characteristic of DC motor: efficiency highest at high speeds
- Robotics: needs HIGH torque:
  - Inertia of mobile robot (high mass!)
  - Driving uphill
  - Robot arm: lift mass (object and robot arm) at long distances (lever!) – gravity!
- Most important property: Number of teeth => Gear Ratio $= \frac{DrivenGearTeeth}{DriveGearTeeth}$
- Torque = Motor Torque * Gear Ratio
- Speed = Motor Speed / Gear Ratio
- Teeth have same size =>
  gear diameter proportional to Number of teeth…



Gears

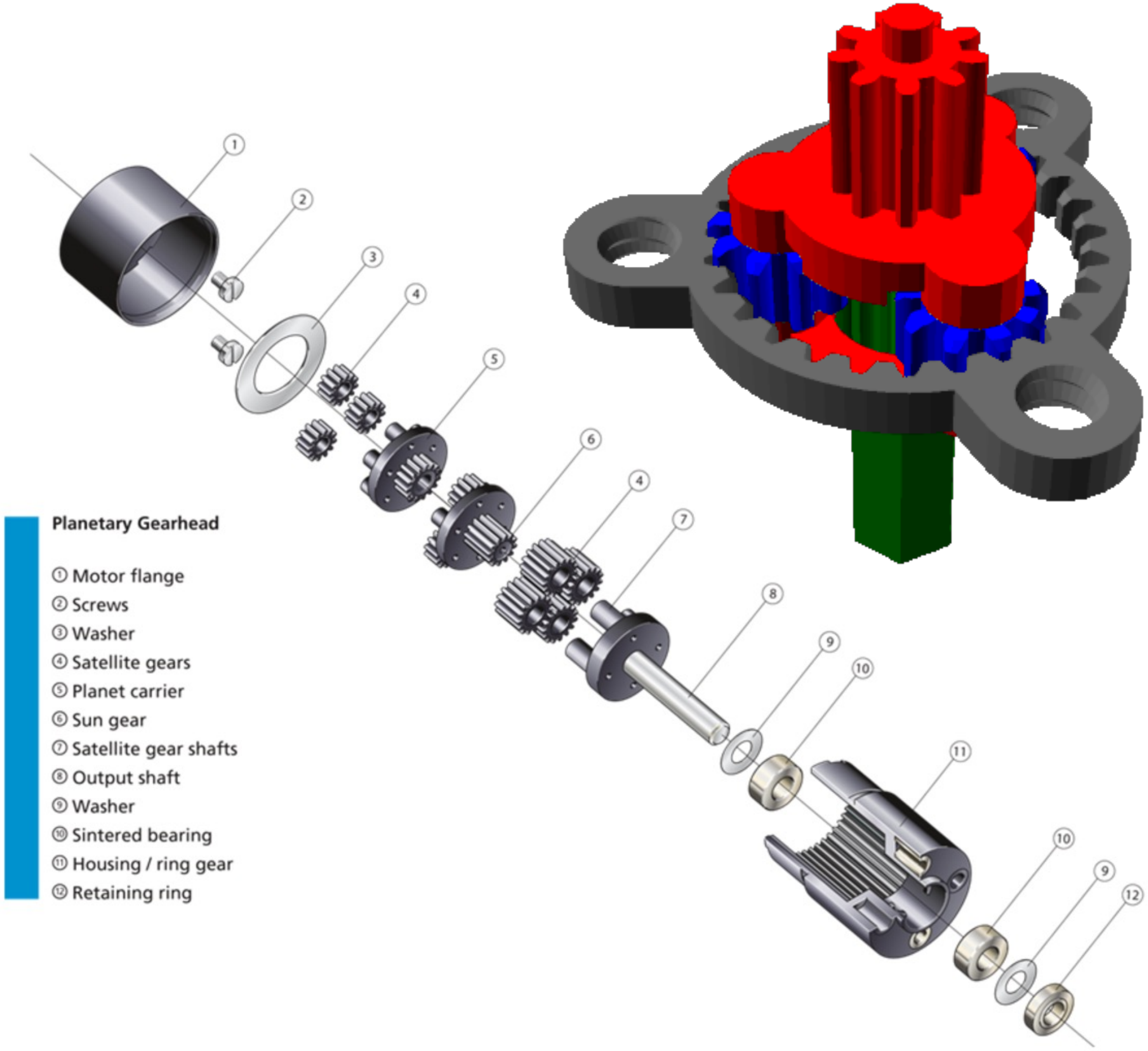
- Must be well designed to provide constant force transmission
  - Low wear/ low noise
- Back drivable: Can the wheel move the motor?
- Spur Gear reverses rotation direction!
- Backlash: when reversing direction: short moment of no force transmission => error in position estimate of wheel!



https://www.youtube.com/watch?v=8s4zm_ajxAA

Planetary Gear

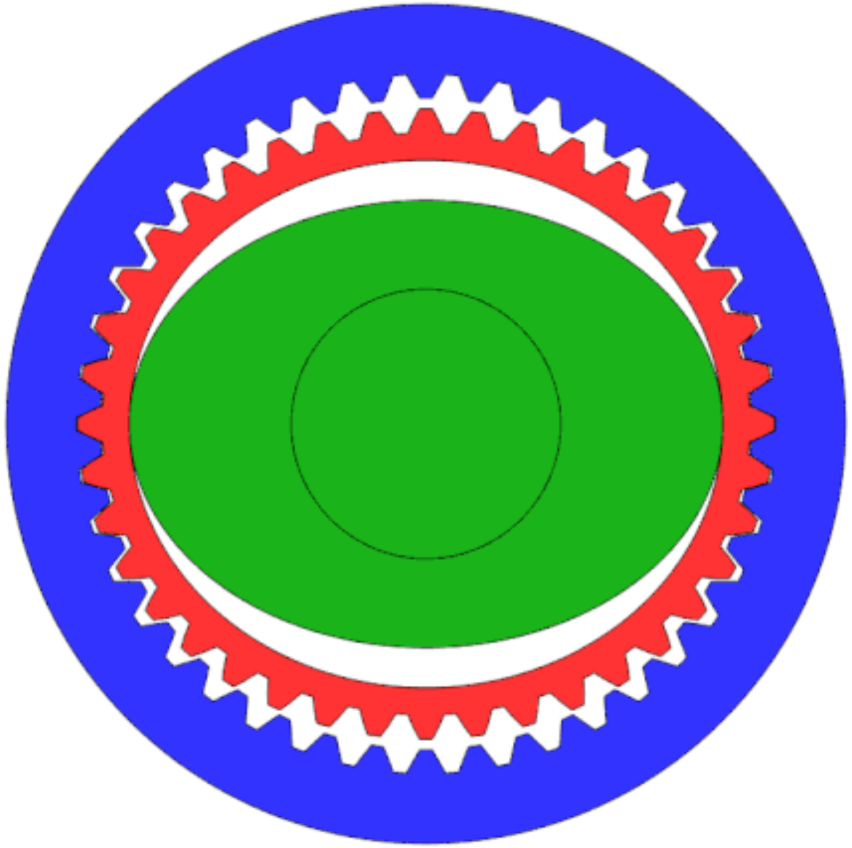
- Aka epicyclic gear train
- Quite common!
- Ratios: 3:1 … 1526:1
- Typical setup:
  - Sun (green) to motor
  - Carrier (red) output
  - Planets (blue): support
  - Ring (black): constraints the planets
  - => Ratio = 1:(1 + N_{Ring}/N_{Sun})



Harmonic Drive

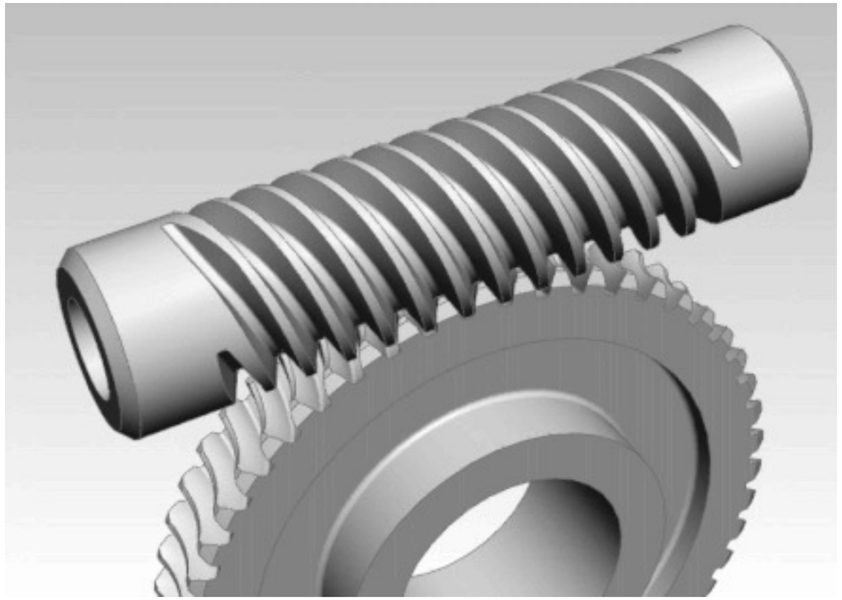
- High reduction in small volume (30:1 to 320:1)
- No backlash
- Light weight

- Used in robotics,
  e.g. robotic arms
  (e.g. our Schunk arm!)



$$\text{reduction ratio} = \frac{\text{flex spline teeth} - \text{circular spline teeth}}{\text{flex spline teeth}}$$

More Gears



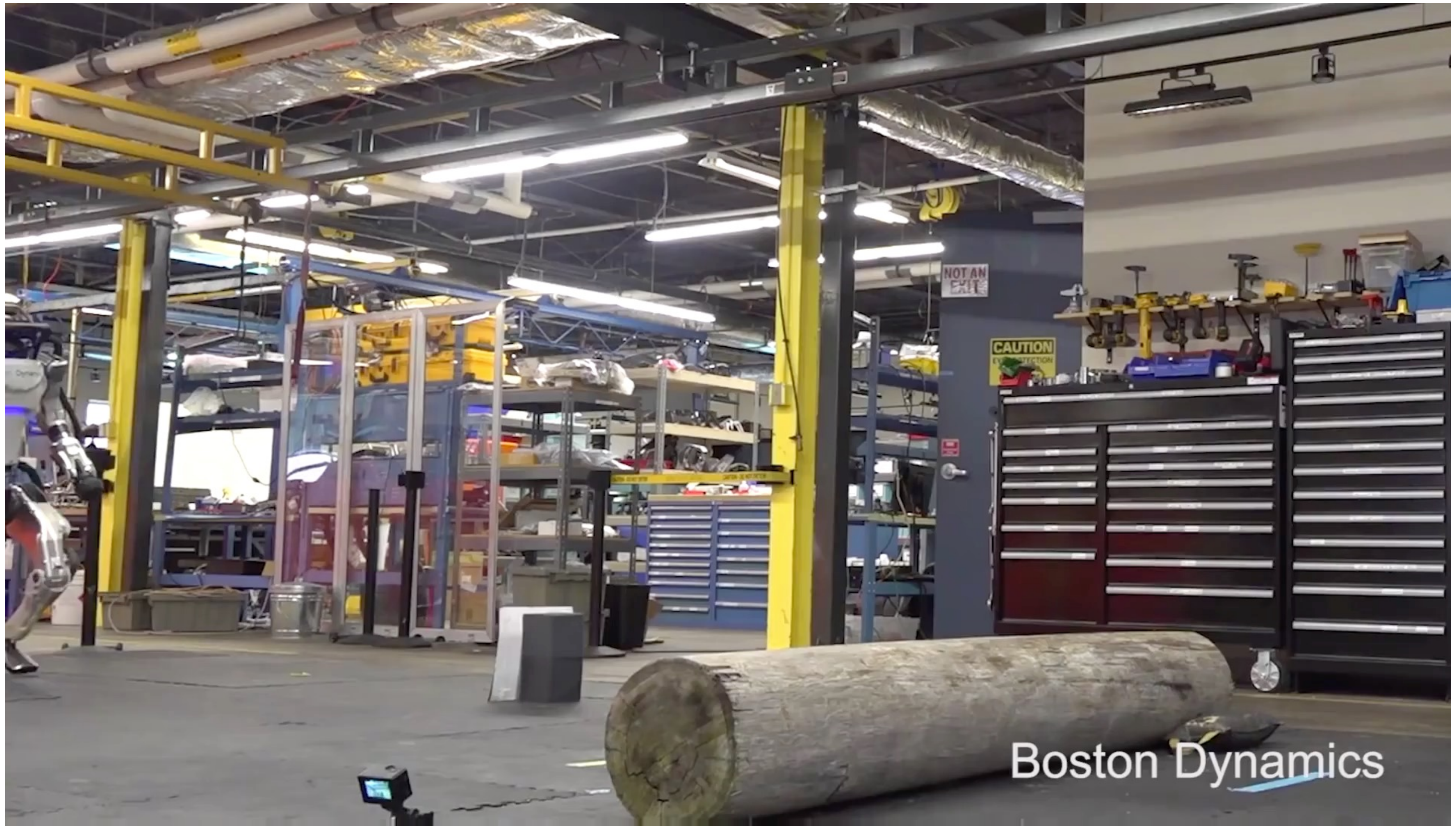
- Rack and pinion
  - linear drive
- Worm drive
  - Very high torque
  - Ratio: N_{Wheel} : 1
  - Locking (not back-drivable) gear)
- Bevel gear
  - Mainly to change direction

Summary: Mechatronics of Controlling a Wheel

1. Use Encoder and PID to control the speed of the motor
2. Send PWM signals to Motor Driver using dedicated circuits in CPU
3. Use Motor Driver to send high voltage & high current to motor
4. Use DC Brushed or Brushless motor to drive gear
5. Use Gear to get the required torque/ speed
6. Connect wheel to gear

ALTERNATIVES

Hydraulics



- 28 Hydraulic actuated joints
- Why?
  - Compact actuators with high torque – do not get hot!
  - Low mass
  - One central, highly efficient motor to pressurize the hydraulic fluid

- Actuation controlled via controlling valves

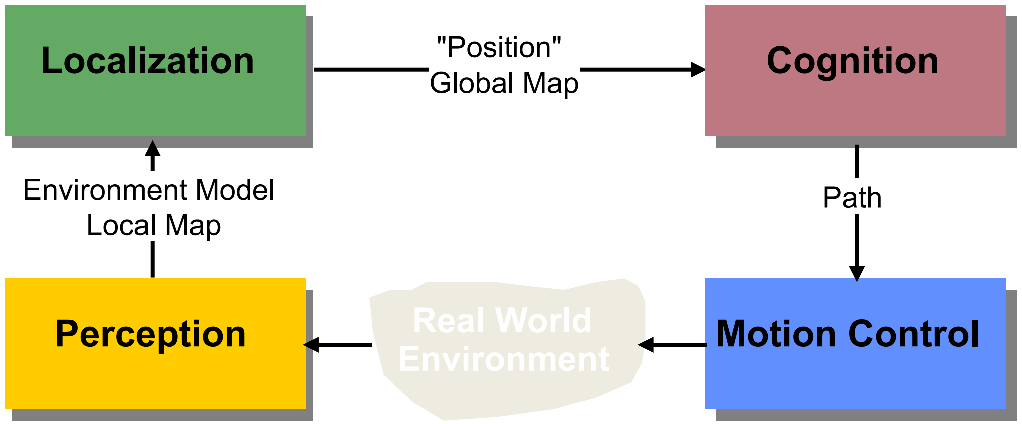
Synthetic Muscles

- Electroactive polymer: Apply voltage => change shape by 30% OR: …

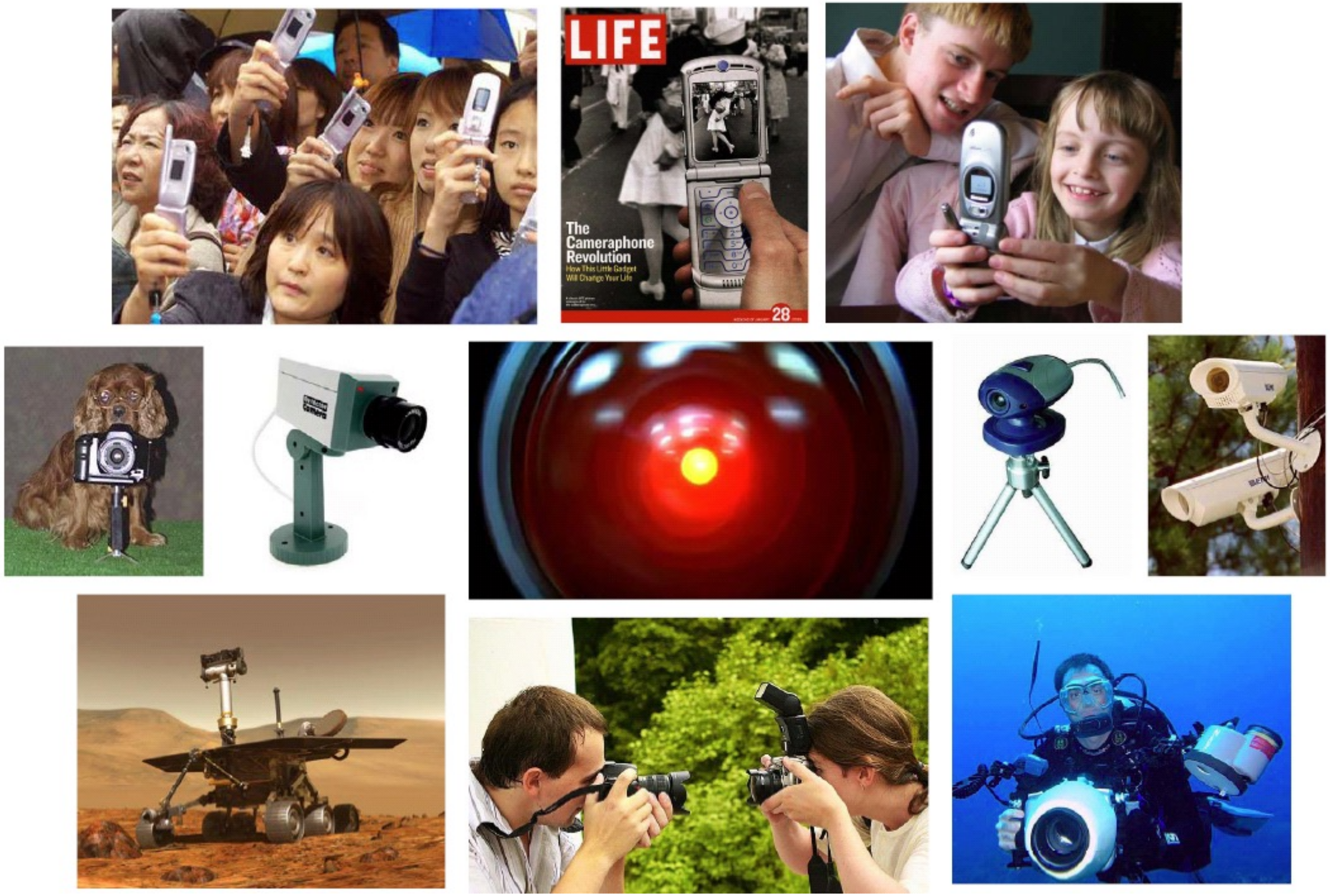
Others

- Piezoelectric actuation
  - Small motions only
  - Very fast and precise
- Pneumatic actuator
  - Uses compressible gas
- Thermal-driven actuation

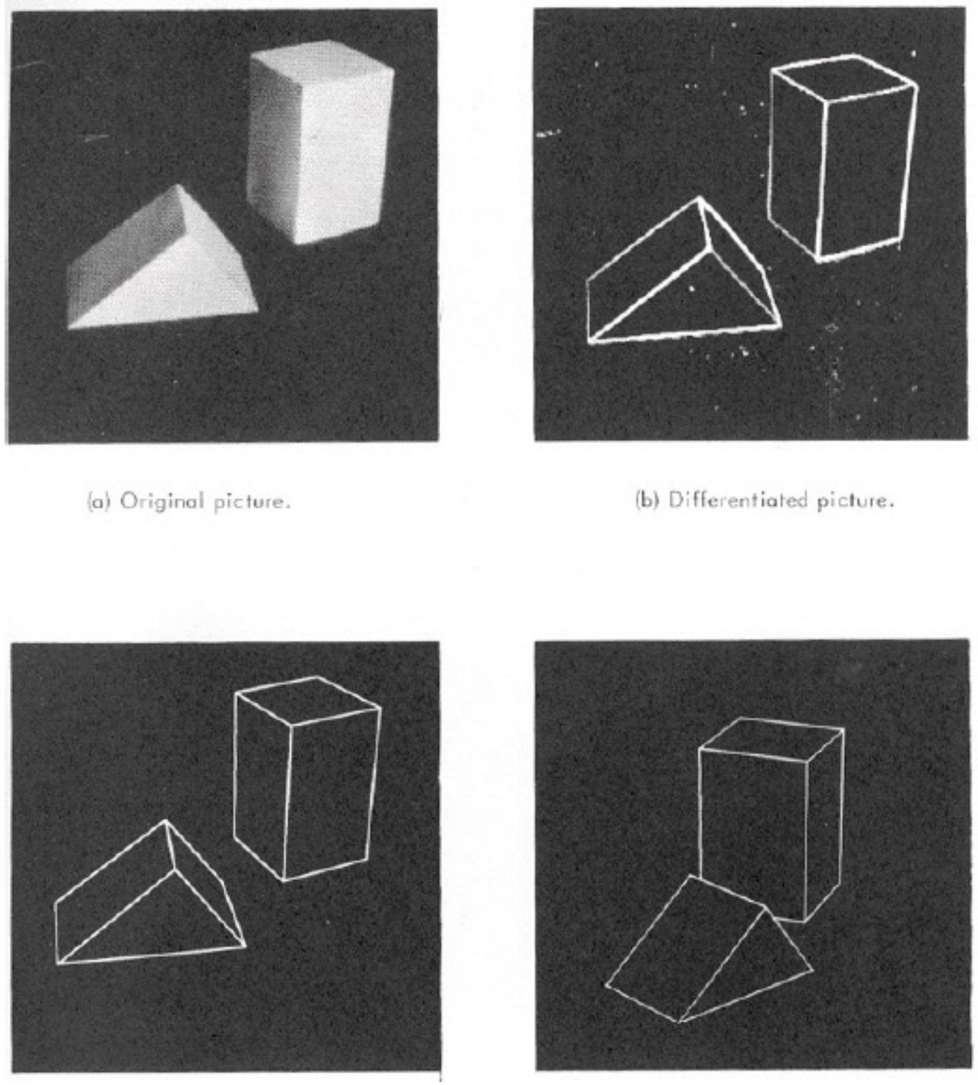
VISION


Localization
"Position"
Global Map
Cognition
Environment Model
Local Map
Path
Perception
Real World
Environment
Motion Control

Computer Vision

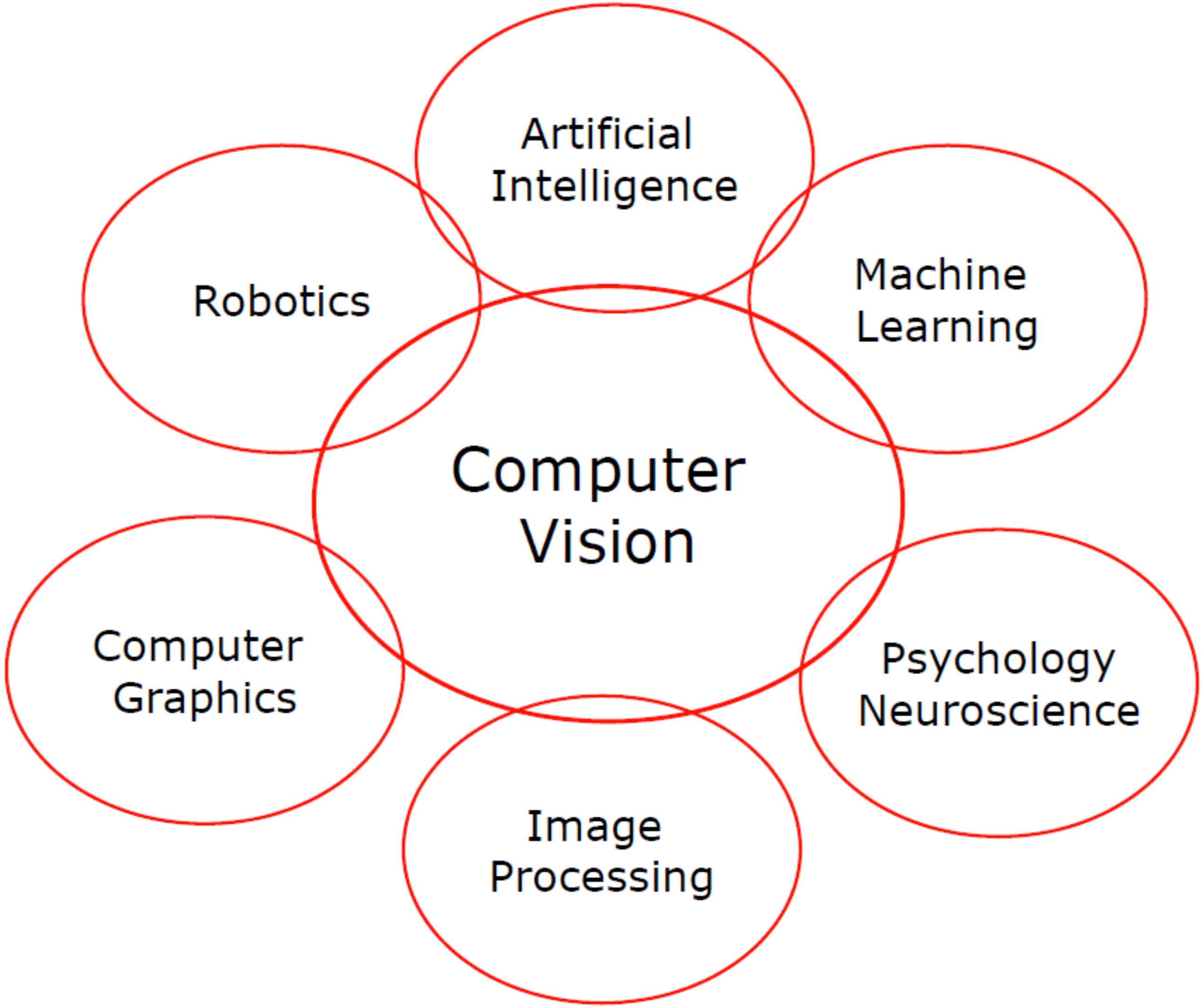


Origins of Computer Vision



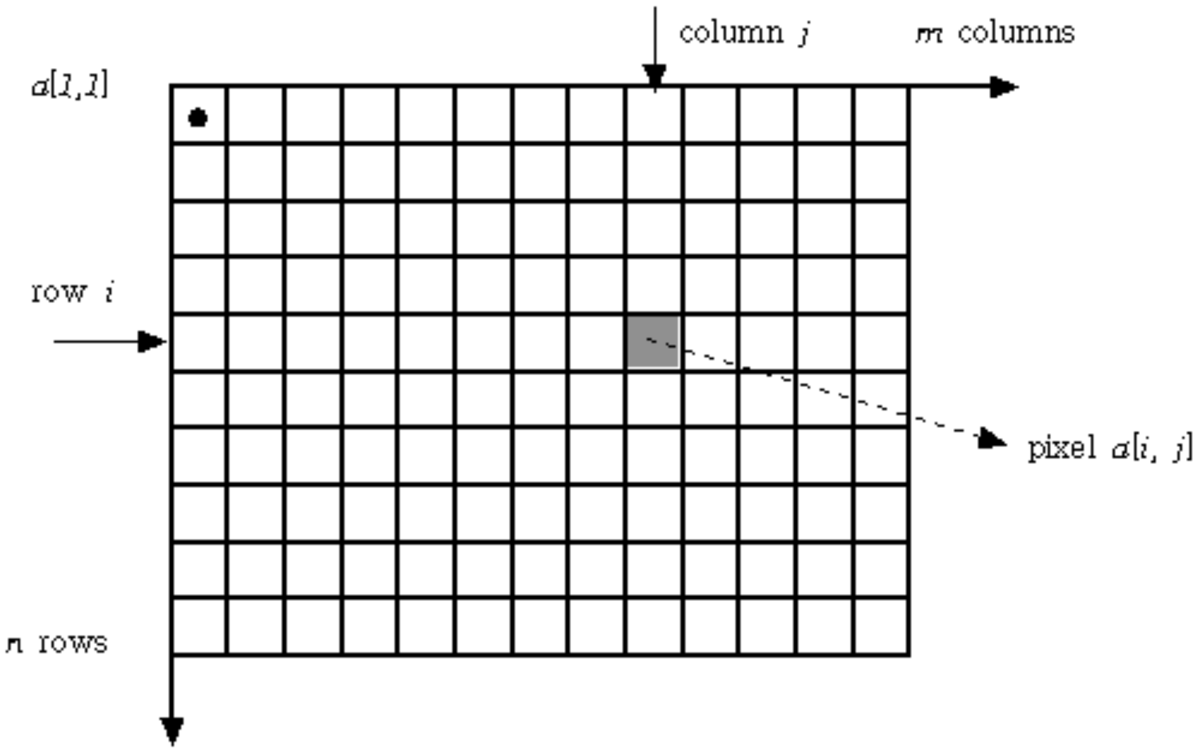
L. G. Roberts, *Machine Perception of Three Dimensional Solids*, Ph.D. thesis, MIT Department of Electrical Engineering, 1963.

Connection to other disciplines



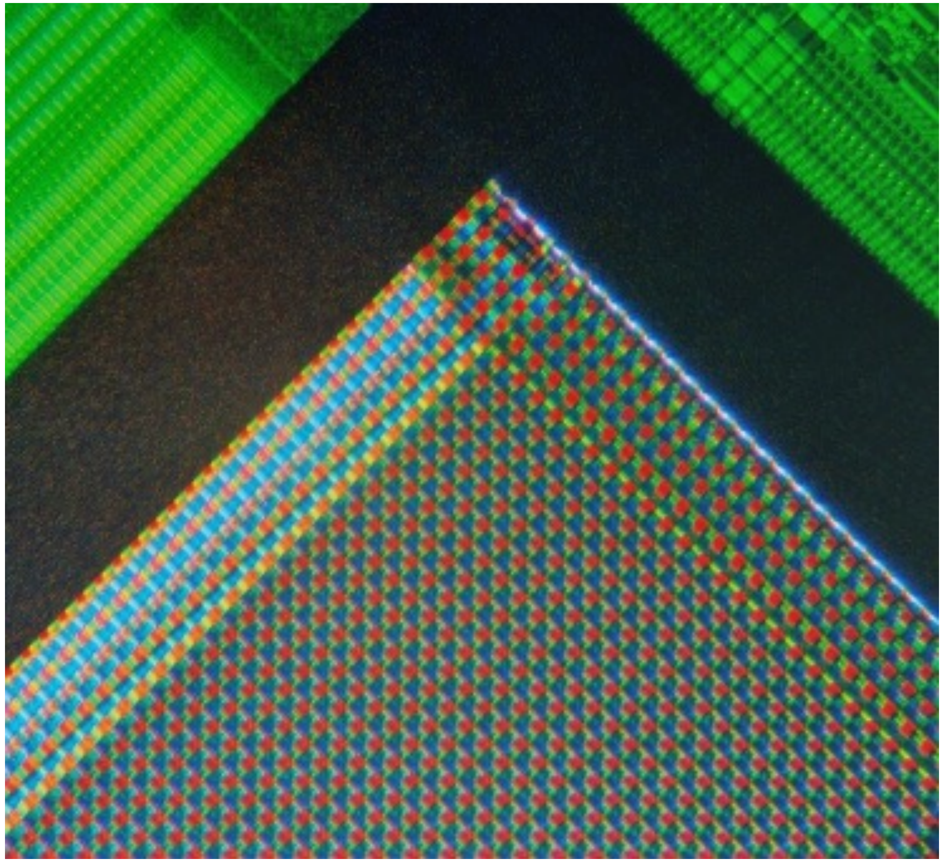
Image

- Image : a two-dimensional array of pixels
- The indices [i, j] of pixels : integer values that specify the rows and columns in pixel values

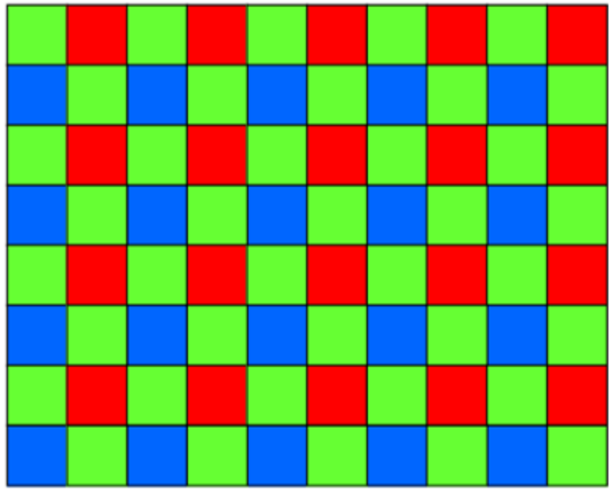


Digital Color Camera

- Bayer Pattern:
  - 50% green, 25% red and 25% blue =>
  - RGBG or GRGB or RGGB.
  - 1 Byte per square
  - 4 squared per 1 pixel
  - More green: eyes are more sensitive to green (nature!)

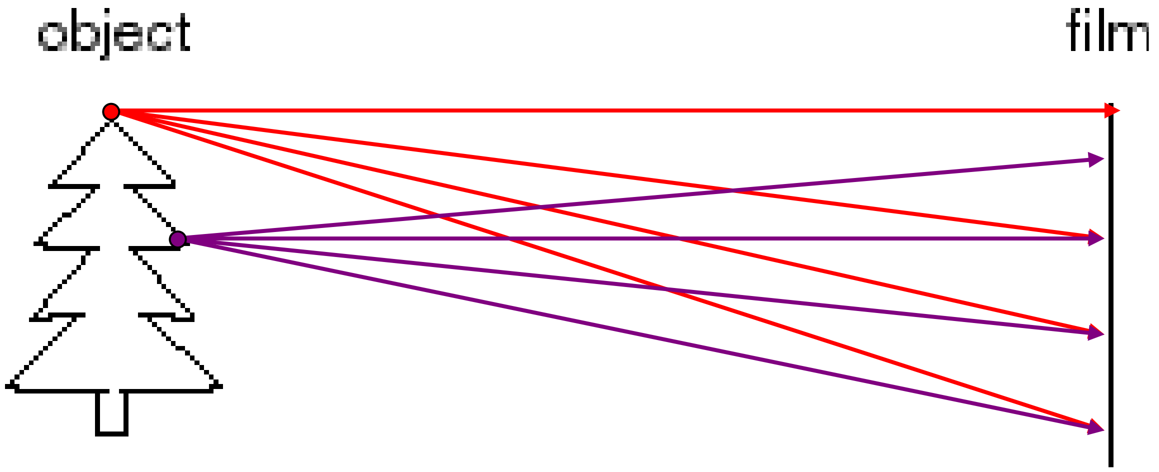


A micrograph of the corner of the photosensor array of a ‘webcam’ digital camera. (Wikimedia)



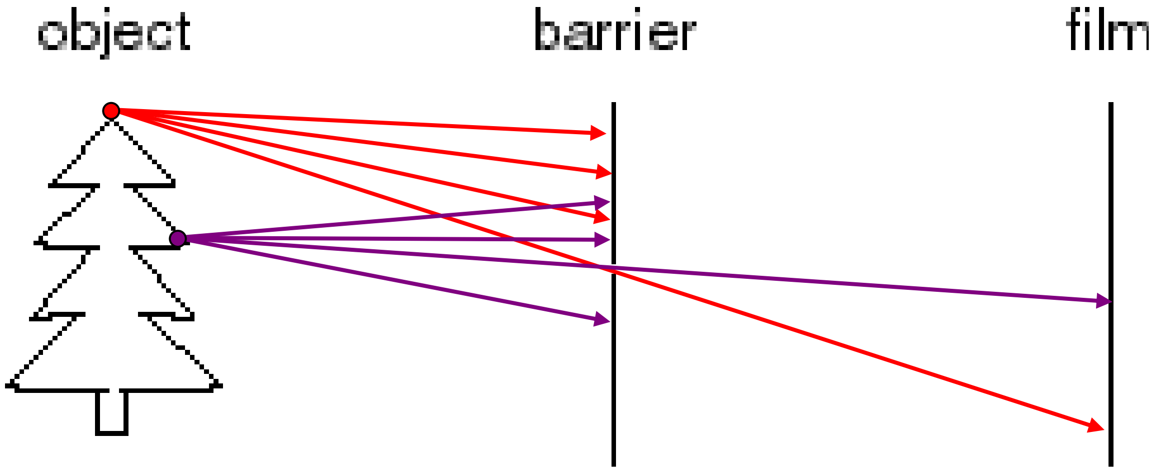
https://en.wikipedia.org/wiki/Bayer_filter

How do we see the world?



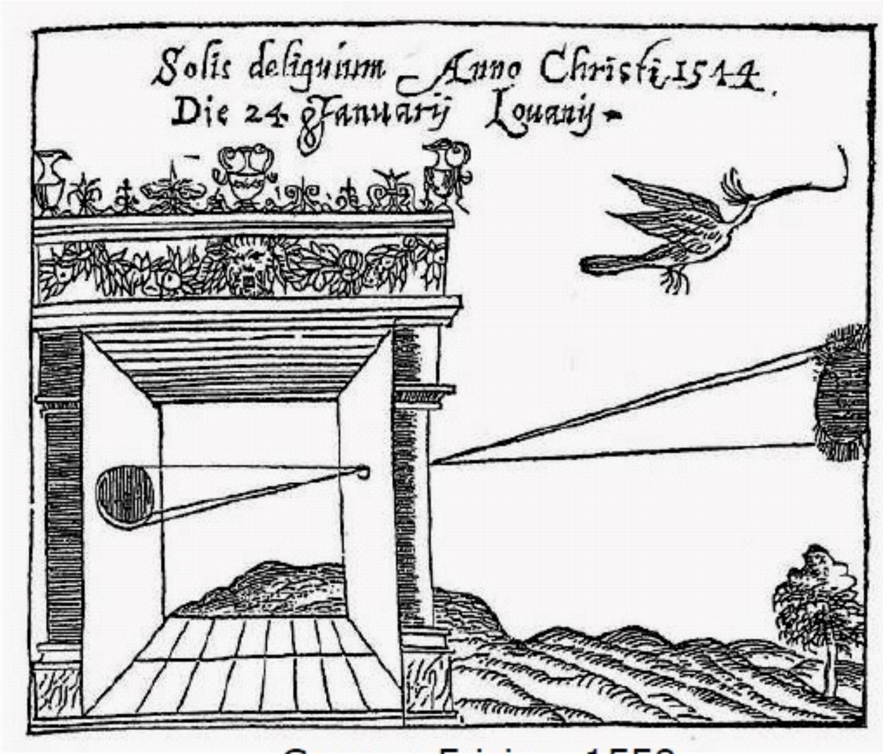
- Let’s design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**

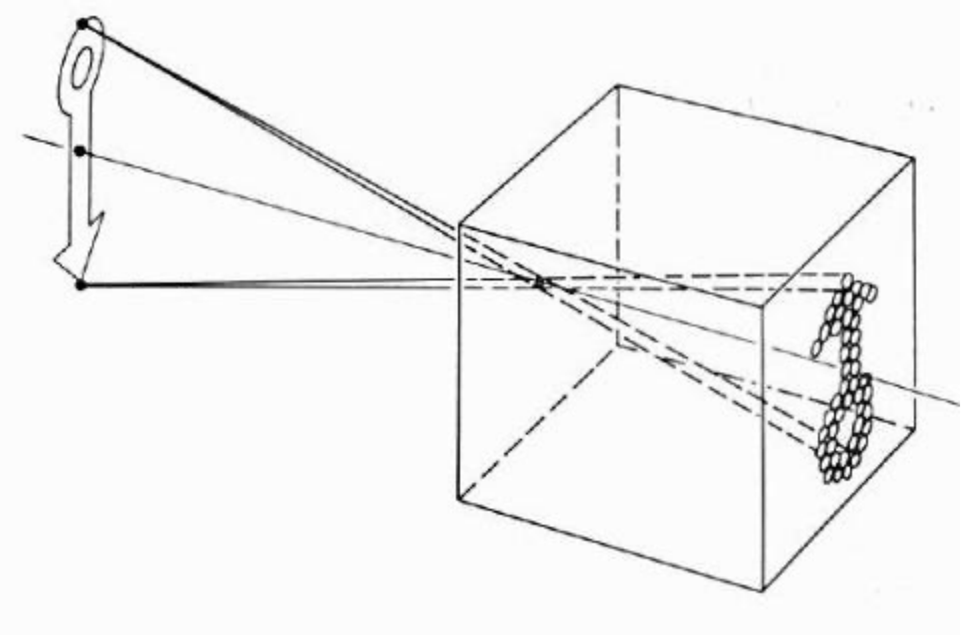
Camera obscura



Gemma Frisius, 1558

- Basic principle known to Mozi (470-390 BC), Aristotle (384-322 BC)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)
- Depth of the room (box) is the effective focal length

Pinhole camera model



- Pinhole model:
  - Captures **pencil of rays** – all rays through a single point
  - The point is called **Center of Projection**
  - The image is formed on the **Image Plane**

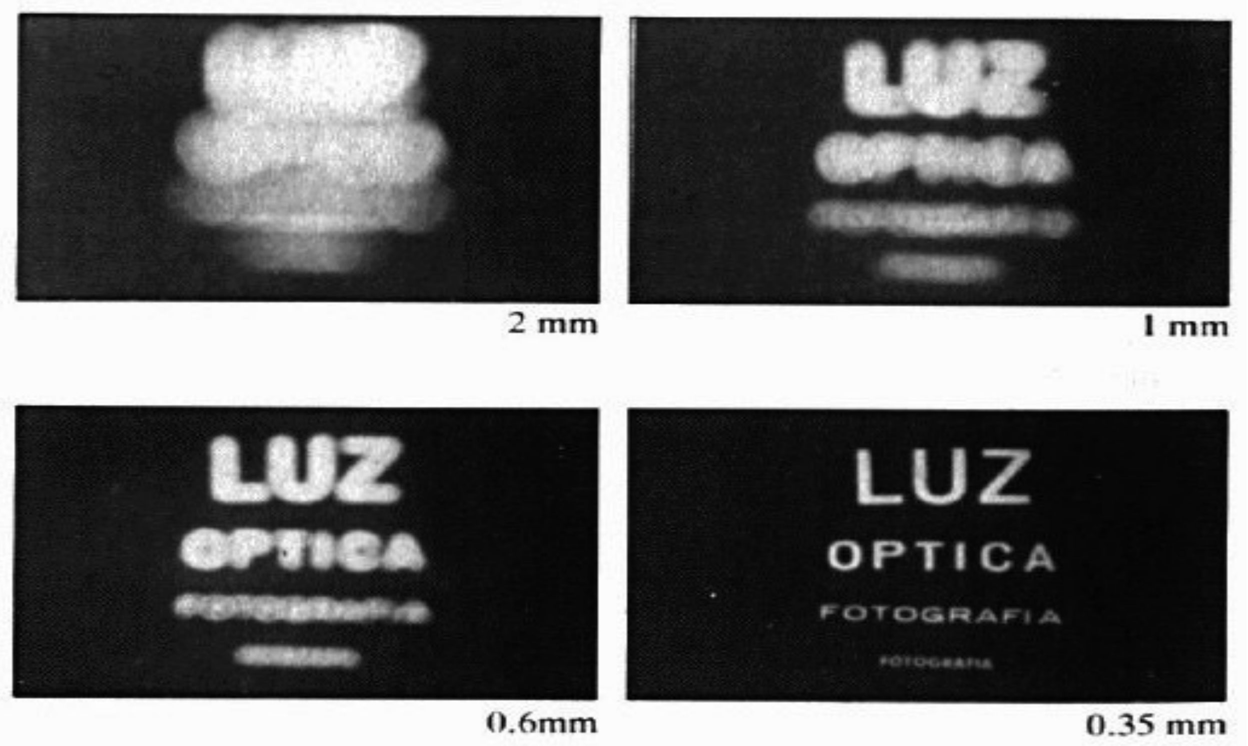
Slide by Steve Seitz

Home-made pinhole camera

Why so blurry?

http://www.debevec.org/Pinhole/

Shrinking the aperture



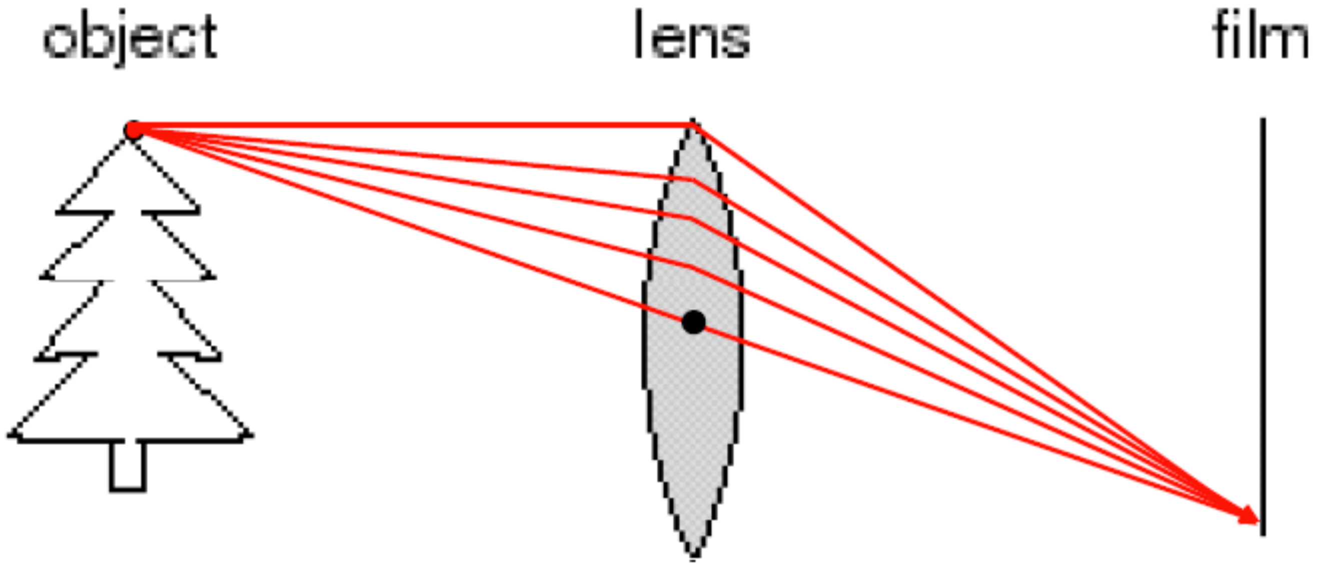
- Why not make the aperture as small as possible?
  - Less light gets through (must increase the exposure)
  - Diffraction effects…

Slide by Steve Seitz

Shrinking the aperture

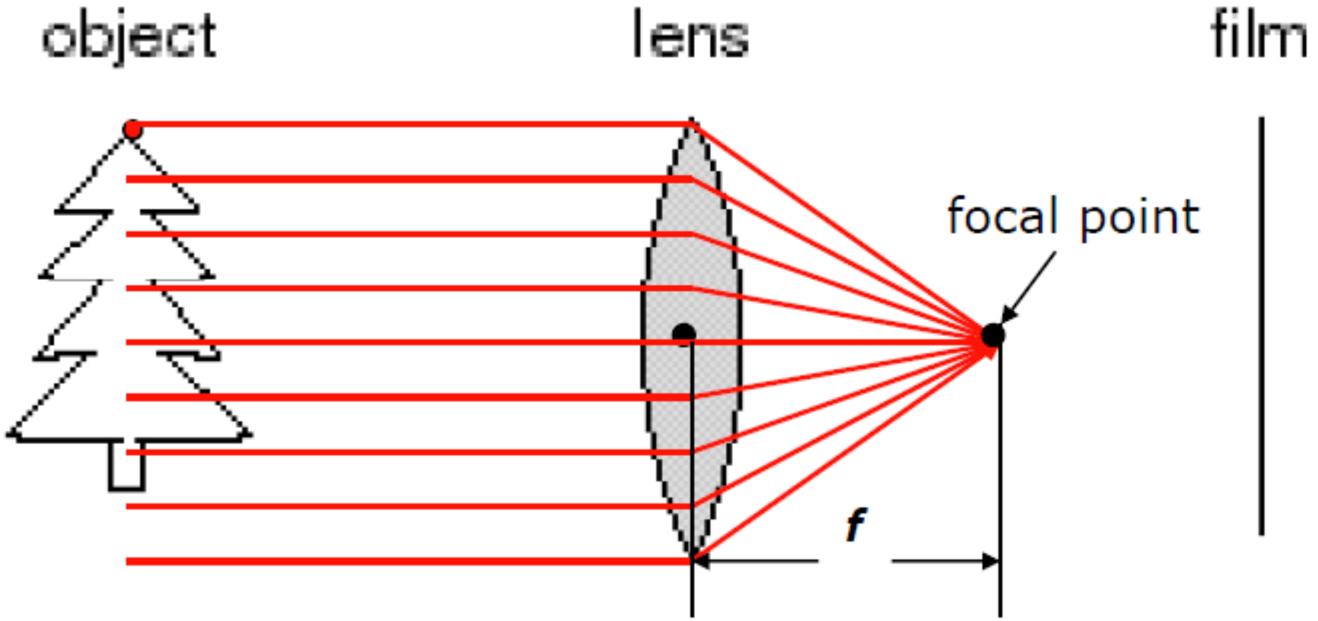
2 mm
1 mm
0.6mm
0.35 mm
0.15 mm
0.07 mm

Solution: adding a lens



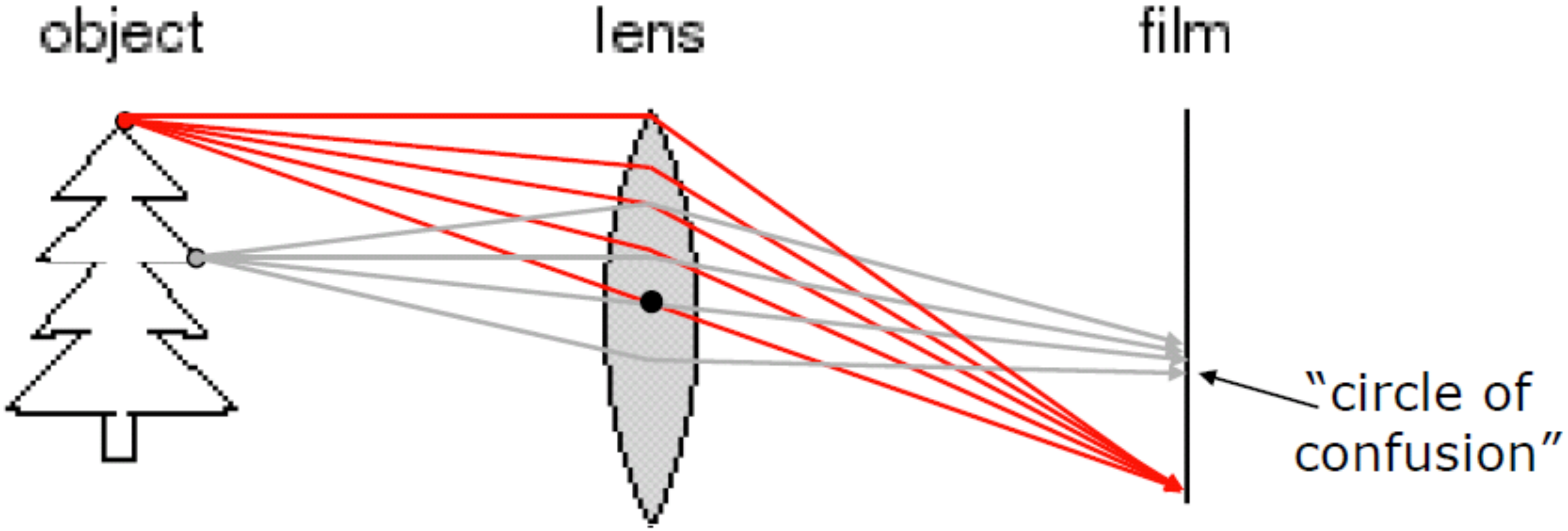
- A lens focuses light onto the film
  - Rays passing through the center are not deviated

Solution: adding a lens



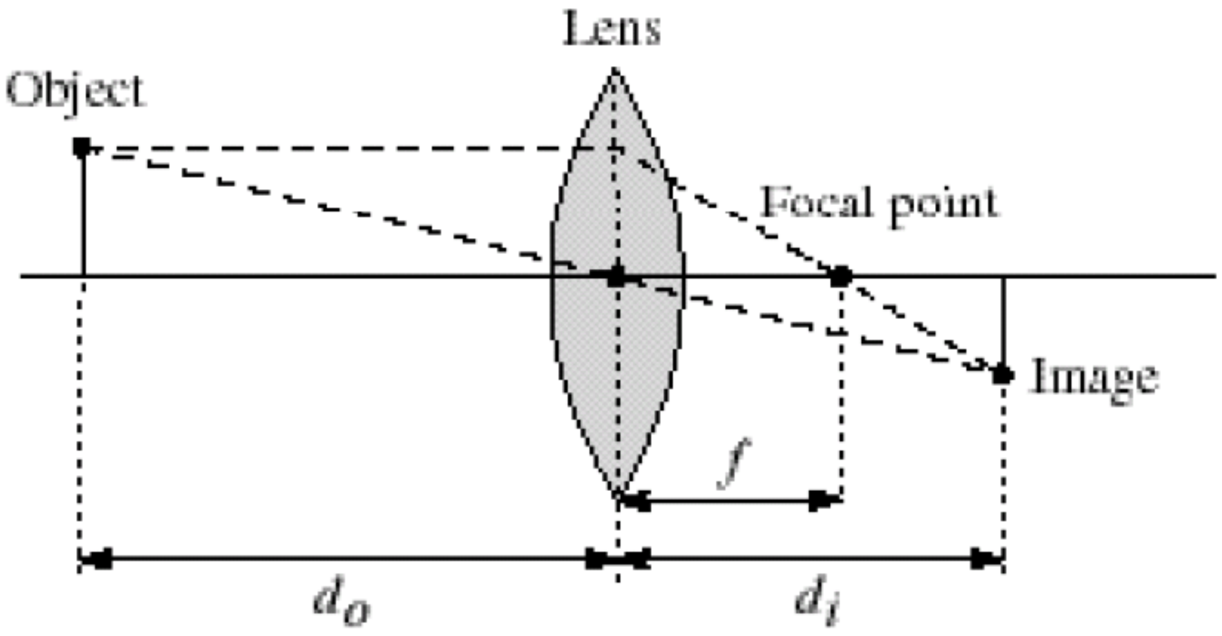
- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

Solution: adding a lens



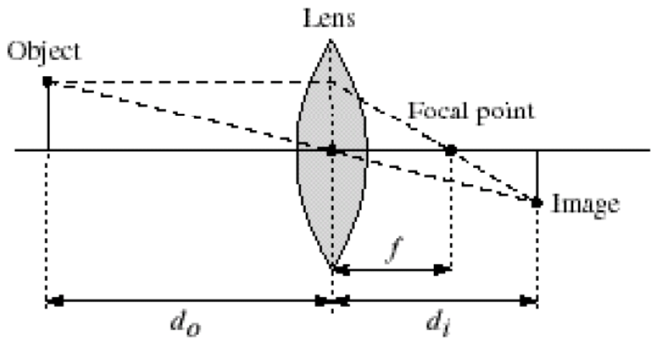
- A lens focuses light onto the film
  - There is a specific distance at which objects are "in focus"
    - other points project to a "circle of confusion" in the image

Thin lenses



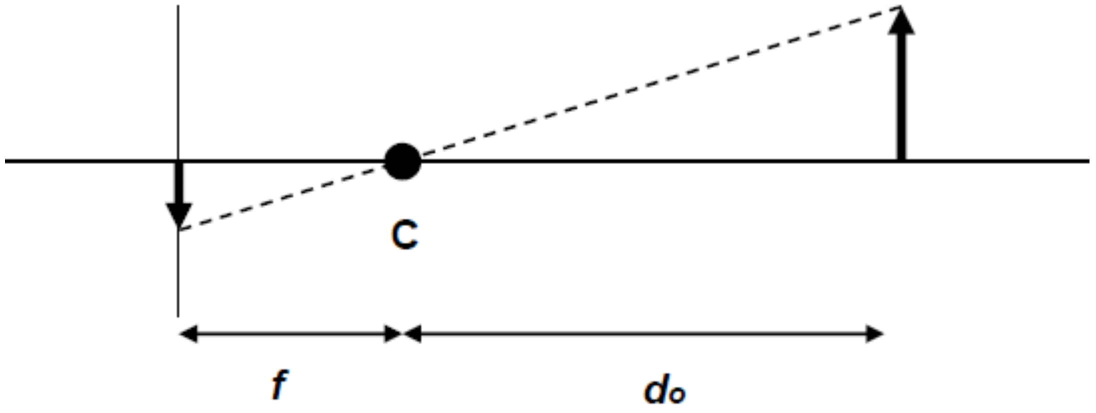
- Thin lens equation: $$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}$$
  - Any object point satisfying this equation is in focus
  - This formula can also be used to estimate roughly the distance to the object (“Depth from Focus”)

Pin-hole approximation



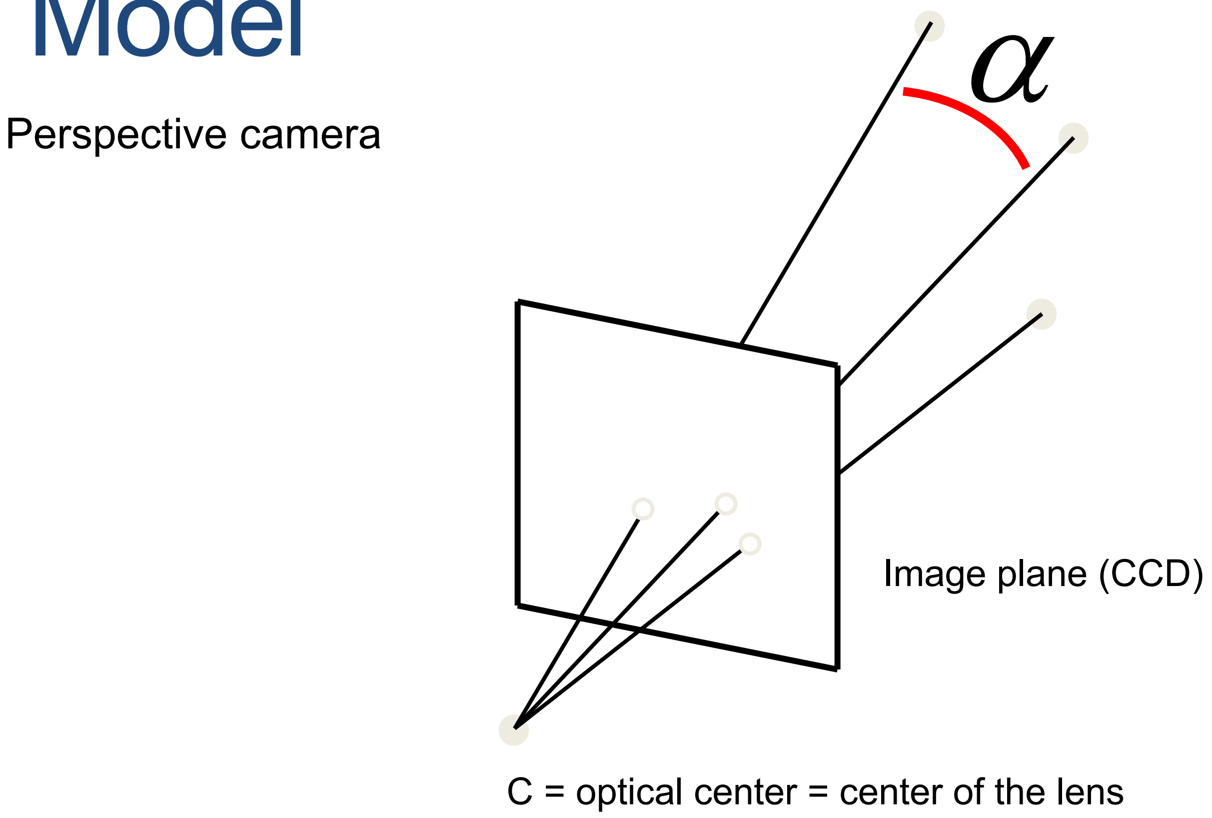
$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

$$d_0 >> d_i \Rightarrow \frac{1}{d_i} \approx \frac{1}{f} \Rightarrow d_i \approx f$$



C = “optical center”

Pin-hole Model



- For convenience, the image plane is usually represented in front so that the image preserves the same orientation (i.e. not flipped)
- **Notice: a camera does not measure distances but angles! Therefore it is a "bearing sensor"**

Perspective Projection onto the image plane

- To project a 3D scene point P = (x,y,z) [meters] onto the camera image plane p=(u,v) [pixels] we need to consider:
    - Pixelization: size of the pixel and position of the CCD with respect to the optical center
    - Rigid body transformation between camera and scene
- u = v = 0: where z-Axis passes trhough center of lens – z-Azis prependicular to lens (coincident with optical axis)

$$u = \frac{f}{z} \cdot x$$

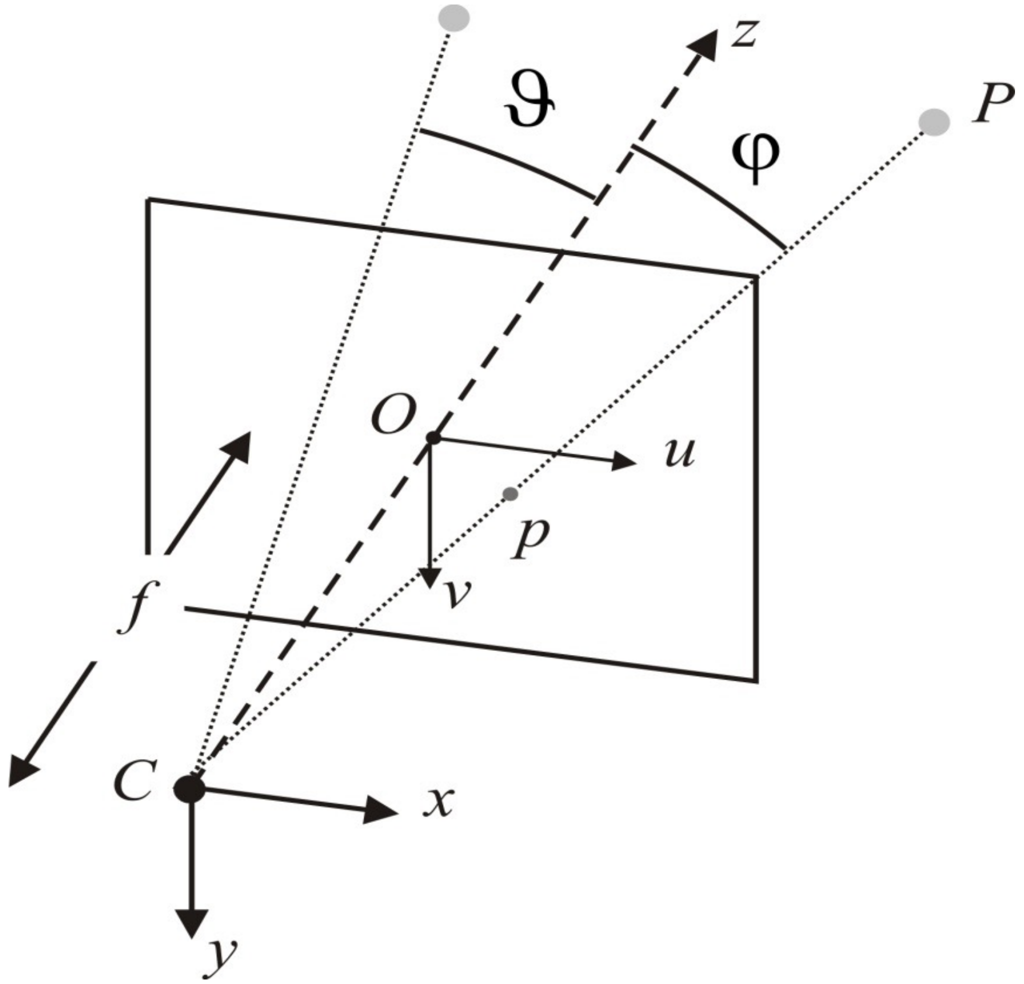
$$v = \frac{f}{z} \cdot y$$

Simple case
(without pixelization)

$$u = k_u \frac{f}{z} \cdot x + u_0$$

$$v = k_v \frac{f}{z} \cdot y + v_0$$

With pixelization
u_0, v_0 are the coordinates of the optical center
Ku and Kv are in [pxl/m]



Projection onto the image plane

- Observe that we can also rewrite this

$$u = k_u \frac{f}{z} \cdot x + u_0$$

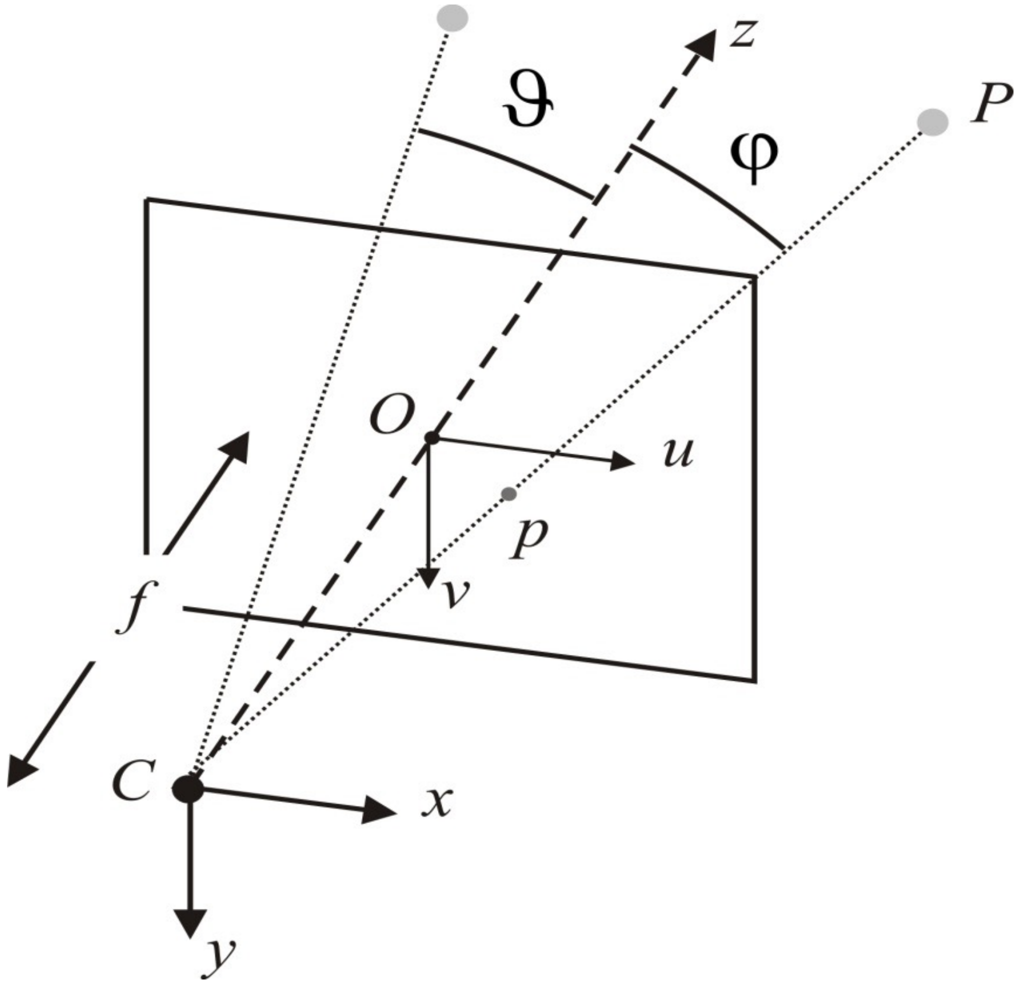
$$v = k_v \frac{f}{z} \cdot y + v_0$$

in matrix form (λ - homogeneous coordinates)

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} k_u f & 0 & u_0 \\ 0 & k_v f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Or alternatively

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



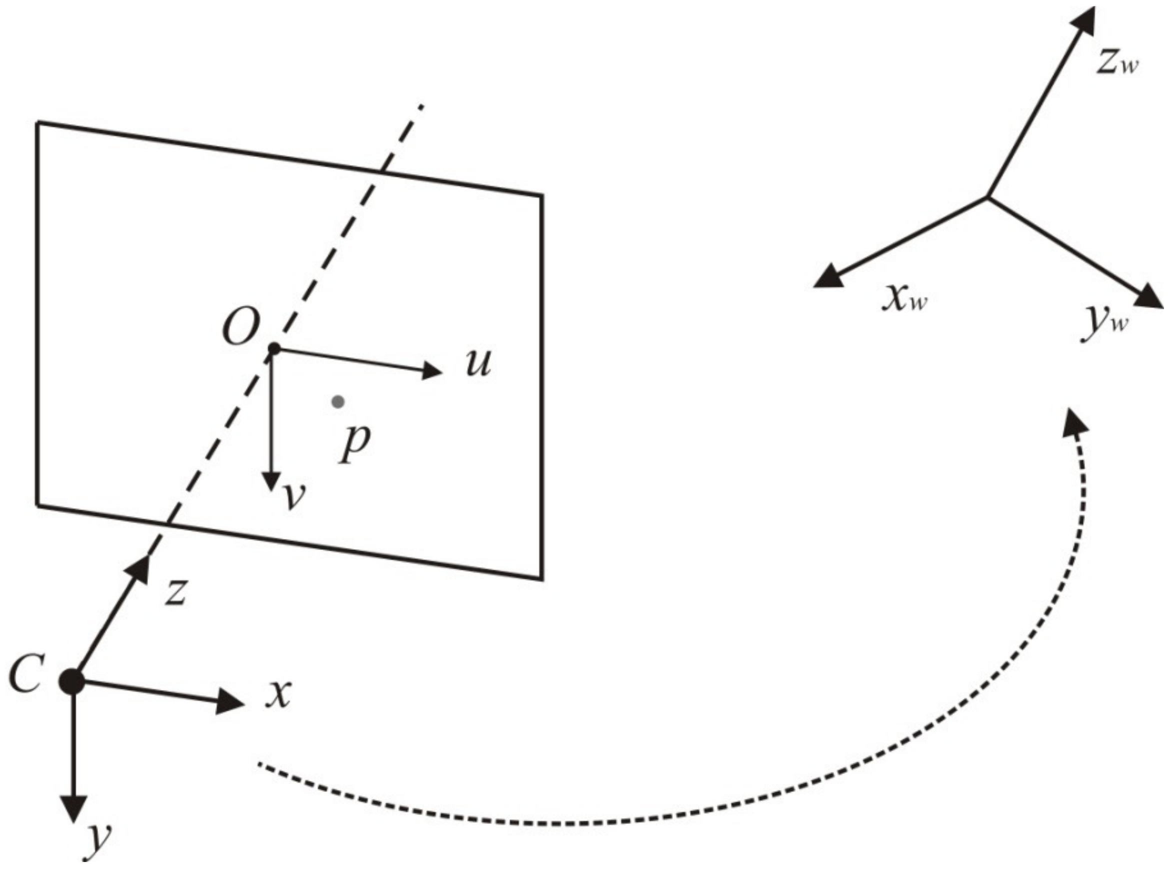
Projection onto the image plane

- Rigid body transformation from the World to the Camera reference frame

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T$$

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A \cdot R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T$$

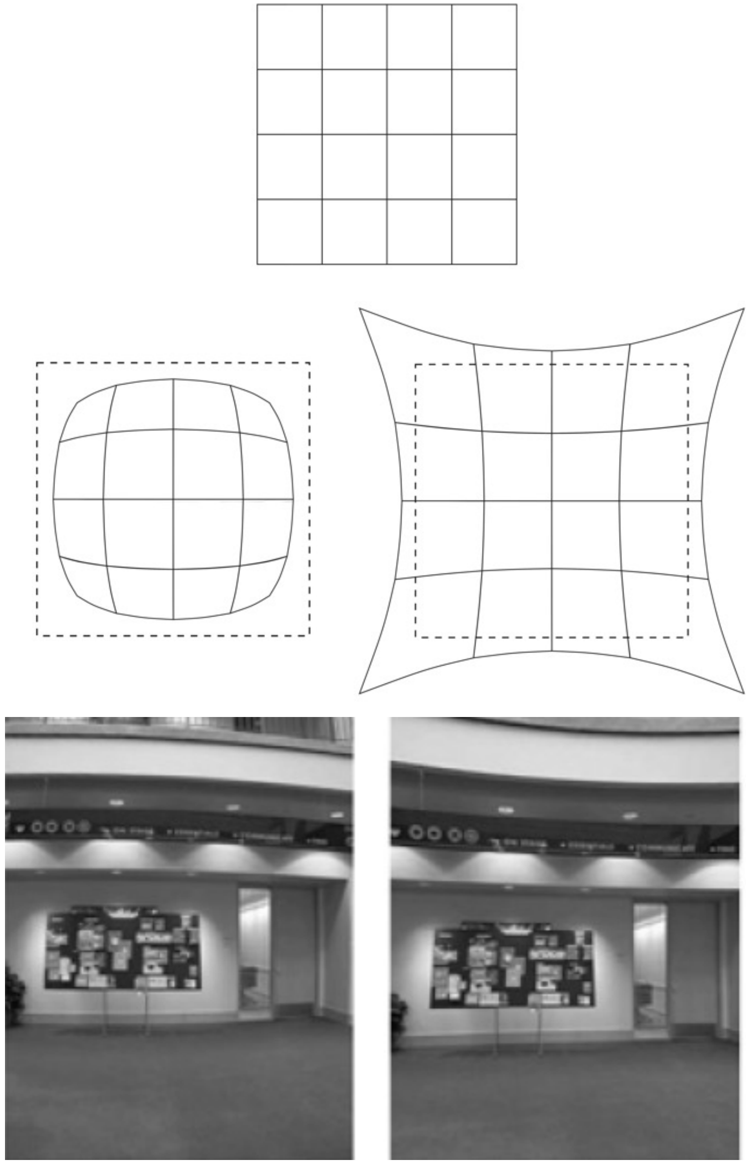


Radial distortion

$$\begin{bmatrix} u_d \\ v_d \end{bmatrix} = (1 + k_1 r^2) \begin{bmatrix} u - u_0 \\ v - v_0 \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$$

where

$$r^2 = (u - u_0)^2 + (v - v_0)^2 .$$



Barrel distortion   Pincushion distortion

Camera Calibration

- How many parameters do we need to model a camera?

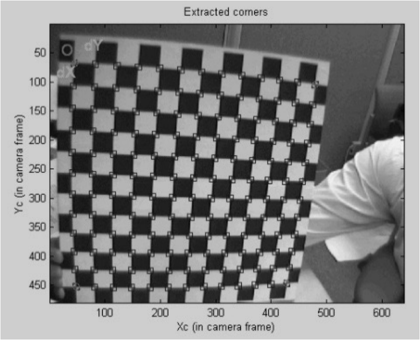
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T \qquad \begin{bmatrix} u_d \\ v_d \end{bmatrix} = (1 + k_1 \rho^2) \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

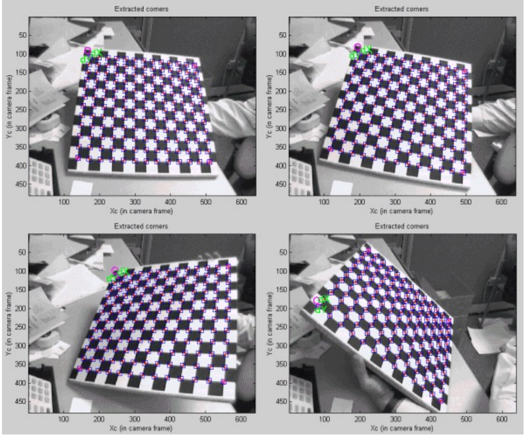
- 5 “intrinsic” parameters: α_u, α_v, u_0, v_0, k_1
- Camera pose?
- 6 “extrinsic” parameters (or 0 if the world and the camera frames coincide)

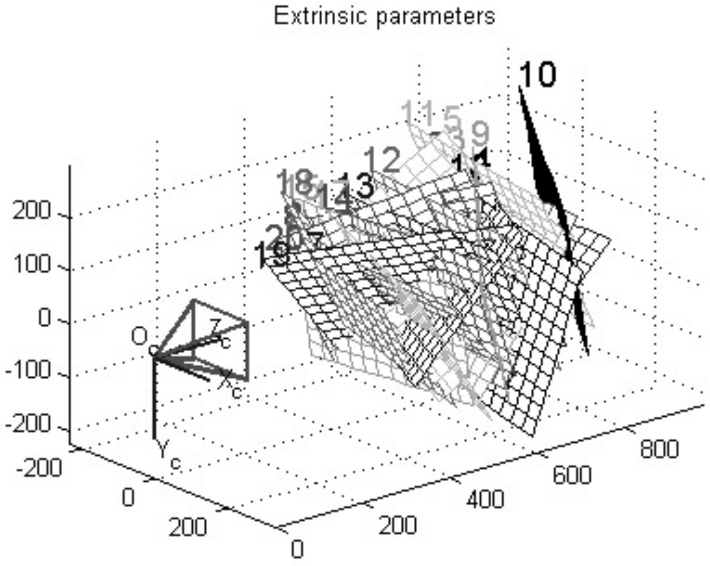
Camera Calibration: how does it work?

- Calibration: measuring accurately **intrinsic** + **extrinsic** parameters of the camera model.
- Parameters: govern mapping from scene points to image points
- Idea: known:
  - pixel coordinates of image points *p*
  - 3D coordinates of the corresponding scene points *P*
  - => compute the unknown parameters A, R, T by solving the perspective projection equation

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T$$



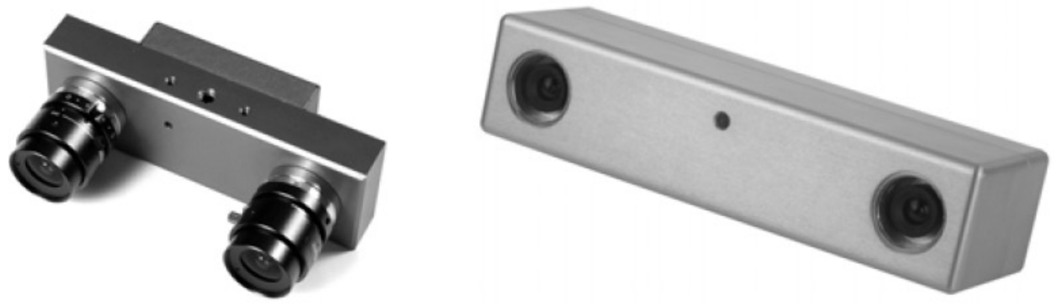




STEREO VISION

How do we measure distances with cameras?

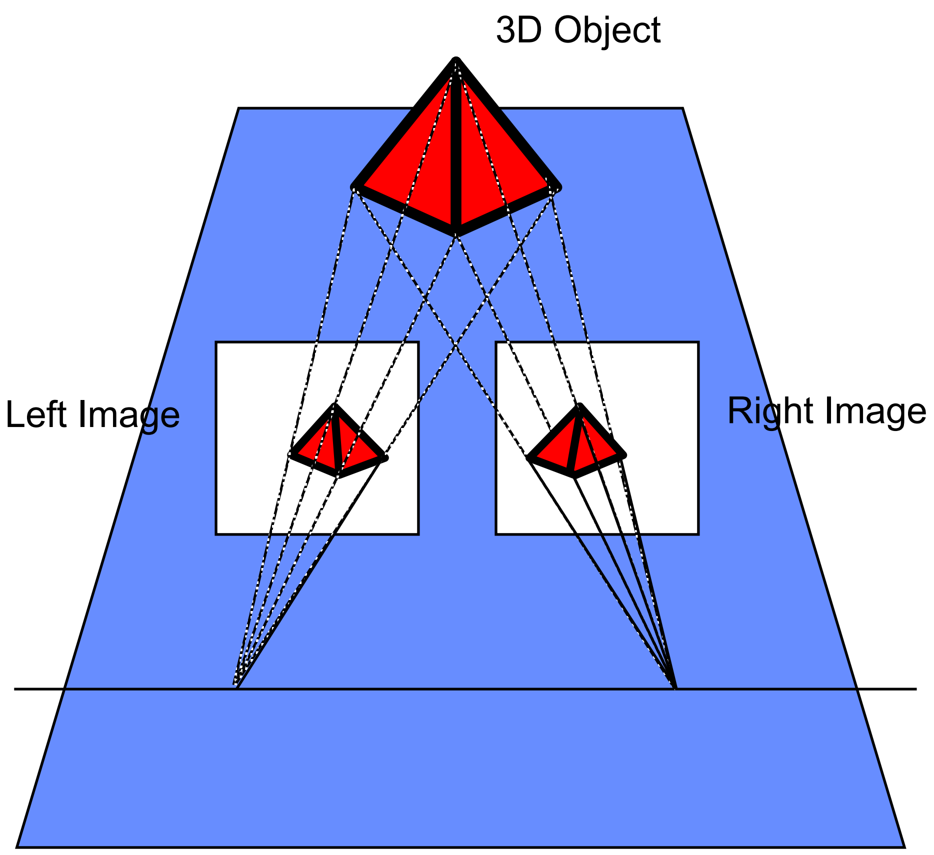
- Structure from stereo (Stereo-vision):
  - use two cameras with known relative position and orientation



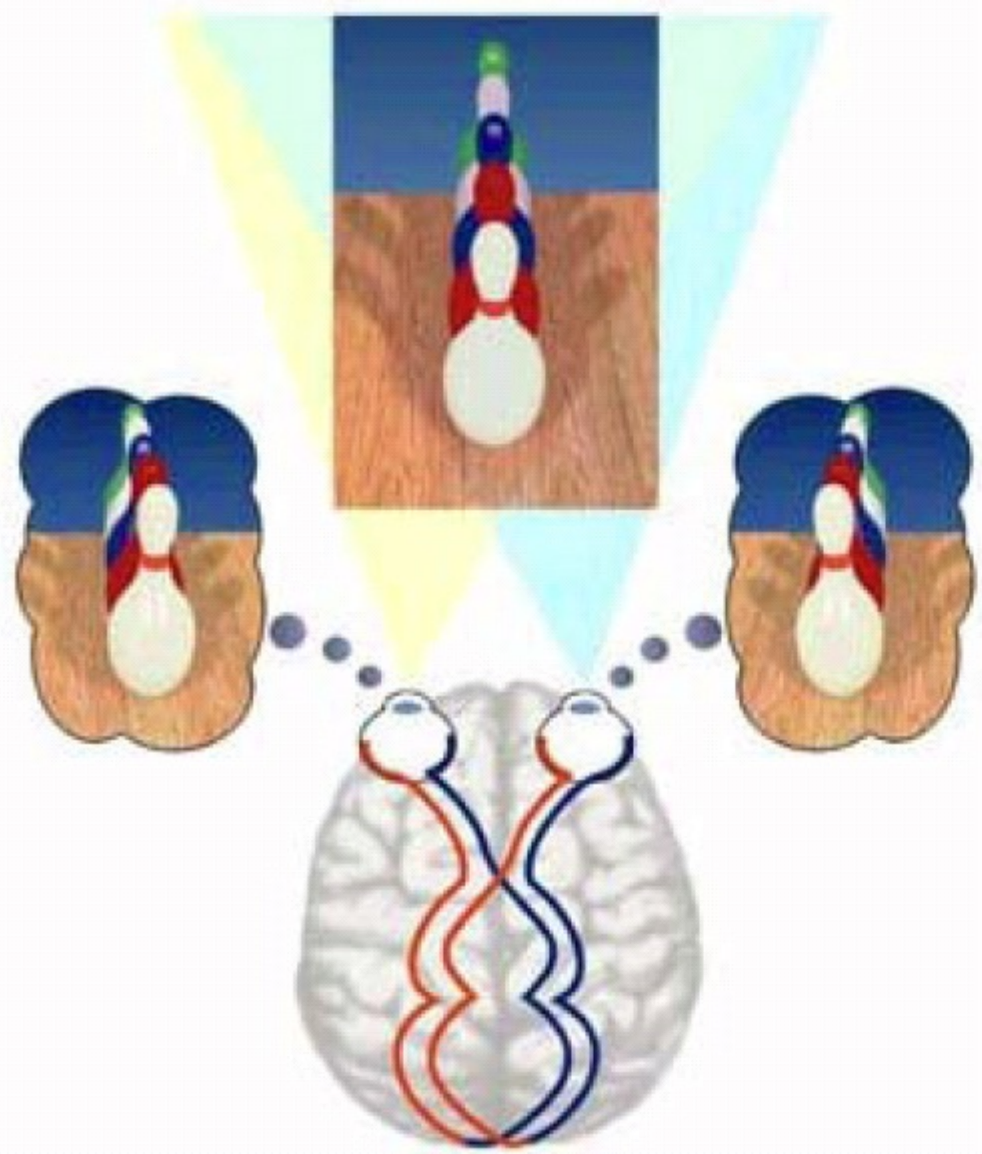
- Structure from motion:
  - use a single moving camera: both 3D structure and camera motion can be estimated up to a scale

Stereo Vision

- Allows to reconstruct a 3D object from two images taken at different locations

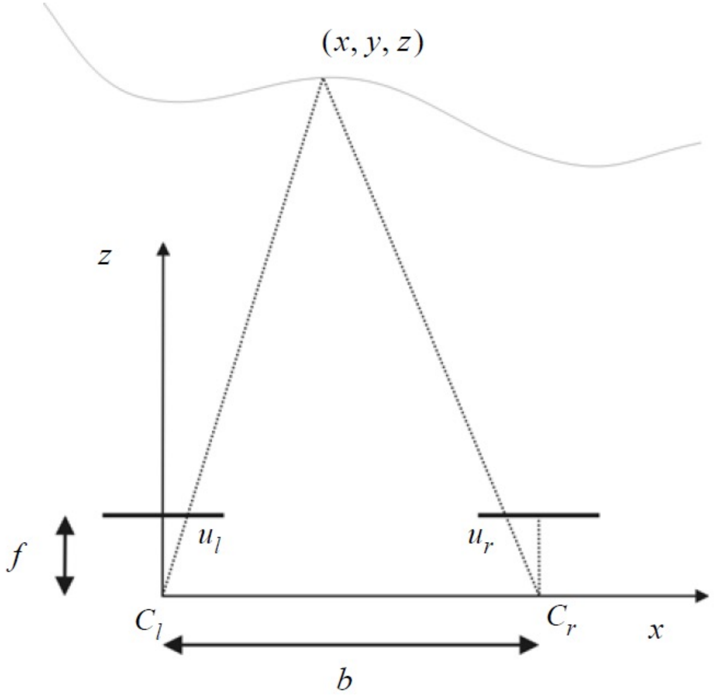


Disparity in the human retina



Stereo Vision - The simplified case

- The simplified case is an ideal case. It assumes that both cameras are identical and are aligned on a horizontal axis



$$\frac{f}{z} = \frac{u_l}{x}, \qquad \frac{f}{z} = \frac{u_r}{b - x}$$

$$\Rightarrow \quad z = b\frac{f}{u_l - u_r}$$

Distance

- ***b*** = baseline, distance between the optical centers of the two cameras
- ***f*** = focal length
- $\boldsymbol{u_l}$-$\boldsymbol{u_r}$ = disparity

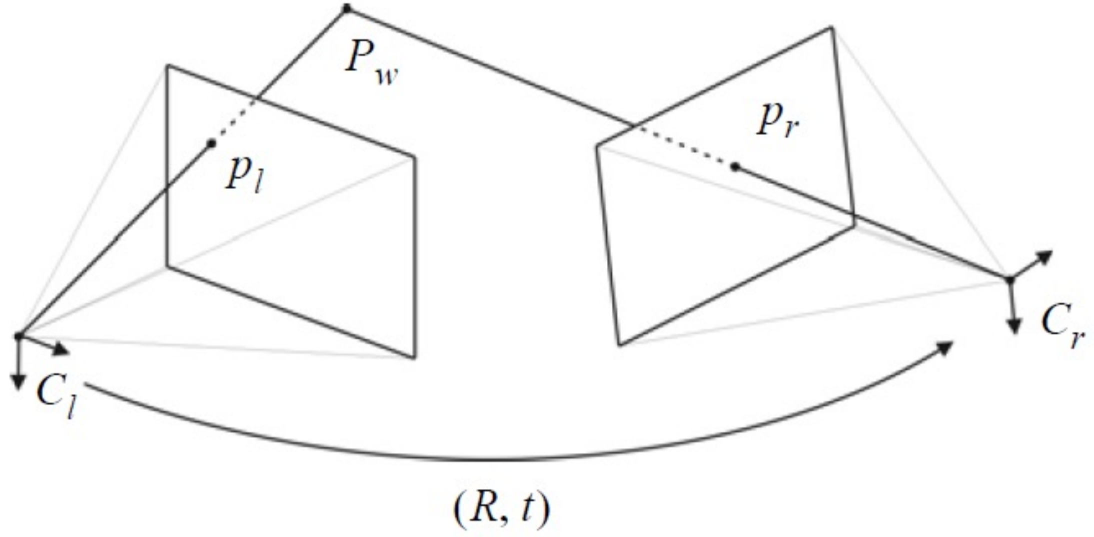
Stereo Vision: how to improve accuracy?

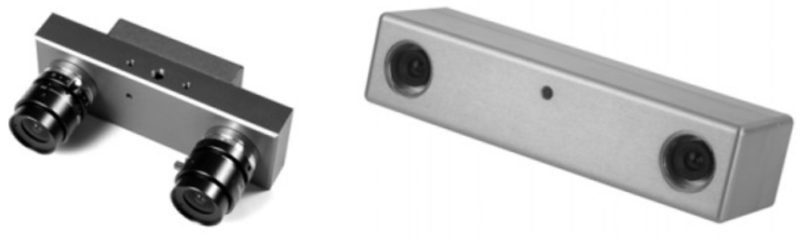
$$z = b\frac{f}{u_l - u_r}$$

1. Distance is inversely proportional to disparity (**u_l-u_r**)
   - closer objects can be measured more accurately
2. Disparity is proportional to ***b***
   - For a given disparity error, the accuracy of the depth estimate increases with increasing baseline ***b***.
   - However, as ***b*** is increased, some objects may appear in one camera, but not in the other.
3. Increasing image resolution improves accuracy

Stereo Vision – the general case

- Two identical cameras do not exist in nature!
- Aligning both cameras on a horizontal axis is very hard, also with the most expensive stereo cameras!

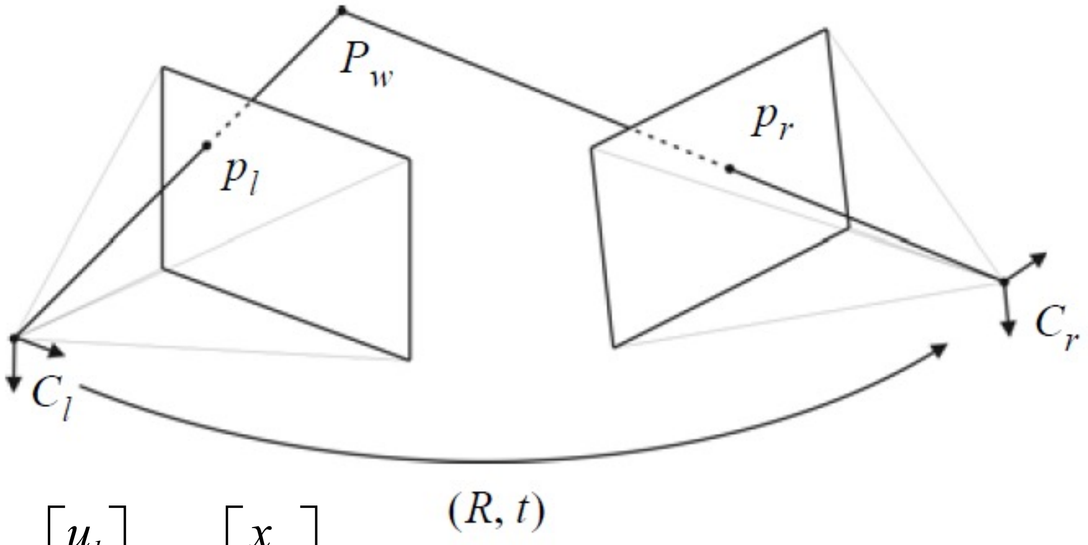




- In order to be able to use a stereo camera, we need first to estimate the relative pose between the cameras, that is, **Rotation** and **Translation**
- However, as the two cameras are not identical, we need to estimate:
  focal length, **image center**, **radial distortion**

Stereo Vision – the general case

- To estimate the 3D position we just construct the system of equations of the left and right camera



$$\lambda_l \begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = A_l \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

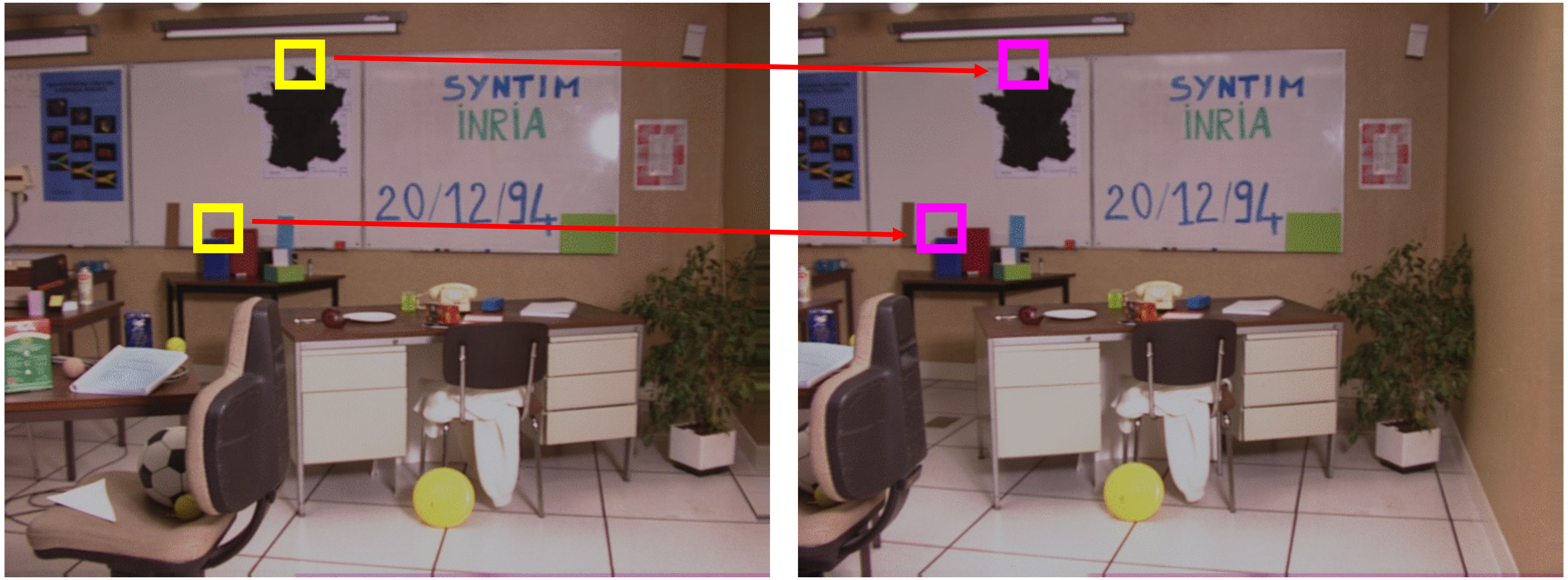
Left camera; world frame coincides with the left camera frame

$$\lambda_r \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = A_r \cdot R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T$$

Right camera

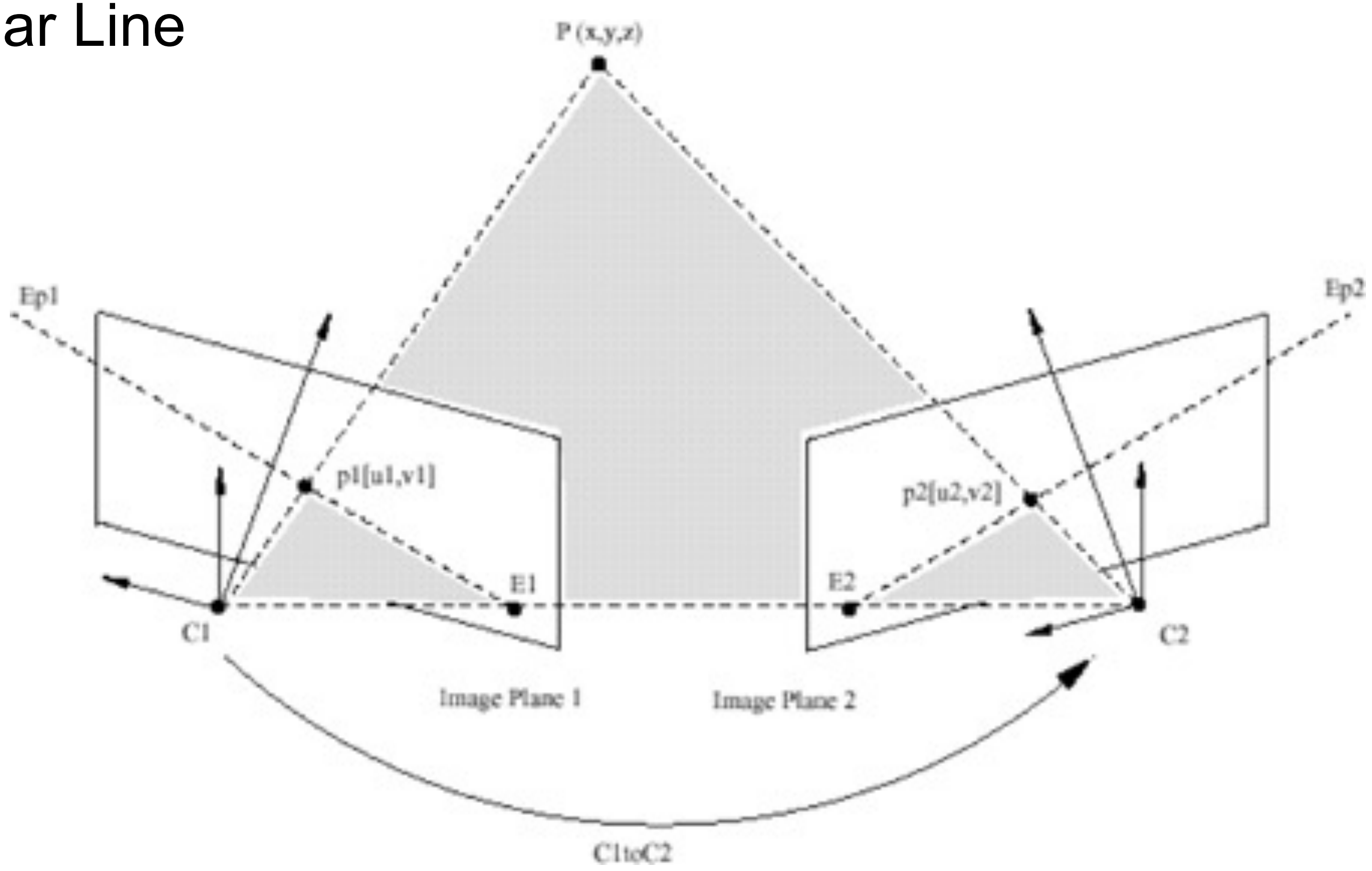
Stereo Vision: Correspondence Problem

- Matching between points in the two images which are projection of the same 3D real point
- Correspondence search could be done by comparing the observed points with all other points in the other image. Typical similarity measures are the Correlation and image Difference.
- This image search can be computationally very expensive! Is there a way to make the correspondence search 1 dimensional?



Correspondence Problem: Epipolar Constraint

- The correspondent of a point in an image must lie on a line in the other image, called Epipolar Line



Correspondence Problem: Epipolar Constraint

- Thanks to the epipolar constraint, conjugate points can be searched along epipolar lines: this reduces the computational cost to 1 dimension!

Epipolar Rectification

- Determines a transformation of each image plane so that pairs of conjugate epipolar lines become collinear and parallel to one of the image axes (usually the horizontal one)

Rotation

↓

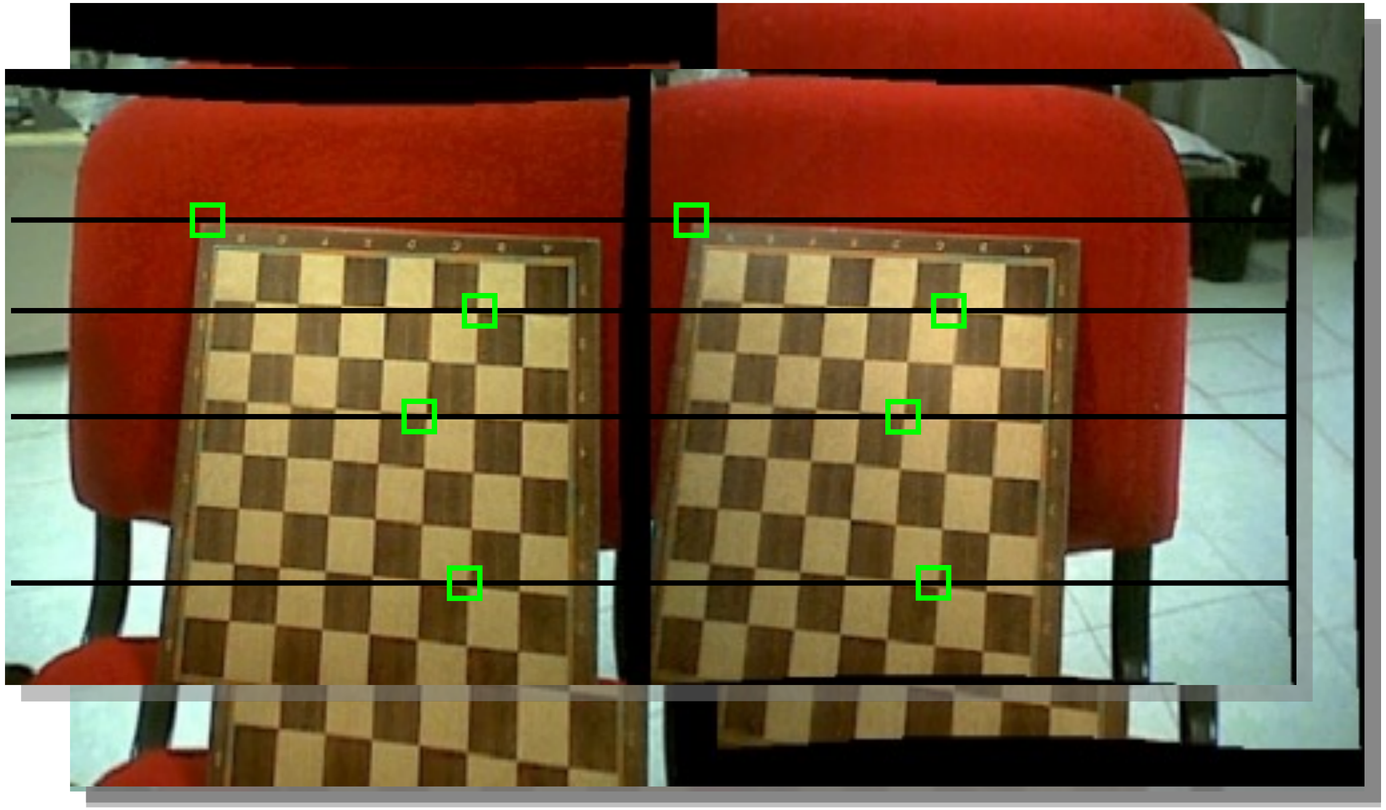
Focal lengths

↓

Lens Distortion

↓

Translation



Stereo Vision Output 1 – Disparity map

- Find the correspondent points of all image pixels of the original images
- For each pair of conjugate points compute the disparity $d = v - v'$
- $d(x,y)$ is called Disparity map.

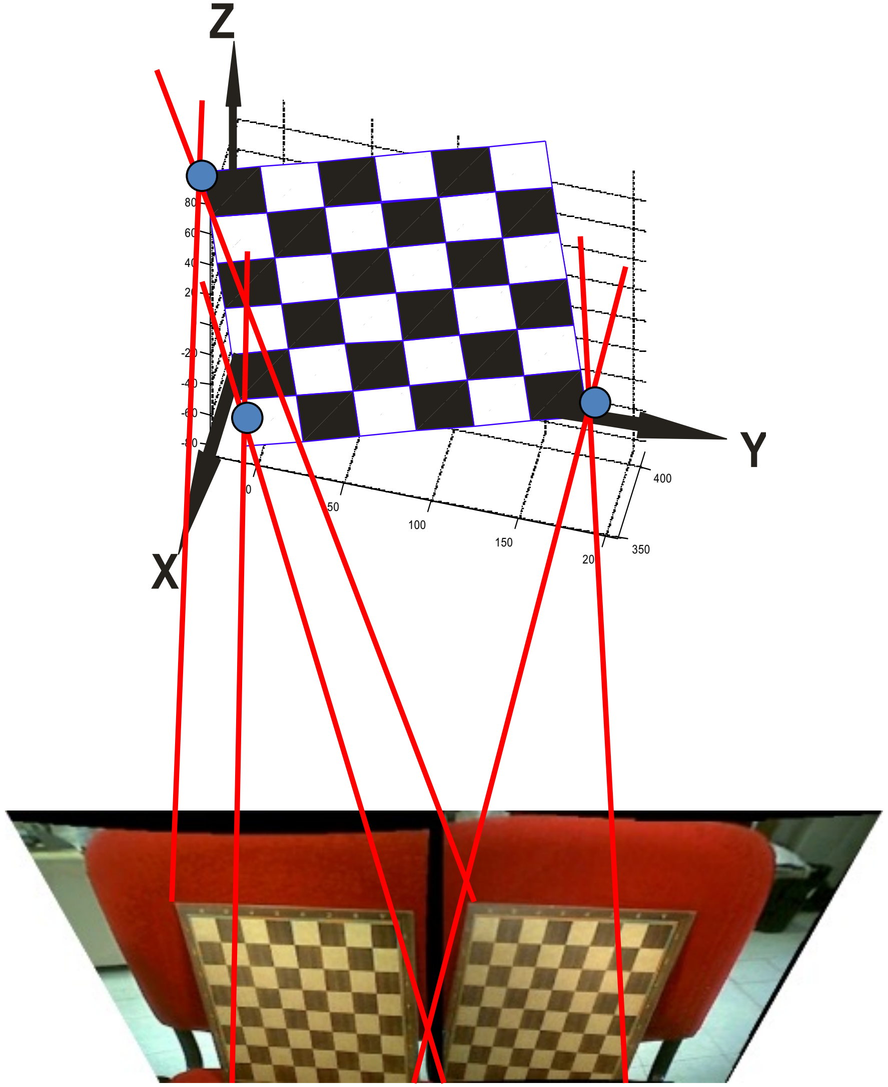
Left image

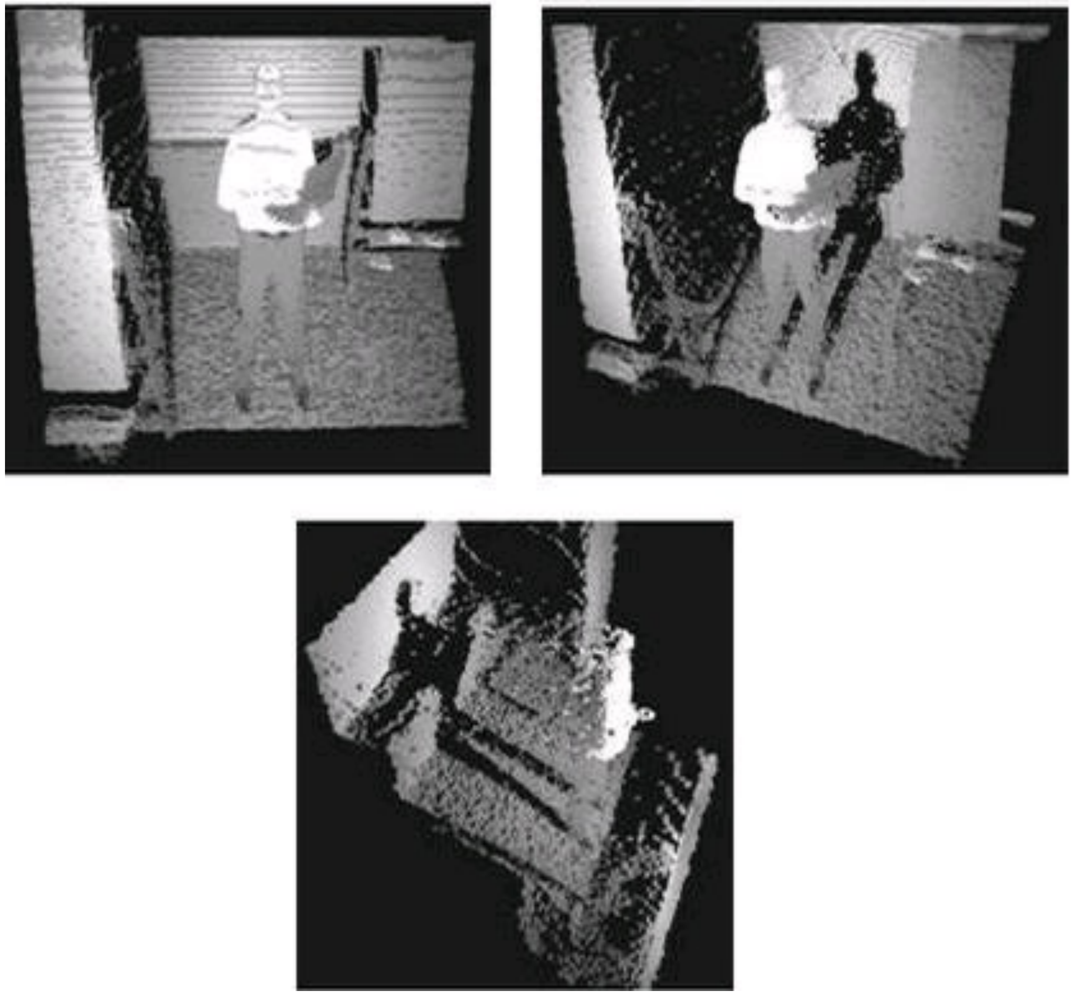
Right image

Disparity map

- Disparity maps are usually visualized as grey-scale images. Objects that are closer to the camera appear lighter, those who are further appear darker.

Stereo Vision Output 2 - 3D Reconstruction via triangulation





Stereo Camera Calibration

Estimates the parameters that manage the 3D – 2D transformation

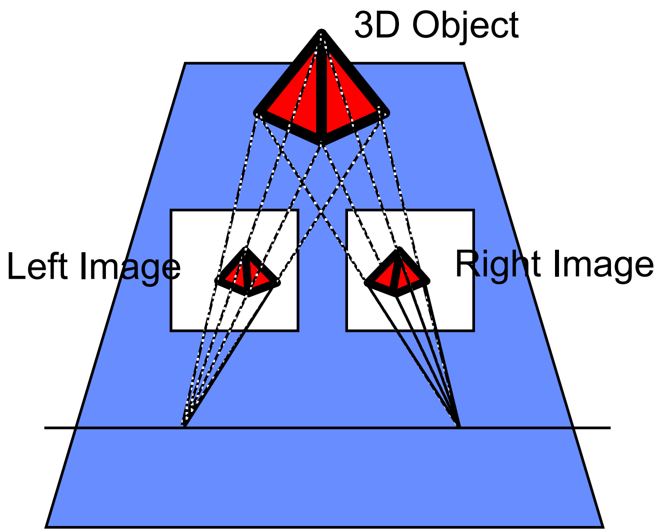
$$\begin{cases} \begin{bmatrix} u_L \\ v_L \end{bmatrix} = f\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \\ \begin{bmatrix} u_R \\ v_R \end{bmatrix} = f\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \end{cases}$$

Stereo Camera Calibration

PLUS
5 parameters for each camera in order to compensate for lens distortion (radial & tangential distortion)

$$\begin{cases} \begin{bmatrix} u_{d,L} \\ v_{d,L} \end{bmatrix} = (1 + kc_{1,L}\rho^2 + kc_{2,L}\rho^4 + kc_{5,L}\rho^6) \cdot \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 2kc_{3,L} \cdot u \cdot v + kc_{4,L}(\rho^2 + 2u^2) \\ kc_{3,L}(\rho^2 + 2v^2) + 2kc_{4,L} \cdot u \cdot v \end{bmatrix} \\ \begin{bmatrix} u_{d,R} \\ v_{d,R} \end{bmatrix} = (1 + kc_{1,R}\rho^2 + kc_{2,R}\rho^4 + kc_{5,R}\rho^6) \cdot \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 2kc_{3,R} \cdot u \cdot v + kc_{4,R}(\rho^2 + 2u^2) \\ kc_{3,R}(\rho^2 + 2v^2) + 2kc_{4,R} \cdot u \cdot v \end{bmatrix} \end{cases}$$

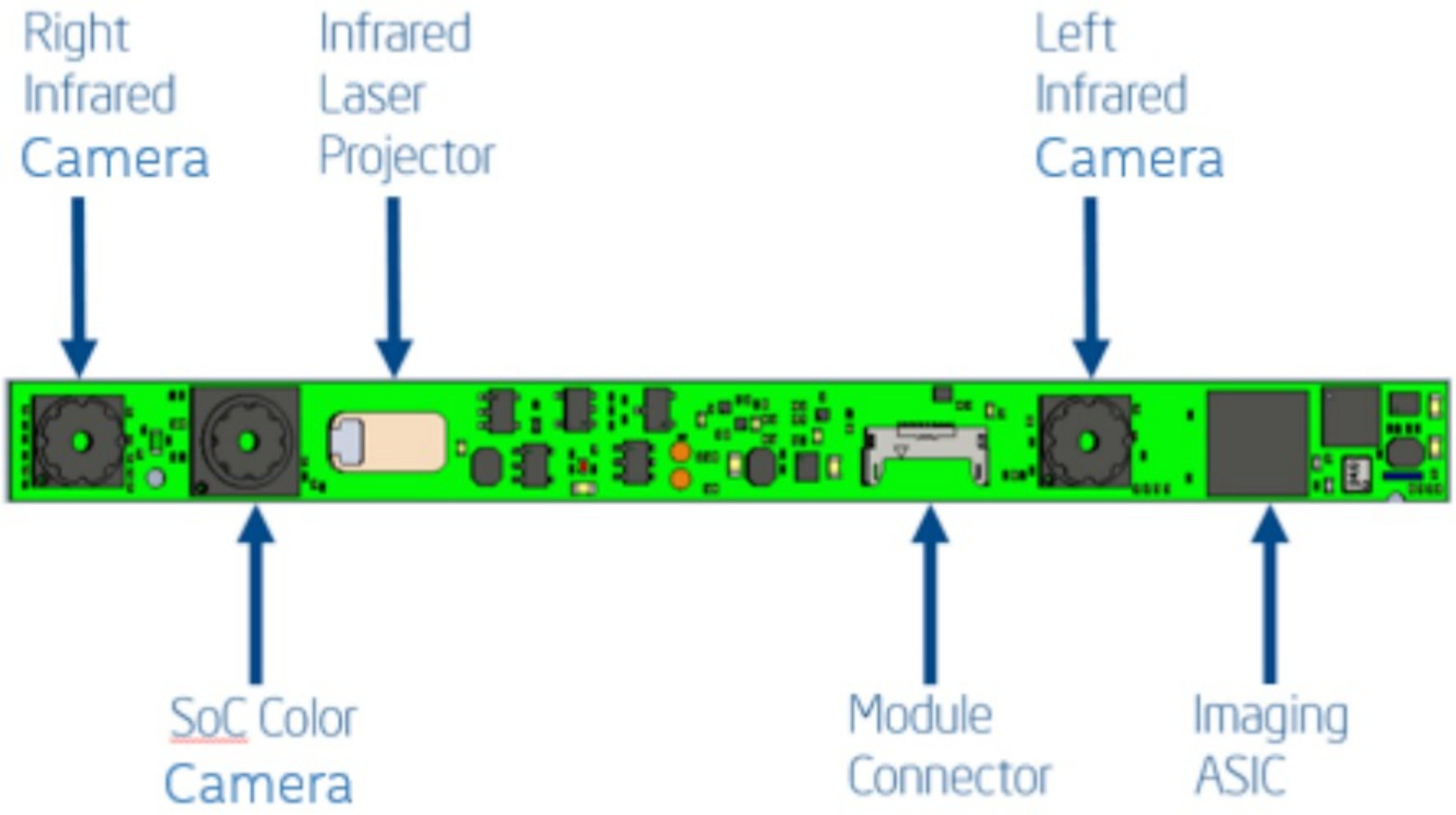
Stereo Vision - summary



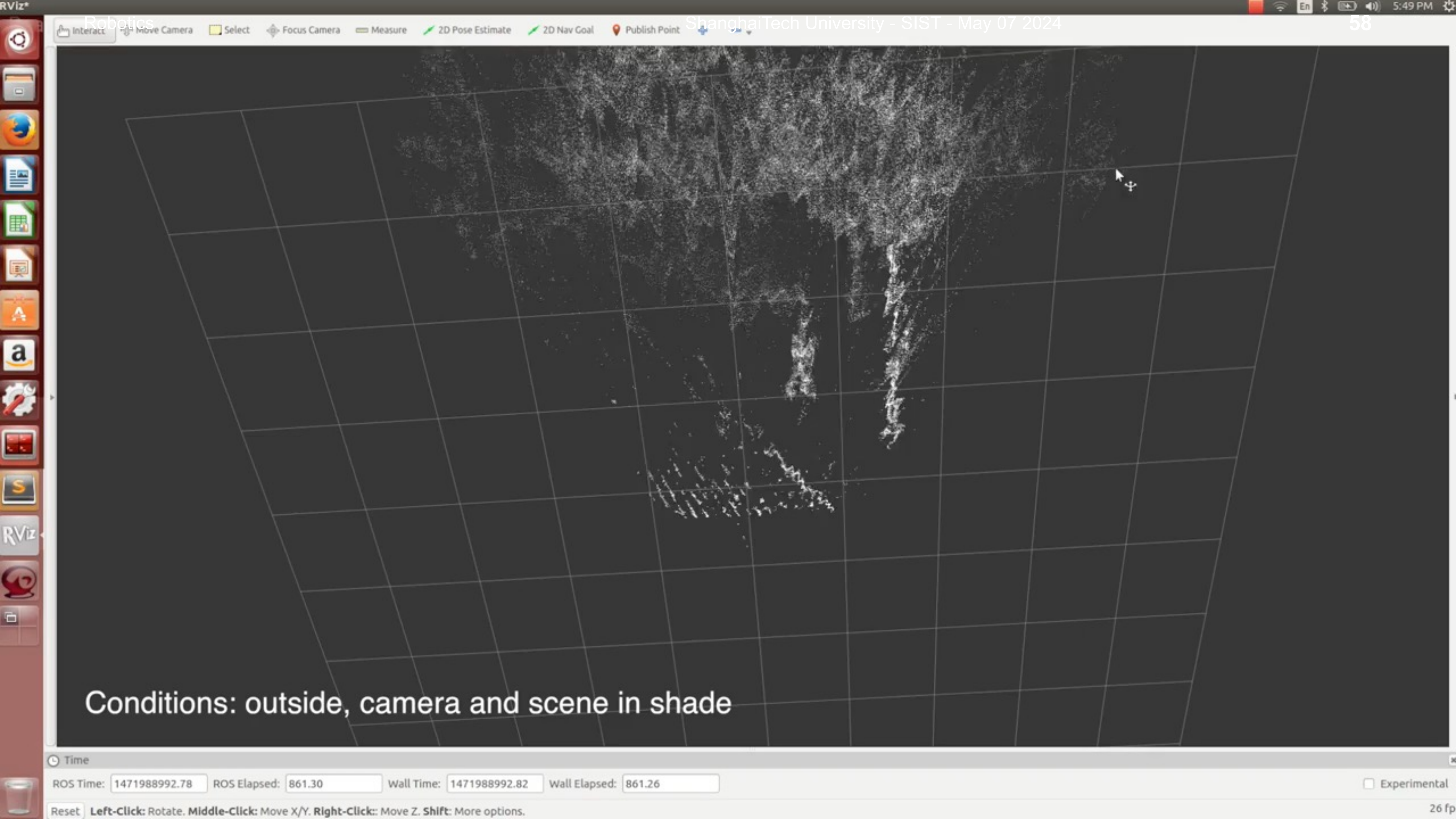
1. Stereo camera calibration -> compute camera relative pose
2. Epipolar rectification -> align images
3. Search correspondences
4. Output: compute stereo triangulation or disparity map
5. Consider baseline and image resolution to compute accuracy!

Device example: intel REALSENSE TECHNOLOGY

- 640x480 stereo infrared cameras
- 1920x1080 HD RGB camera
- Infrared Laser Projector
- Up to 60Hz



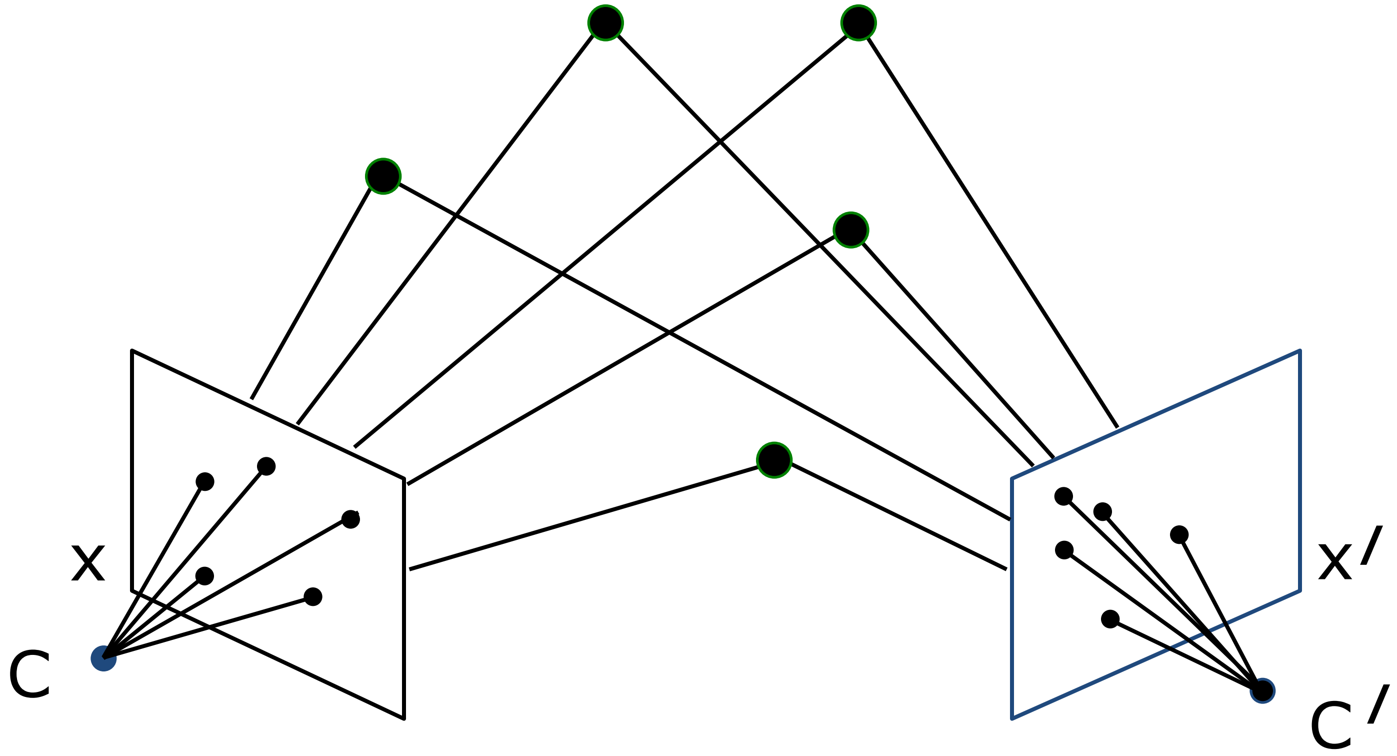

1 CAPTURE
Project invisible light pattern on low texture surfaces like plain walls
Left
Right
Each camera sees a slightly different viewpoint
2 SEARCH
Closer objects shift more than further away objects
ASIC computes depth by calculating shift of every pixel on the image between the left and right images
3 DEPTH



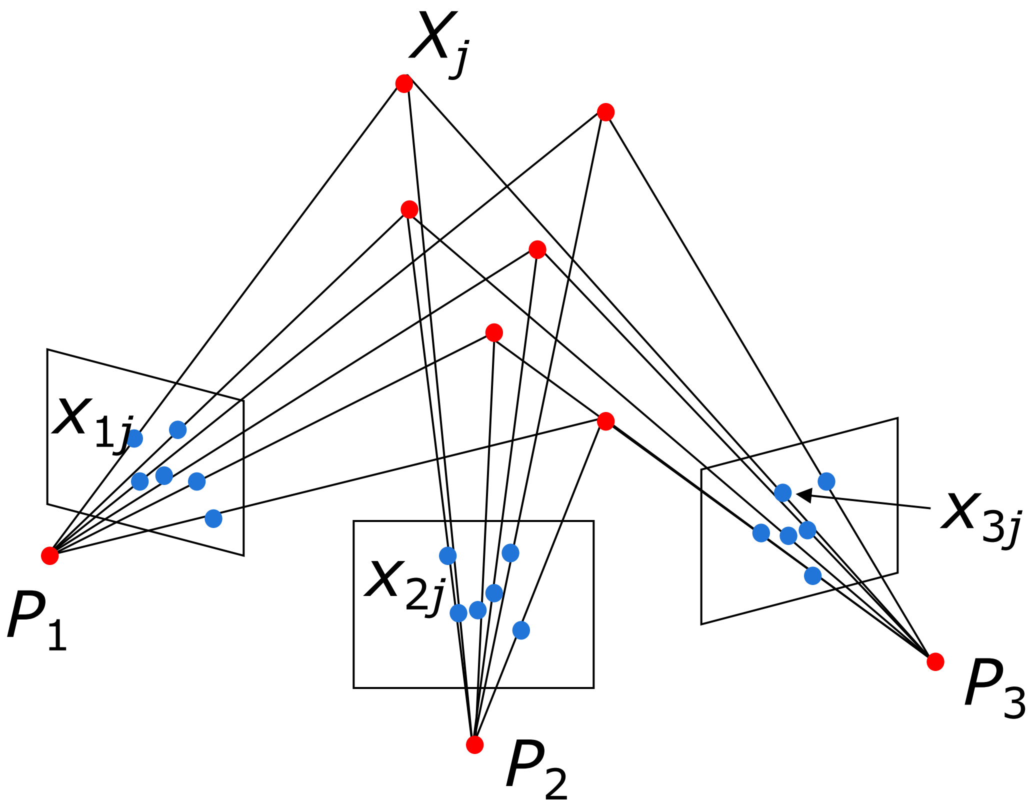
Conditions: outside, camera and scene in shade

Structure from motion

- Given image point correspondences, $\mathbf{x}_i \leftrightarrow \mathbf{x}_i^{/}$,determine R and T
- Rotate and translate camera until stars of rays intersect
- At least 5 point correspondences are needed



Multiple-view structure from motion

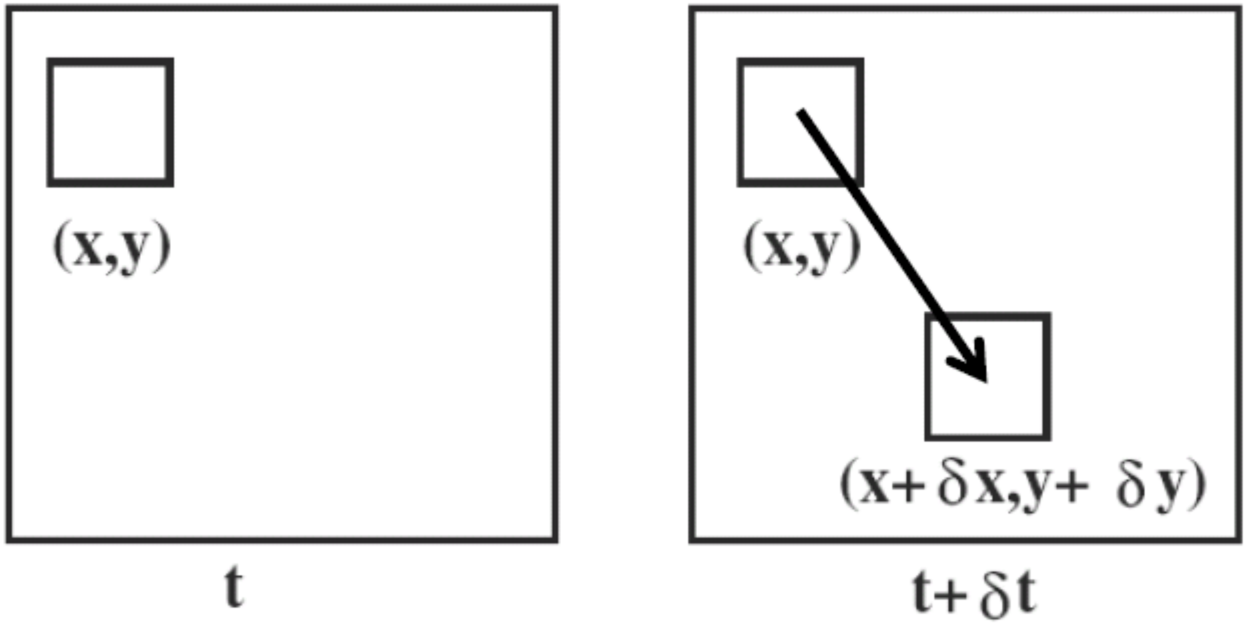


Multiple-view structure from motion

- Results of Structure from motion from 2 million user images from flickr.com - 2,106 images selected, 819,242 points

Optical Flow

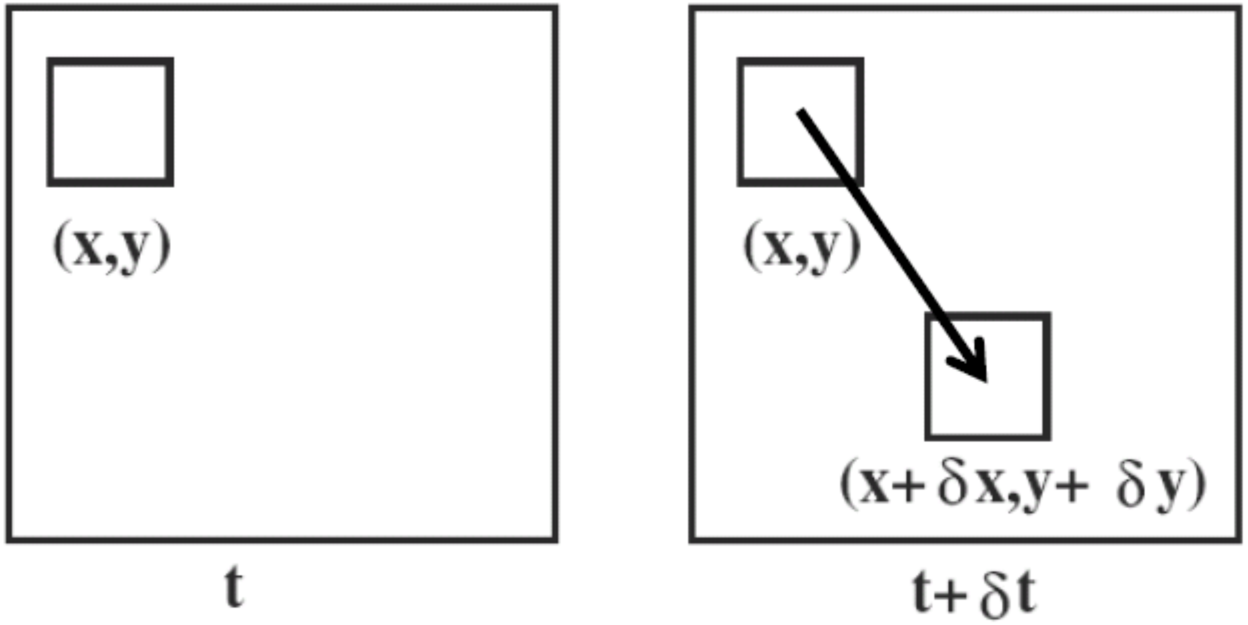
- It computes the motion vectors of all pixels in the image (or a subset of them to be faster)



$$I(x, y, t) = I(x + \delta x, y + \delta y, t + \delta t)$$

- Applications include collision avoidance

Optical Flow



$$\frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = 0$$

$$\frac{\partial I}{\partial x}\frac{\delta x}{\delta t} + \frac{\partial I}{\partial y}\frac{\delta y}{\delta t} + \frac{\partial I}{\partial t}\frac{\delta t}{\delta t} = 0$$

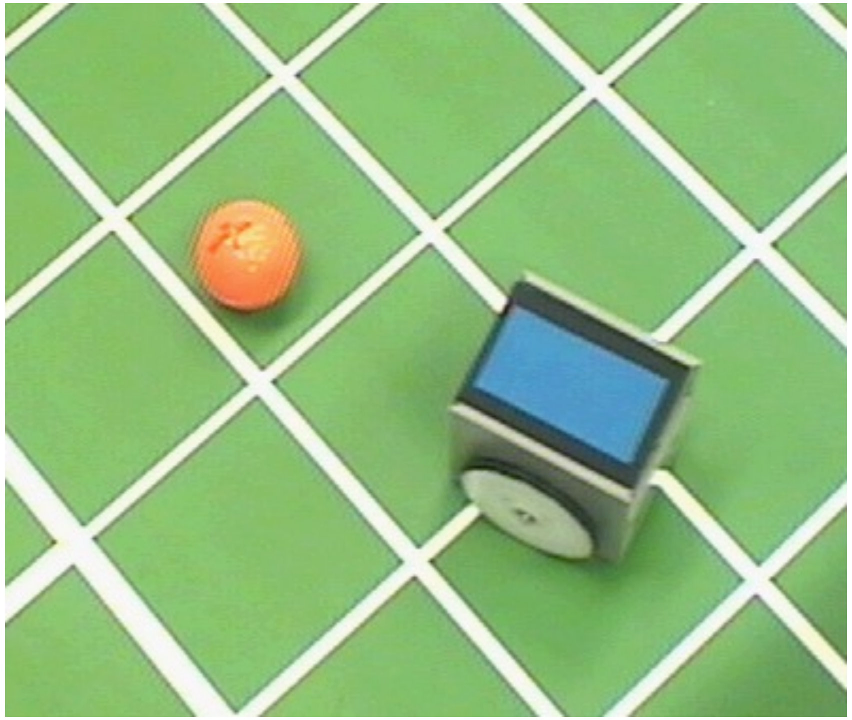
$$\frac{\partial I}{\partial x}v_x + \frac{\partial I}{\partial y}v_y + \frac{\partial I}{\partial t} = 0$$

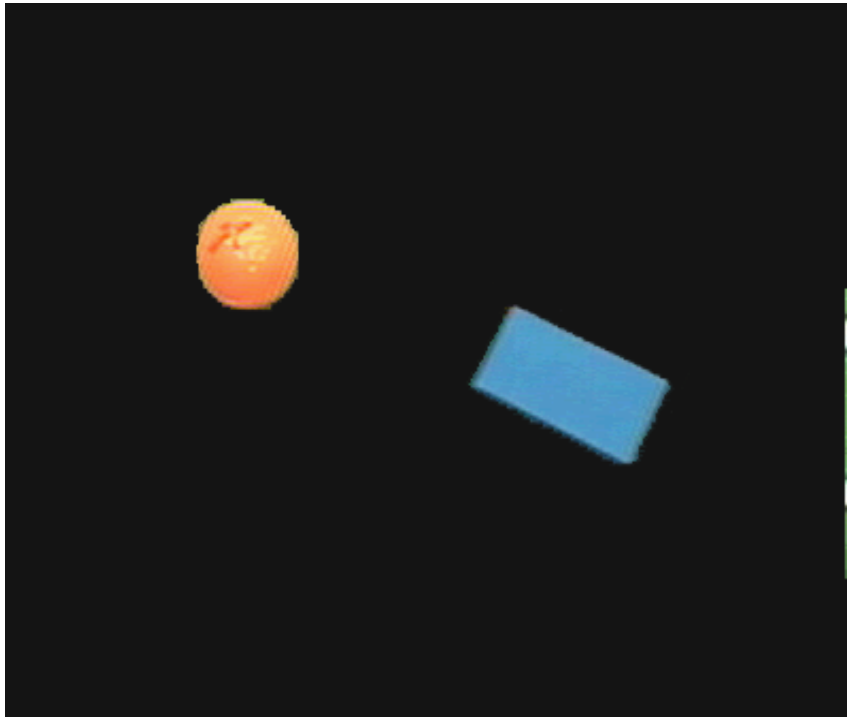
$$I(x, y, t) = I(x + \delta x, y + \delta y, t + \delta t)$$

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t + \cdots$$

Color Tracking

- Motion estimation of ball and robot for soccer playing using color tracking





Color segmentation with fixed thesholds

- Simple: constant thresholding:
  - selection only **iff** RGB values (r,g,b) simultaneously in R, G, and B ranges:
  - six thresholds [Rmin,Rmax], [Gmin,Gmax], [Bmin,Bmax]:

$$R_{min} < r < R_{max} \ \ and \ \ G_{min} < g < G_{max} \ \ and \ \ B_{min} < b < B_{max}$$

- Alternative: YUV color space
  - RGB values encode intensity of each color
  - YUV:
  - U and V together color (or chrominance)
  - Y brightness (or luminosity)
  - bounding box in YUV space => greater stability wrt. changes in illumination