ShanghaiTech University
上海科技大学
School of Information Science and Technology
信息科学与技术学院

Mobile Robotics

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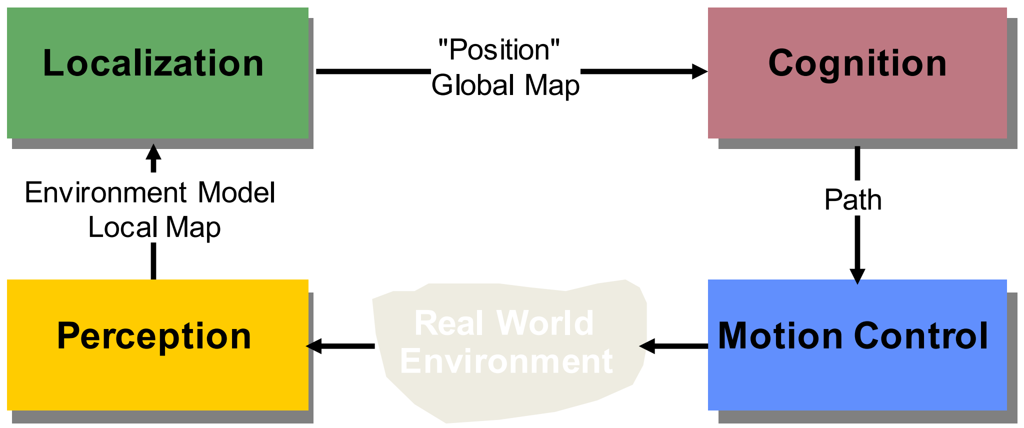
Lecture 7



PERCEPTION

Sensors
Line extraction from laser scans
Uncertainties
Vision

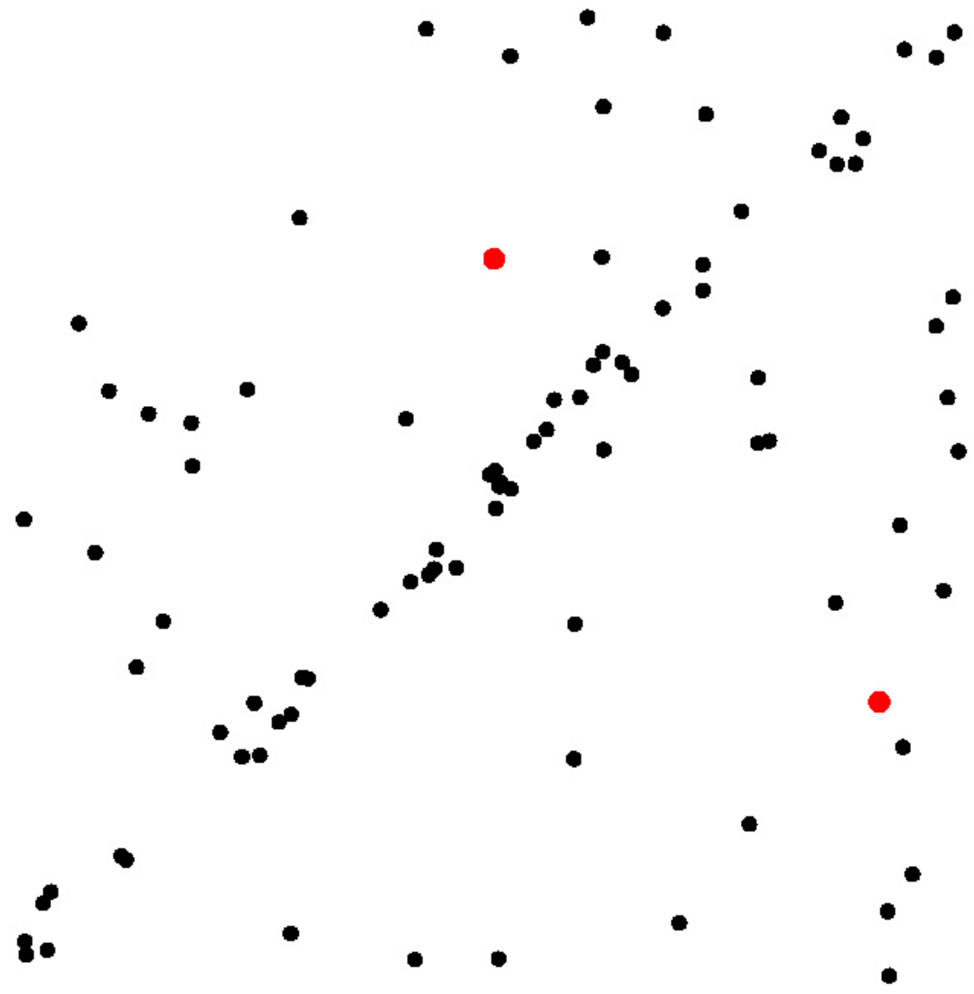
Sildes from Roland Siegwart and Davide Scaramuzza, ETH Zurich

Algorithm 3: RANSAC

- Acronym of Random Sample Consensus.
- It is a generic and robust fitting algorithm of models in the presence of outliers (points which do not satisfy a model)
- RANSAC is not restricted to line extraction from laser data but it can be generally applied to any problem where the goal is to identify the inliers which satisfy a predefined mathematical model.
- Typical applications in robotics are: line extraction from 2D range data (sonar or laser); plane extraction from 3D range data, and structure from motion
- RANSAC is an iterative method and is non-deterministic in that the probability to find a line free of outliers increases as more iterations are used
- Drawback: A nondeterministic method, results are different between runs.

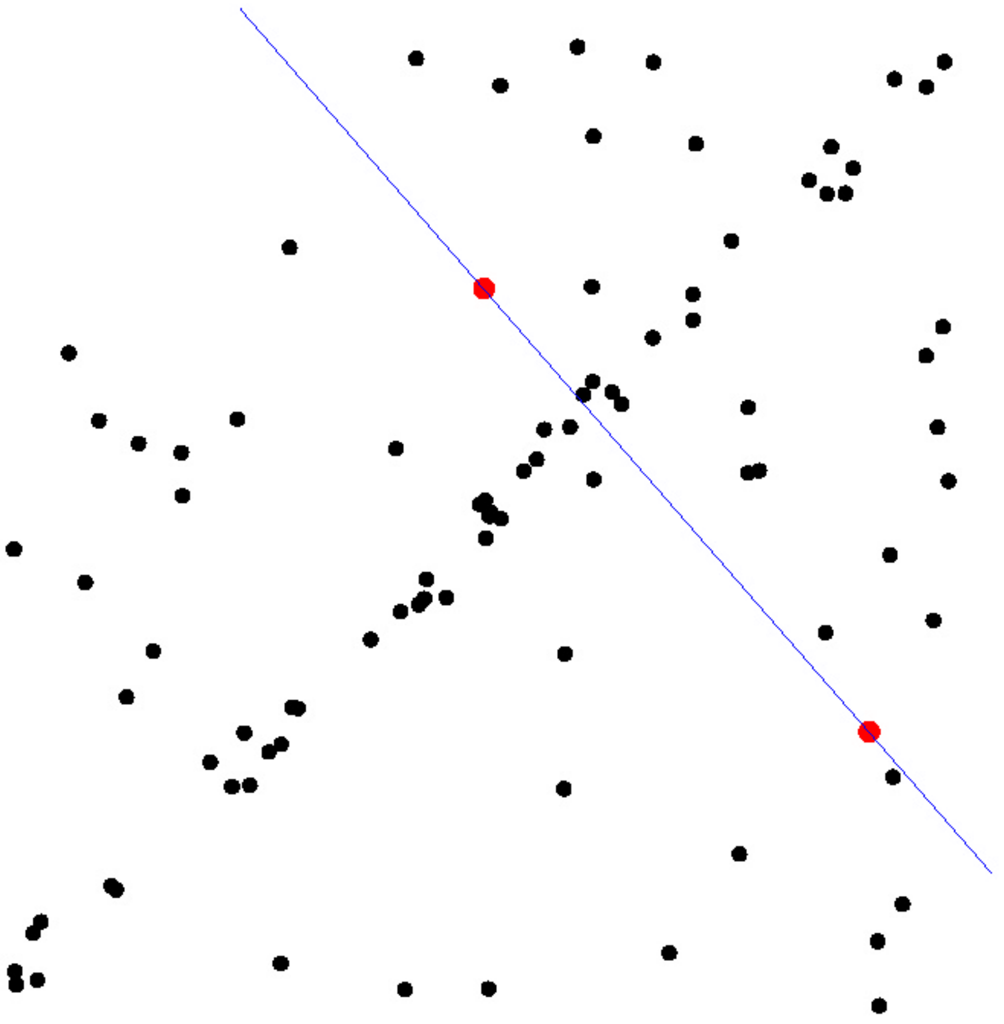
Algorithm 3: RANSAC

Algorithm 3: RANSAC



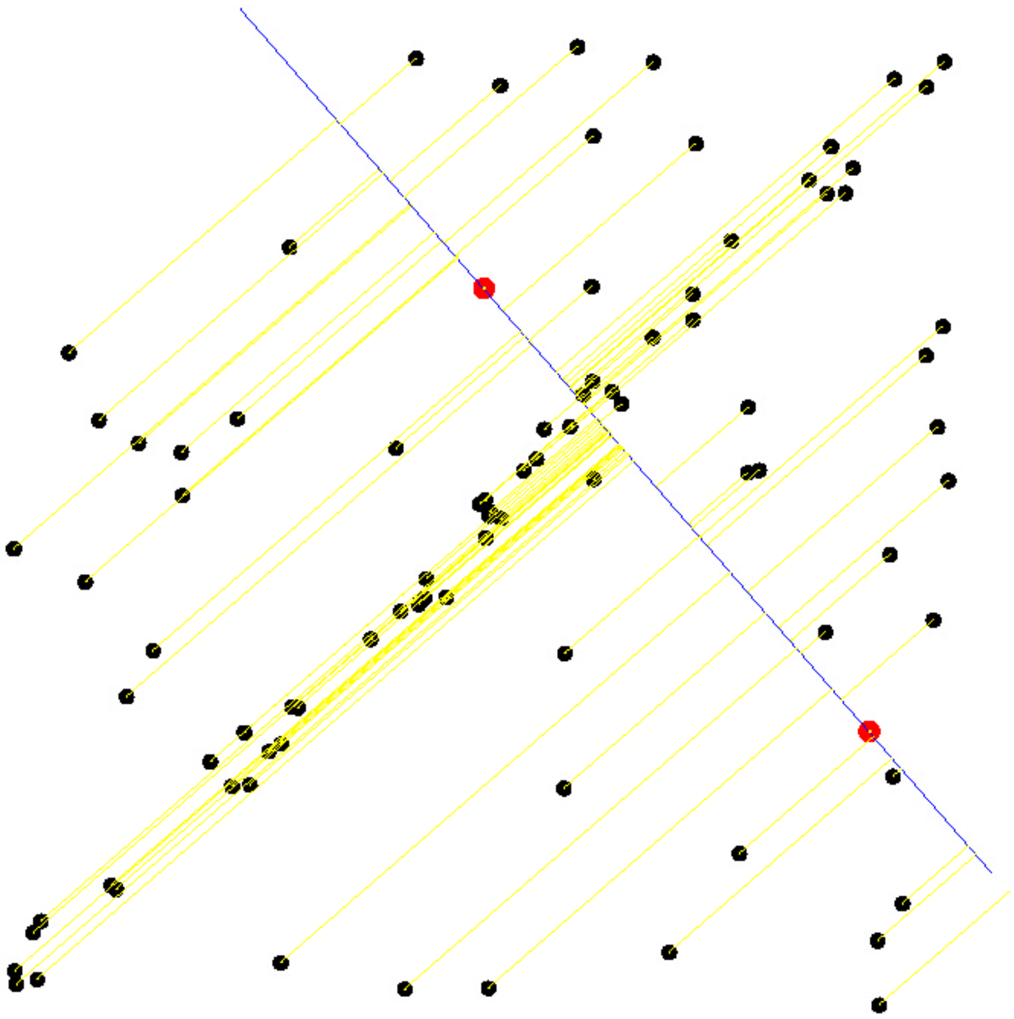
- **Select sample of 2 points at random**

Algorithm 3: RANSAC



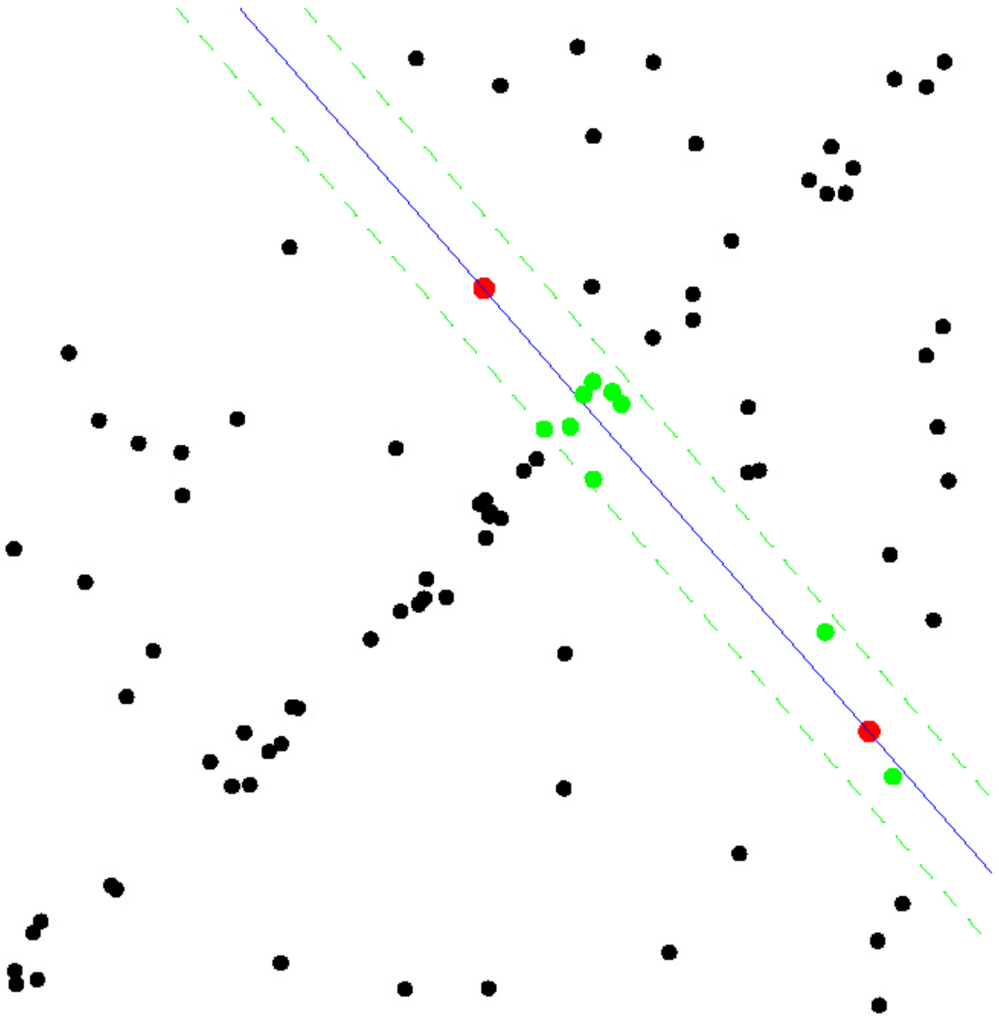
- Select sample of 2 points at random
- **Calculate model parameters that fit the data in the sample**

RANSAC



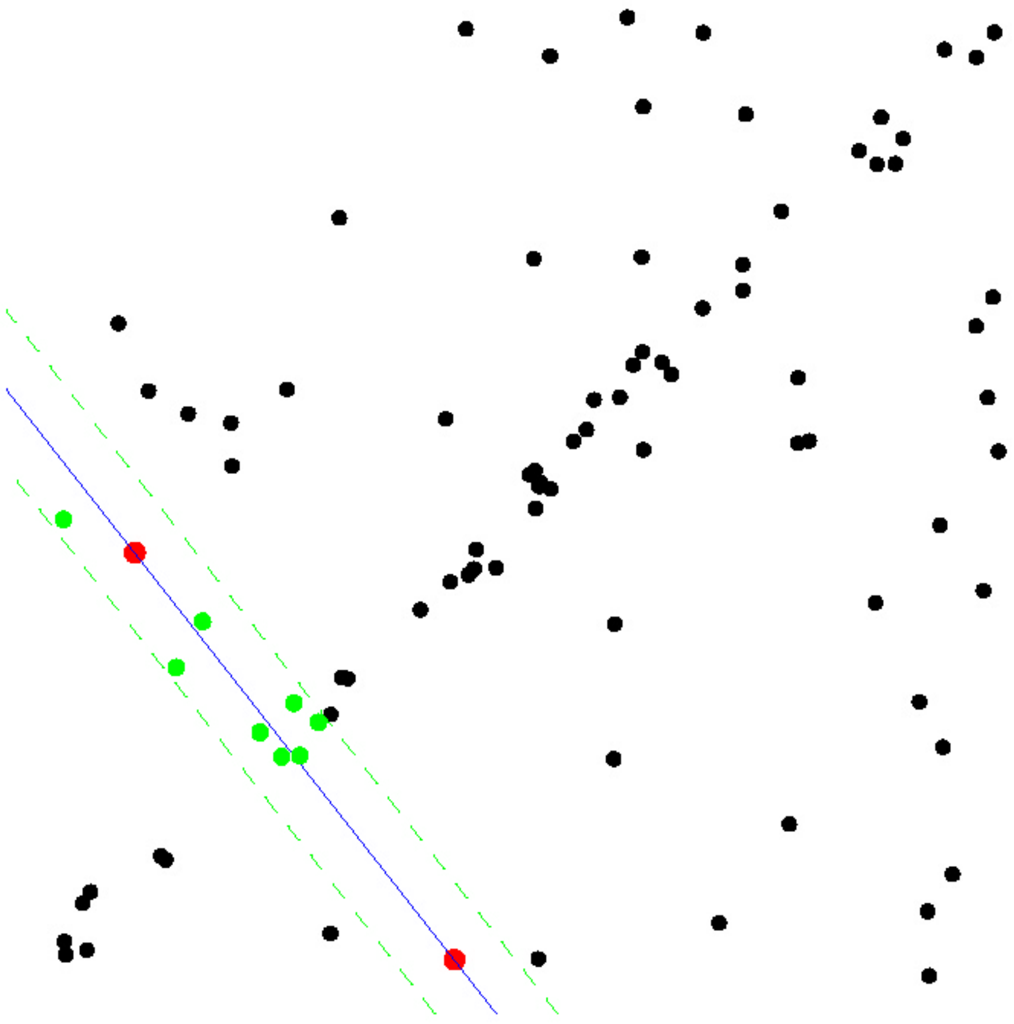
- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- **Calculate error function for each data point**

Algorithm 3: RANSAC



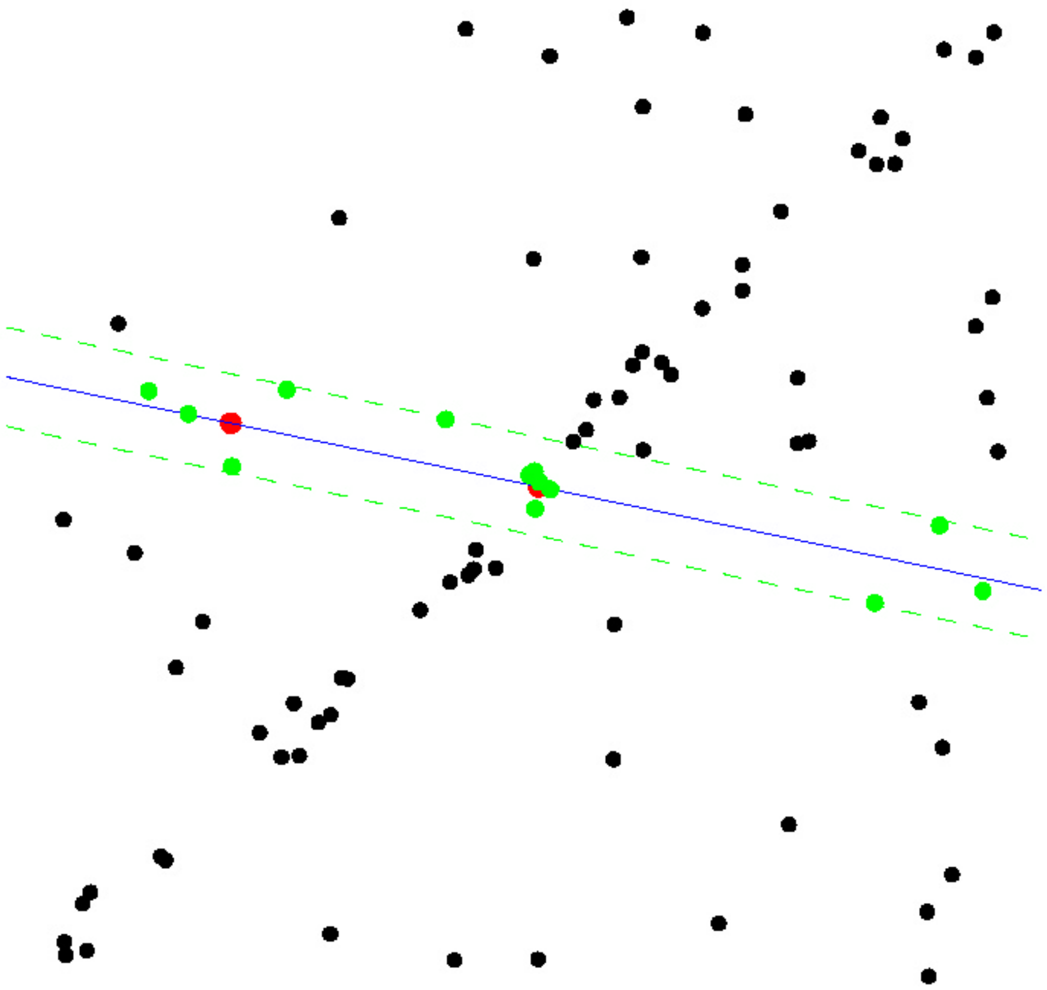
- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- **Select data that support current hypothesis**

Algorithm 3: RANSAC



- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

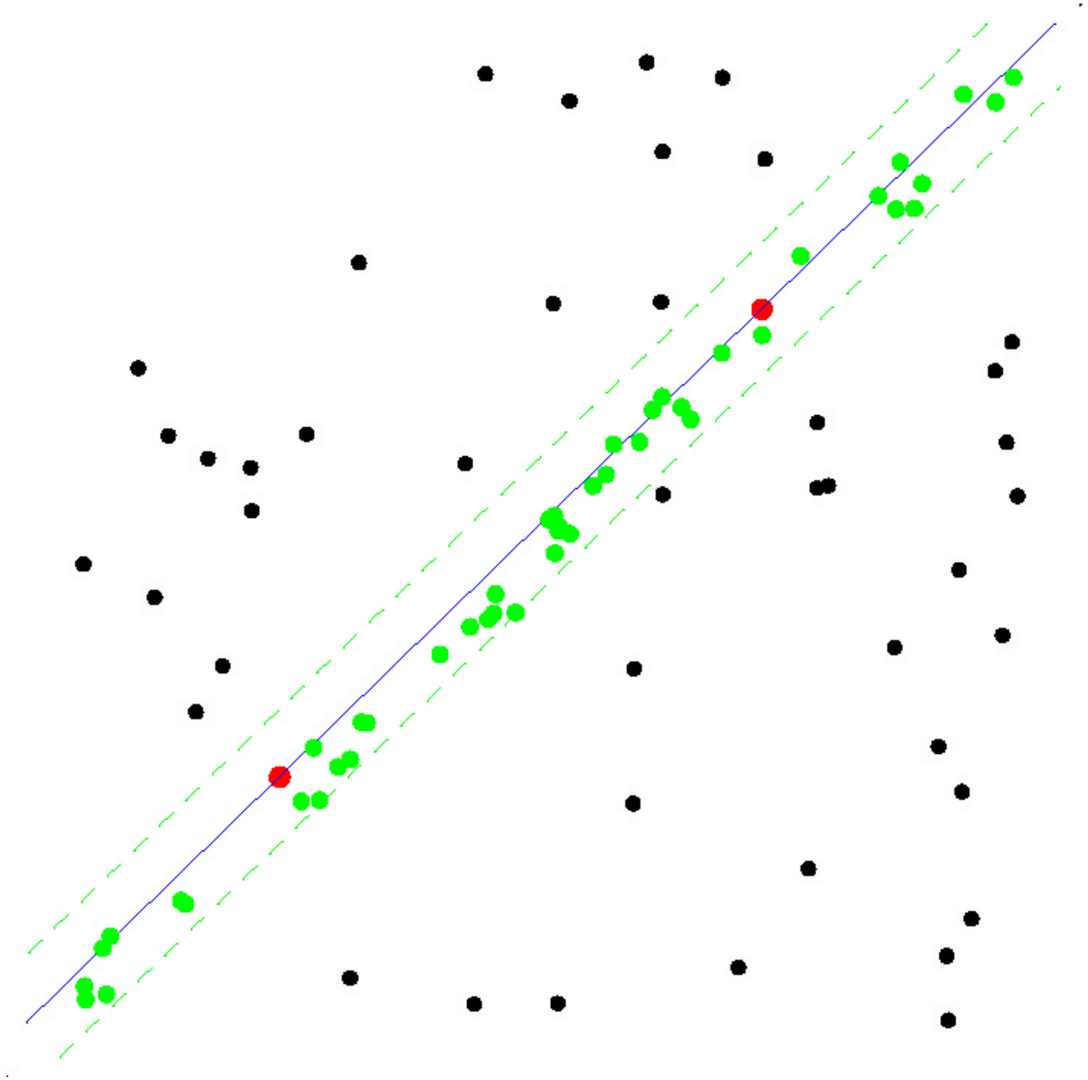
Algorithm 3: RANSAC



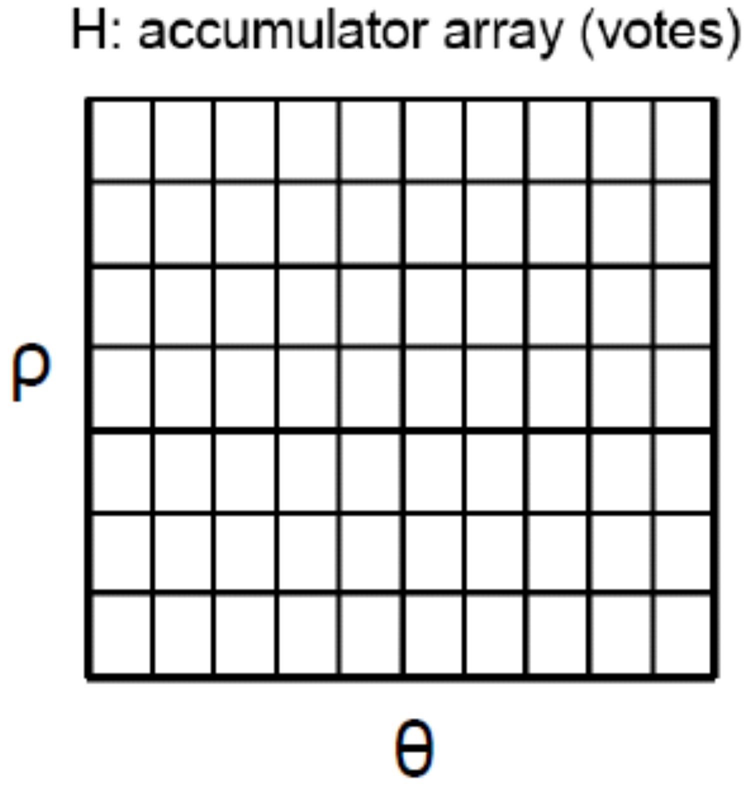
- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

Algorithm 3: RANSAC

ALL-INLIER SAMPLE



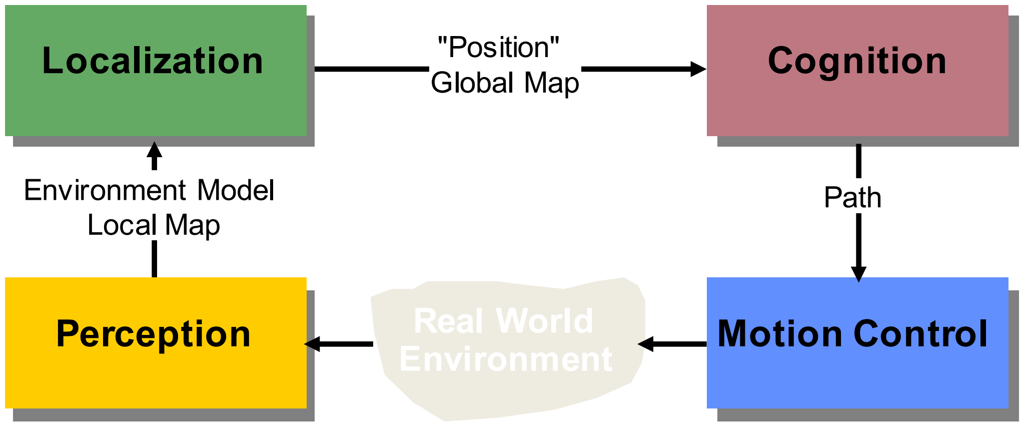
Algorithm 4: Hough-Transform



1. Initialize accumulator H to all zeros
2. For each edge point (x,y) in the image
   - For $\theta = 0$ to 180
     - $\rho = x \cos\theta + y \sin\theta$
     - $H(\theta, \rho) = H(\theta, \rho) + 1$
   - end

   end
3. Find the values of (θ, ρ) where $H(\theta, \rho)$ is a local maximum
4. The detected line in the image is given by $\rho = x \cos\theta + y \sin\theta$

VISION



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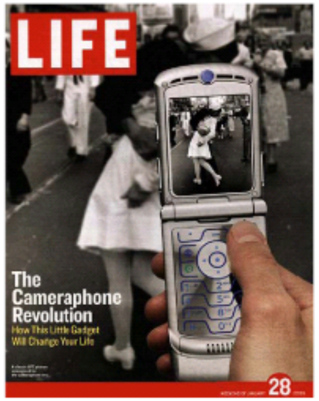
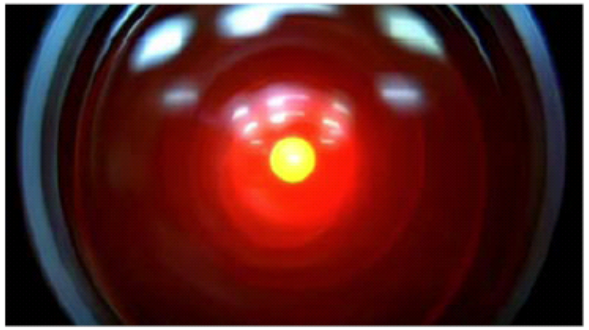
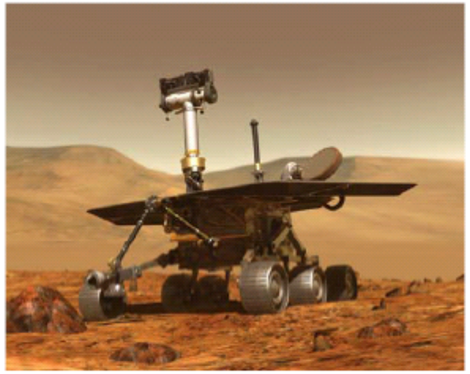
Computer Vision

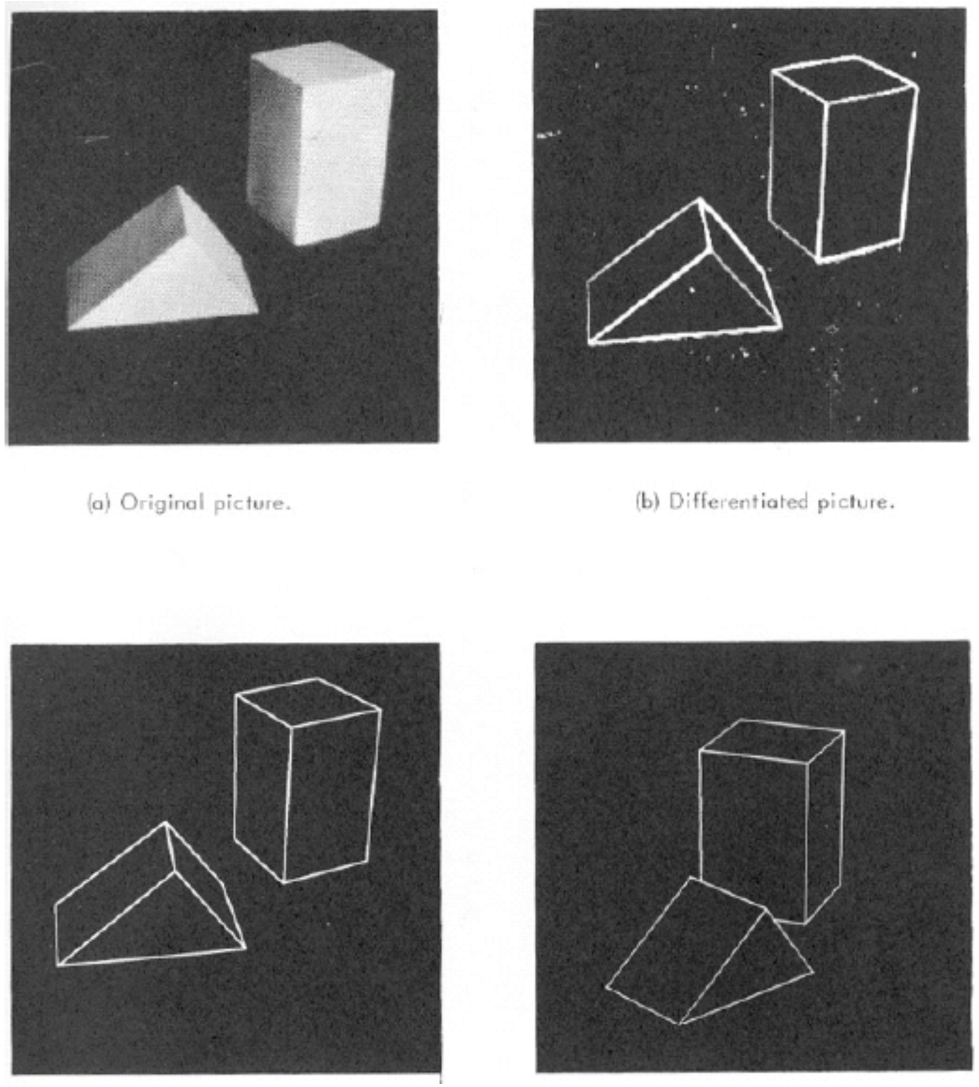





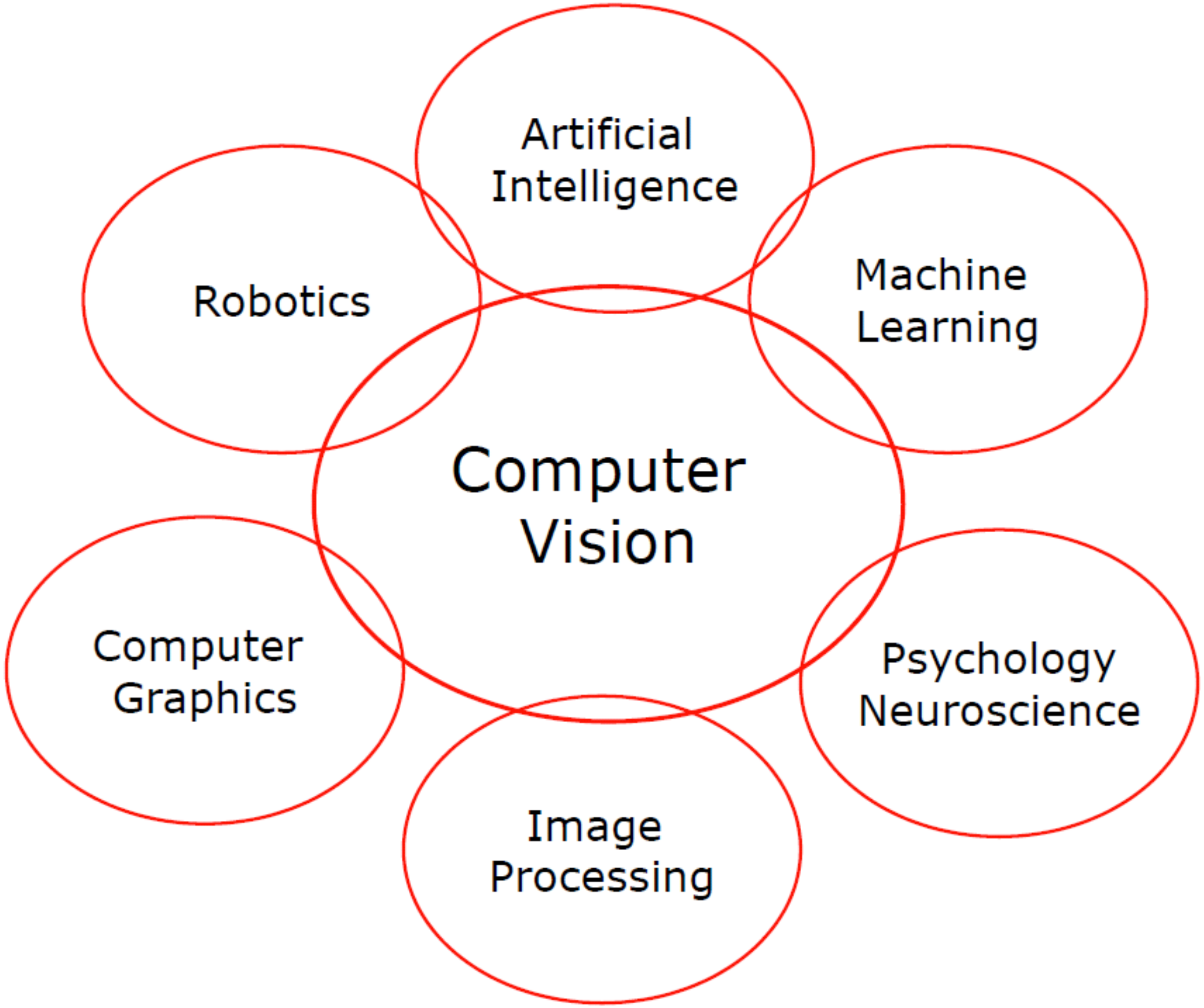



Origins of Computer Vision

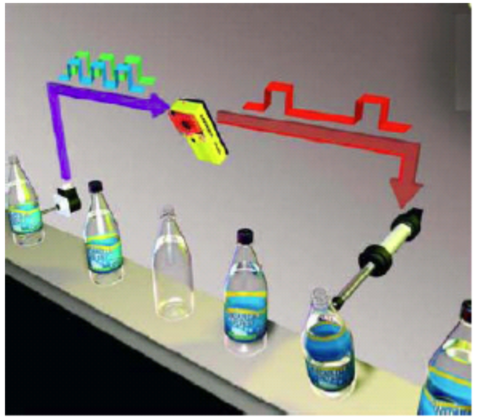


L. G. Roberts, *Machine Perception of Three Dimensional Solids*, Ph.D. thesis, MIT Department of Electrical Engineering, 1963.

Connection to other disciplines

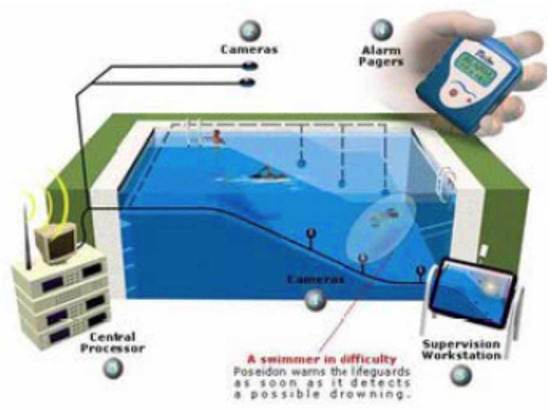


Applications of Computer Vision

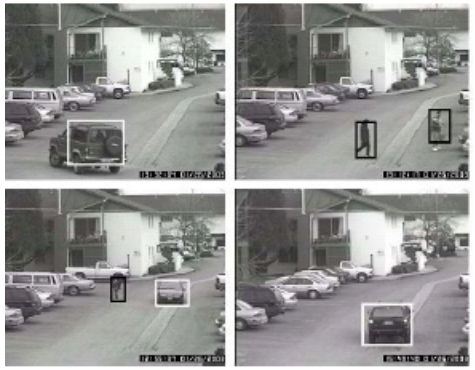


Factory inspection

Reading license plates, checks, ZIP codes



Monitoring for safety (Poseidon)



Surveillance

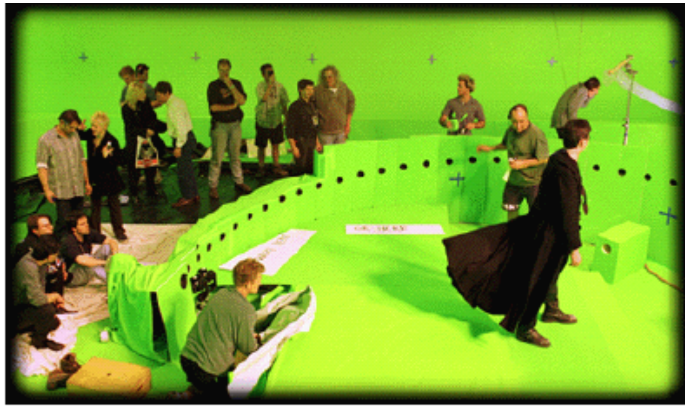
Autonomous driving, robot navigation

Driver assistance (collision warning, lane departure warning, rear object detection)

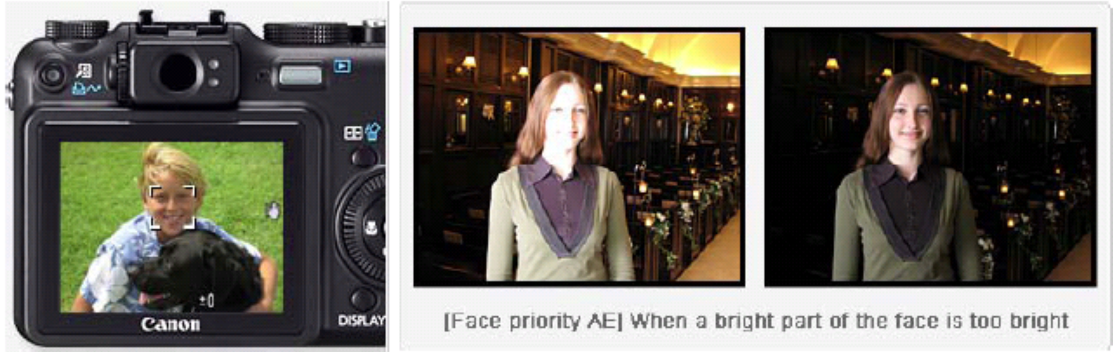
Applications of Computer Vision

Assistive technologies

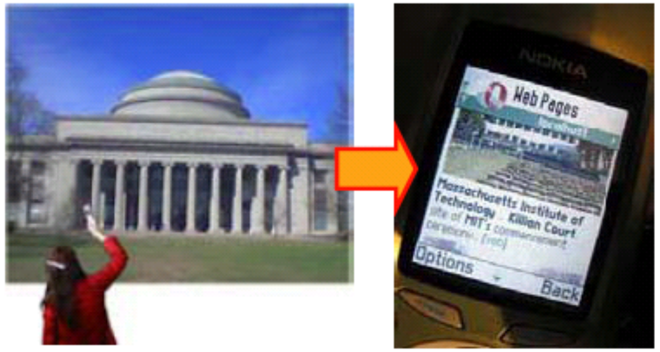
Entertainment (Sony EyeToy)



Movie special effects

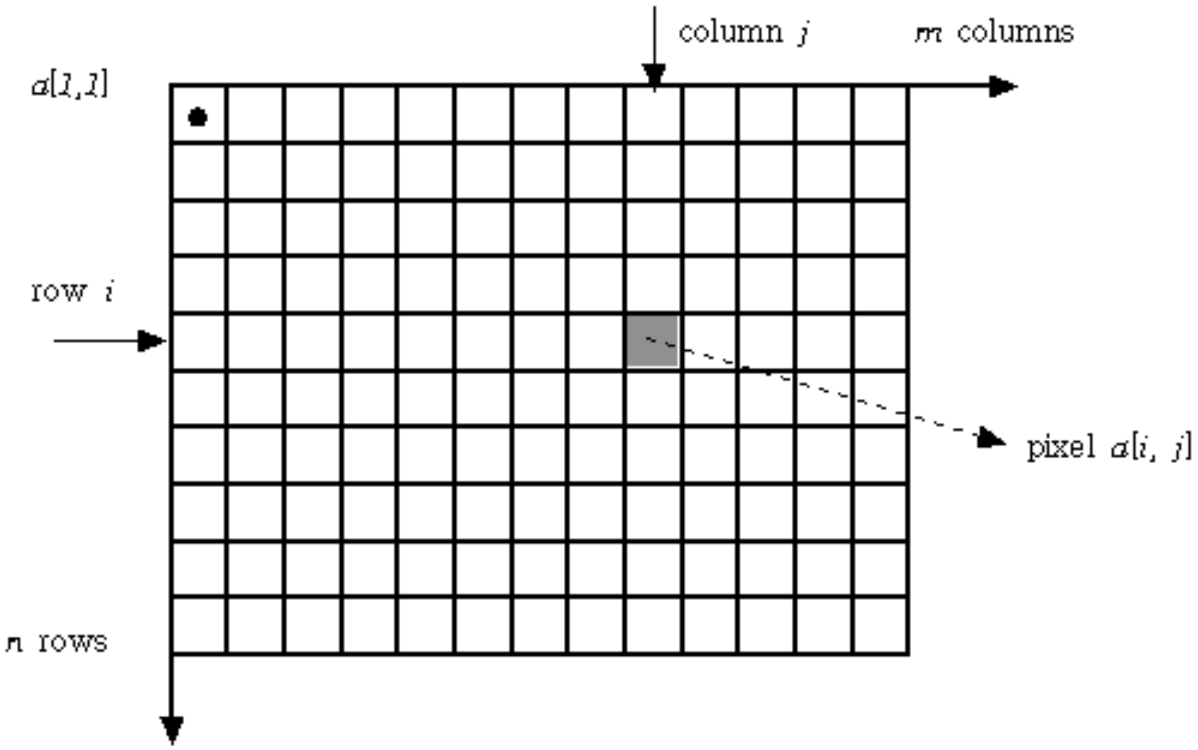


Digital cameras (face detection for setting focus, exposure)



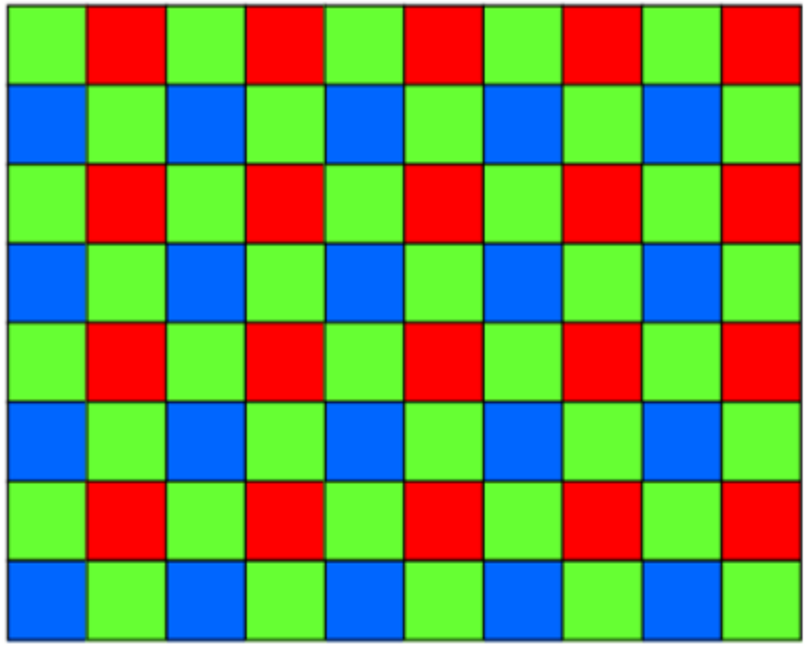
Image

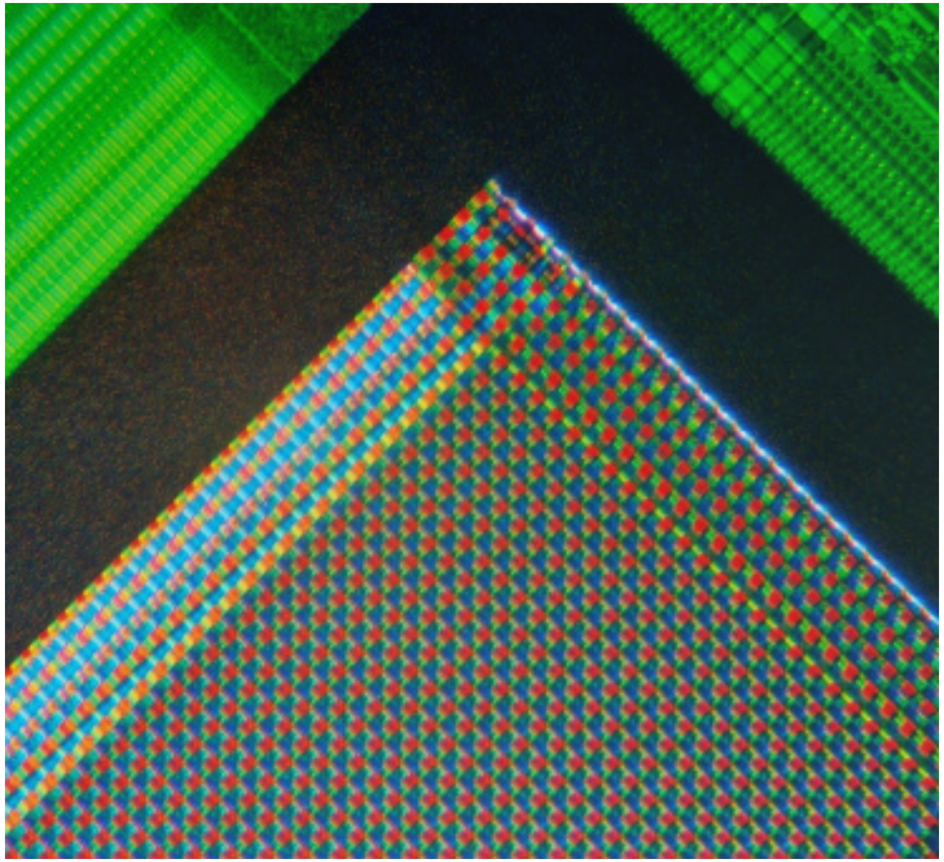
- Image : a two-dimensional array of pixels
- The indices [i, j] of pixels : integer values that specify the rows and columns in pixel values



Digital Color Camera

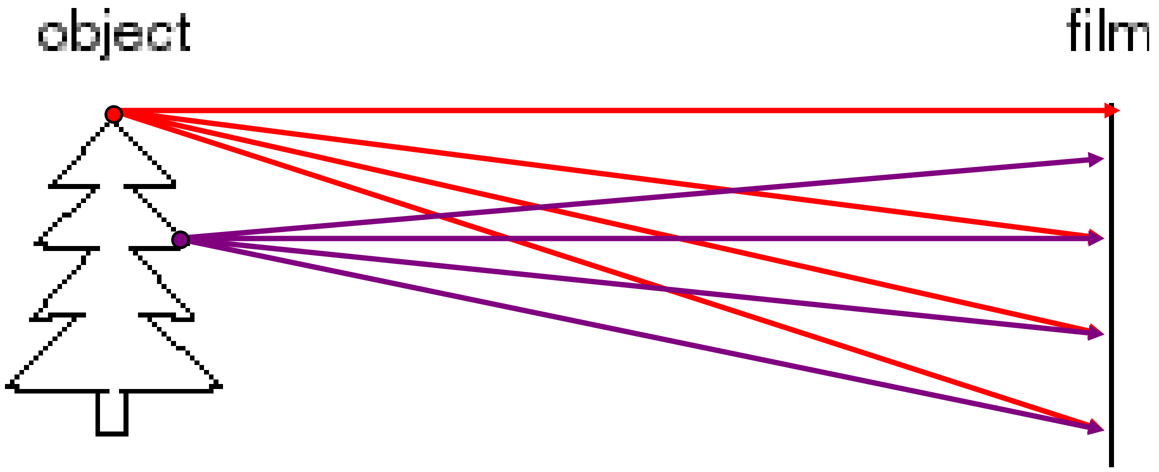
- Bayer Pattern:
  - 50% green, 25% red and 25% blue =>
  - RGBG or GRGB or RGGB.
  - 1 Byte per square
  - 4 squared per 1 pixel
  - More green: eyes are more sensitive to green (nature!)





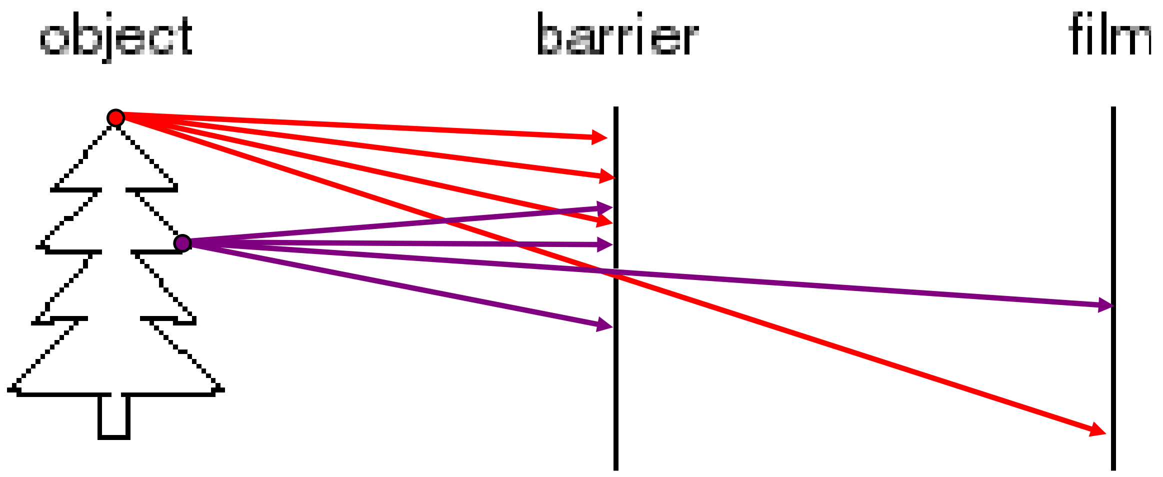
A micrograph of the corner of the photosensor array of a ‘webcam’ digital camera. (Wikimedia)

How do we see the world?



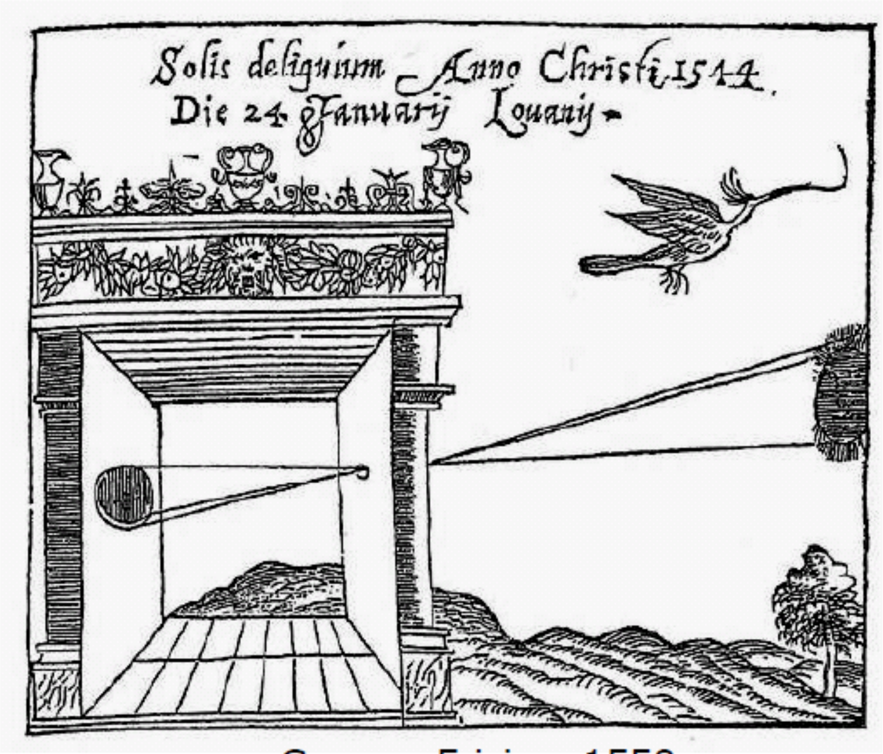
- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**

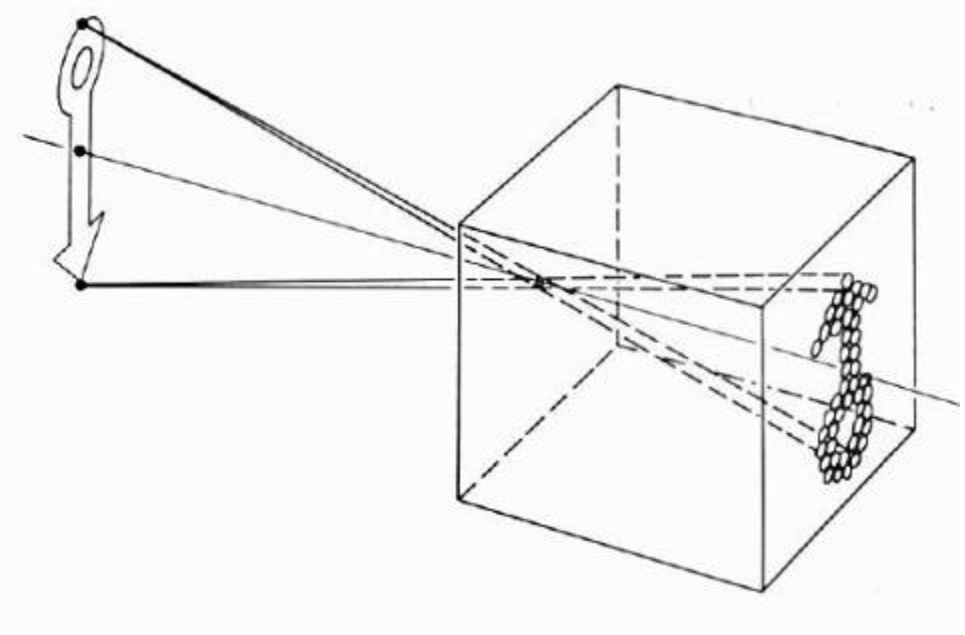
Camera obscura



Gemma Frisius, 1558

- Basic principle known to Mozi (470-390 BC), Aristotle (384-322 BC)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)
- Depth of the room (box) is the effective focal length

Pinhole camera model



- Pinhole model:
  - Captures **pencil of rays** – all rays through a single point
  - The point is called **Center of Projection**
  - The image is formed on the **Image Plane**

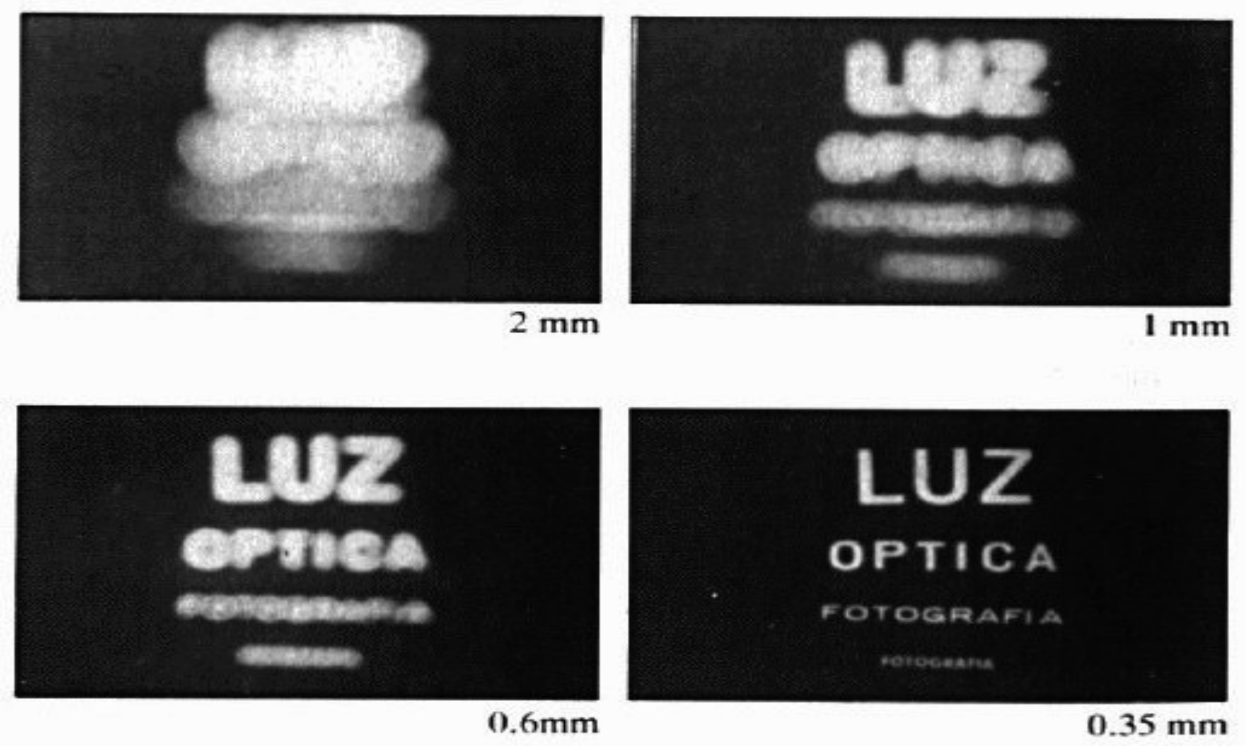
Slide by Steve Seitz

Home-made pinhole camera

Why so blurry?

http://www.debevec.org/Pinhole/

Shrinking the aperture

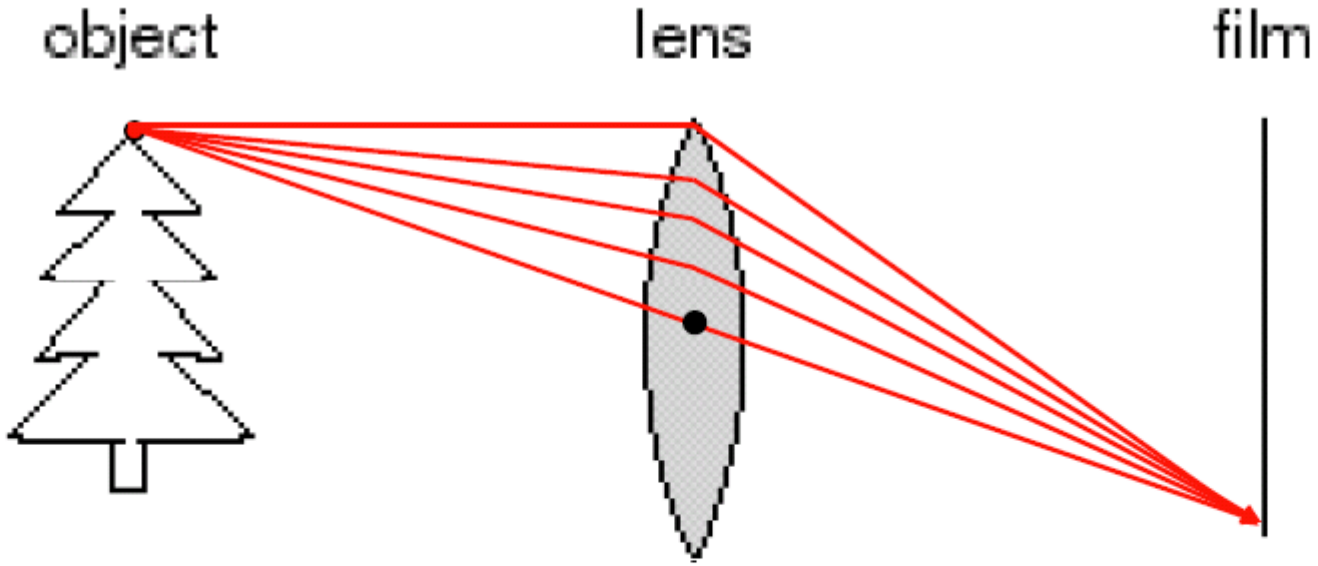


- Why not make the aperture as small as possible?
  - Less light gets through (must increase the exposure)
  - Diffraction effects…

Slide by Steve Seitz

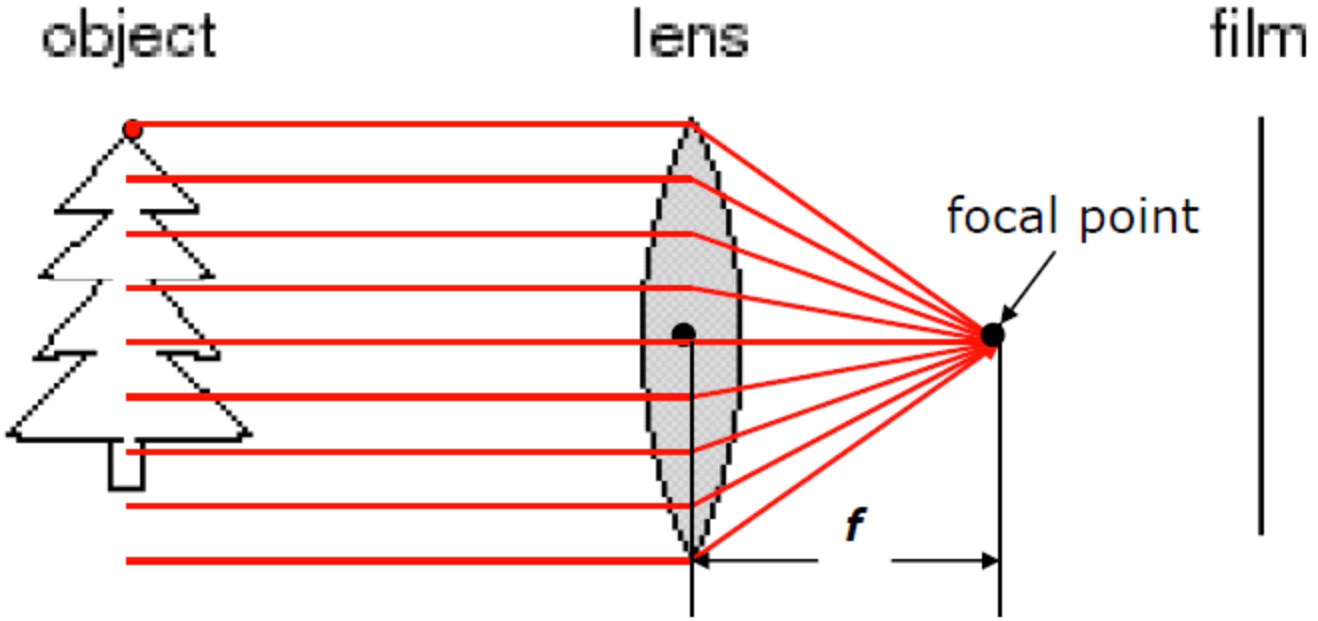
Shrinking the aperture

Solution: adding a lens



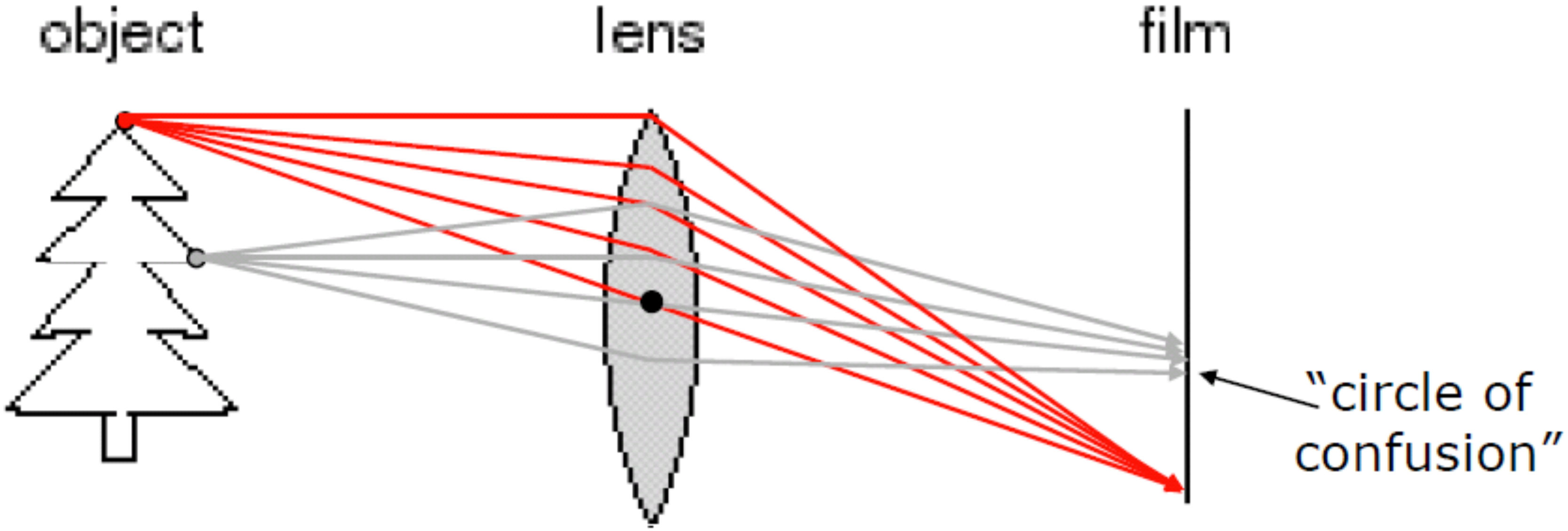
- A lens focuses light onto the film
  - Rays passing through the center are not deviated

Solution: adding a lens



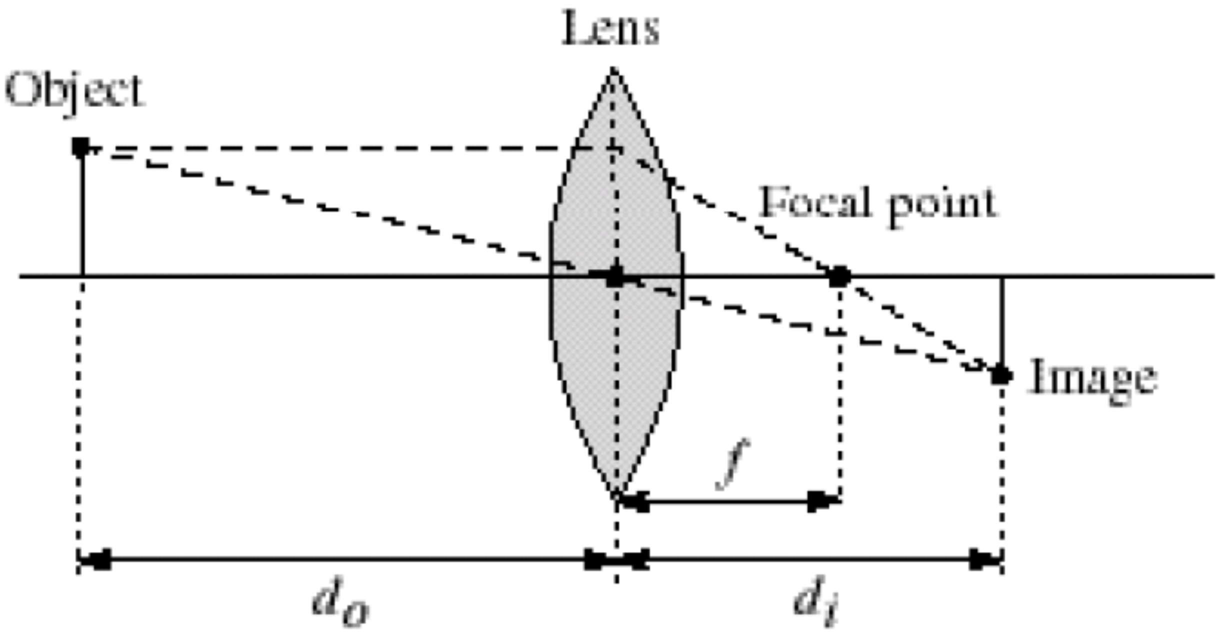
- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

Solution: adding a lens



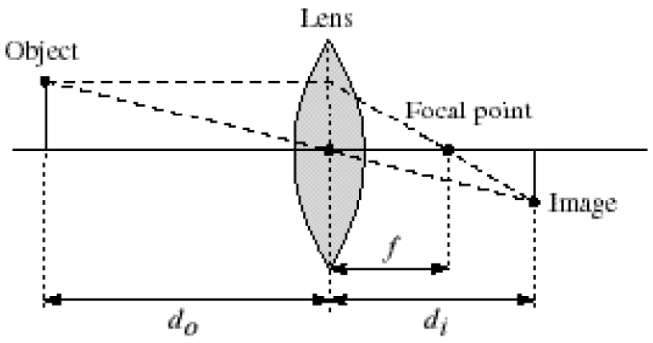
- A lens focuses light onto the film
  - There is a specific distance at which objects are "in focus"
    - other points project to a "circle of confusion" in the image

Thin lenses



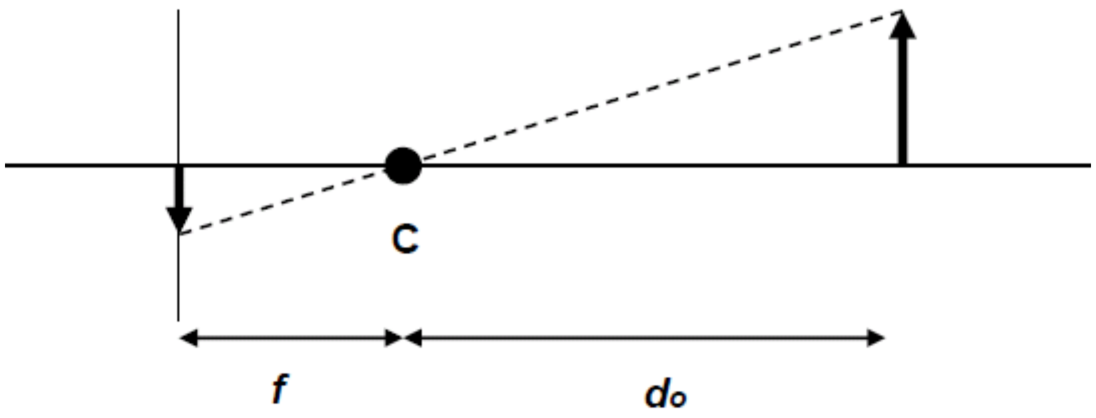
- Thin lens equation: $$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$
  - Any object point satisfying this equation is in focus
  - This formula can also be used to estimate roughly the distance to the object ("Depth from Focus")

Pin-hole approximation



$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

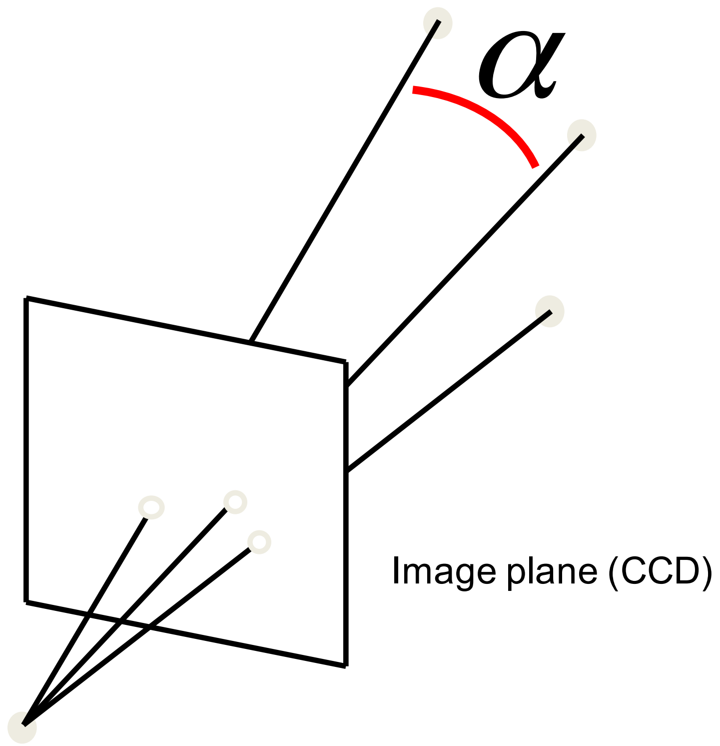
$$d_0 >> d_i \Rightarrow \frac{1}{d_i} \approx \frac{1}{f} \Rightarrow d_i \approx f$$



C = "optical center"

Pin-hole Model

Perspective camera



- For convenience, the image plane is usually represented in front so that the image preserves the same orientation (i.e. not flipped)
- **Notice: a camera does not measure distances but angles! Therefore it is a "bearing sensor"**

Perspective Projection onto the image plane

- To project a 3D scene point P = (x,y,z) [meters] onto the camera image plane p=(u,v) [pixels] we need to consider:
  - Pixelization: size of the pixel and position of the CCD with respect to the optical center
  - Rigid body transformation between camera and scene
- u = v = 0: where z-Axis passes trhough center of lens – z-Azis prependicular to lens (coincident with optical axis)

$$u = \frac{f}{z} \cdot x$$

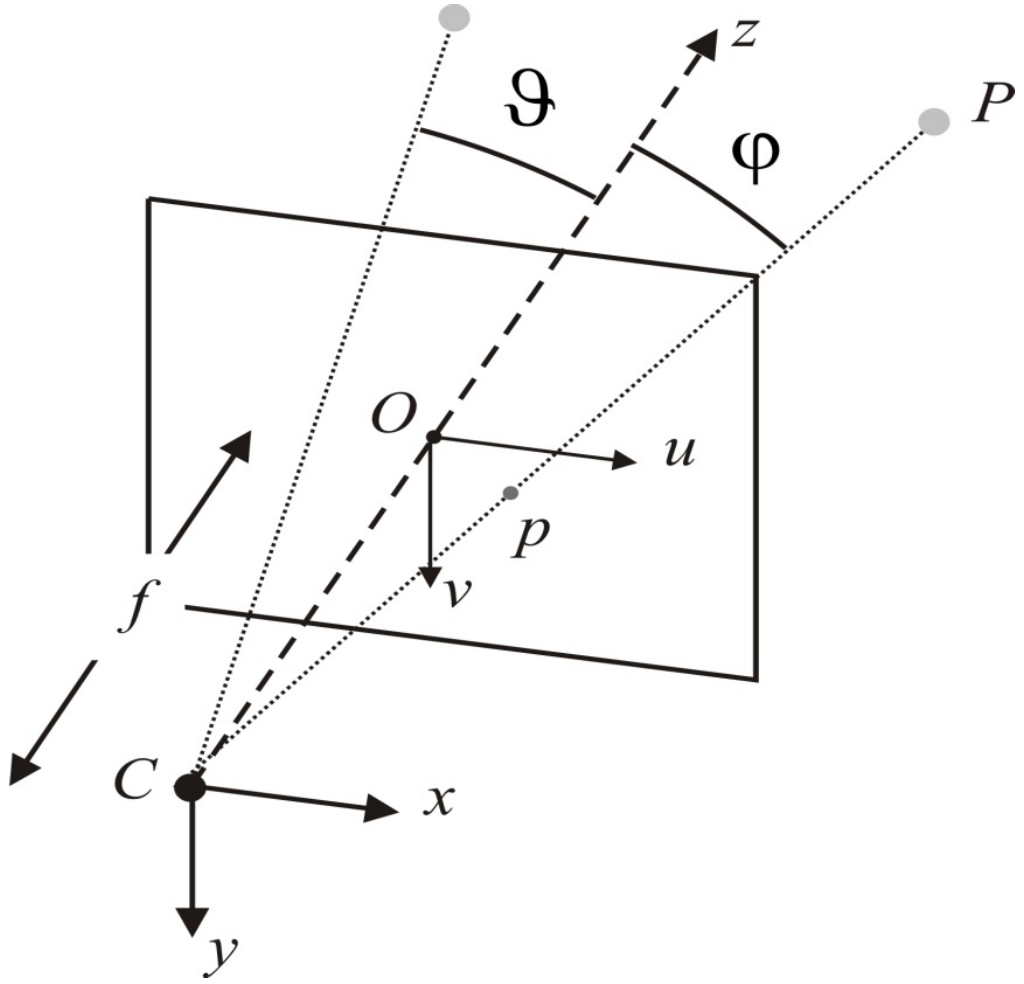
$$v = \frac{f}{z} \cdot y$$

Simple case (without pixelization)

$$u = k_u \frac{f}{z} \cdot x + u_0$$

$$v = k_v \frac{f}{z} \cdot y + v_0$$

With pixelization
u_0, v_0 are the coordinates of the optical center
Ku and Kv are in [pxl/m]



Projection onto the image plane

- Observe that we can also rewrite this

$$u = k_u \frac{f}{z} \cdot x + u_0$$

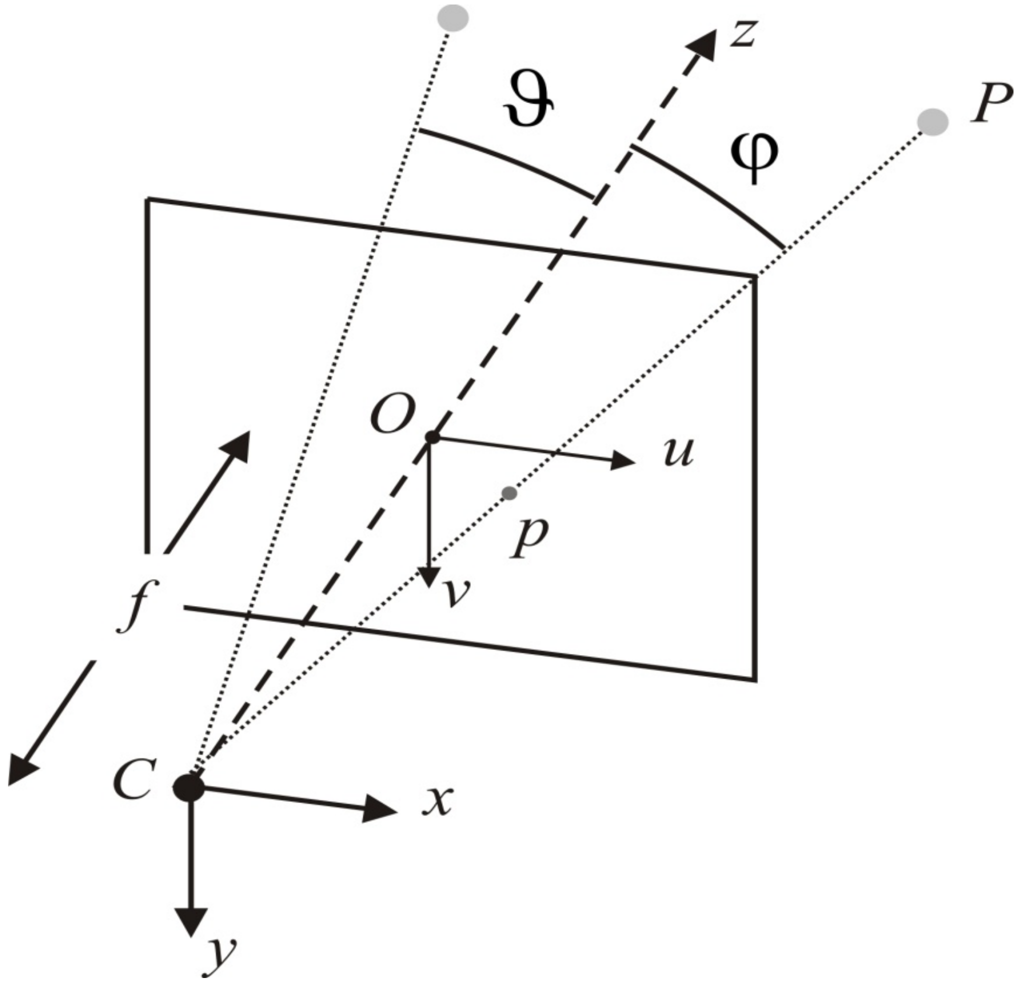
$$v = k_v \frac{f}{z} \cdot y + v_0$$

in matrix form (λ - homogeneous coordinates)

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} k_u f & 0 & u_0 \\ 0 & k_v f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Or alternatively

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



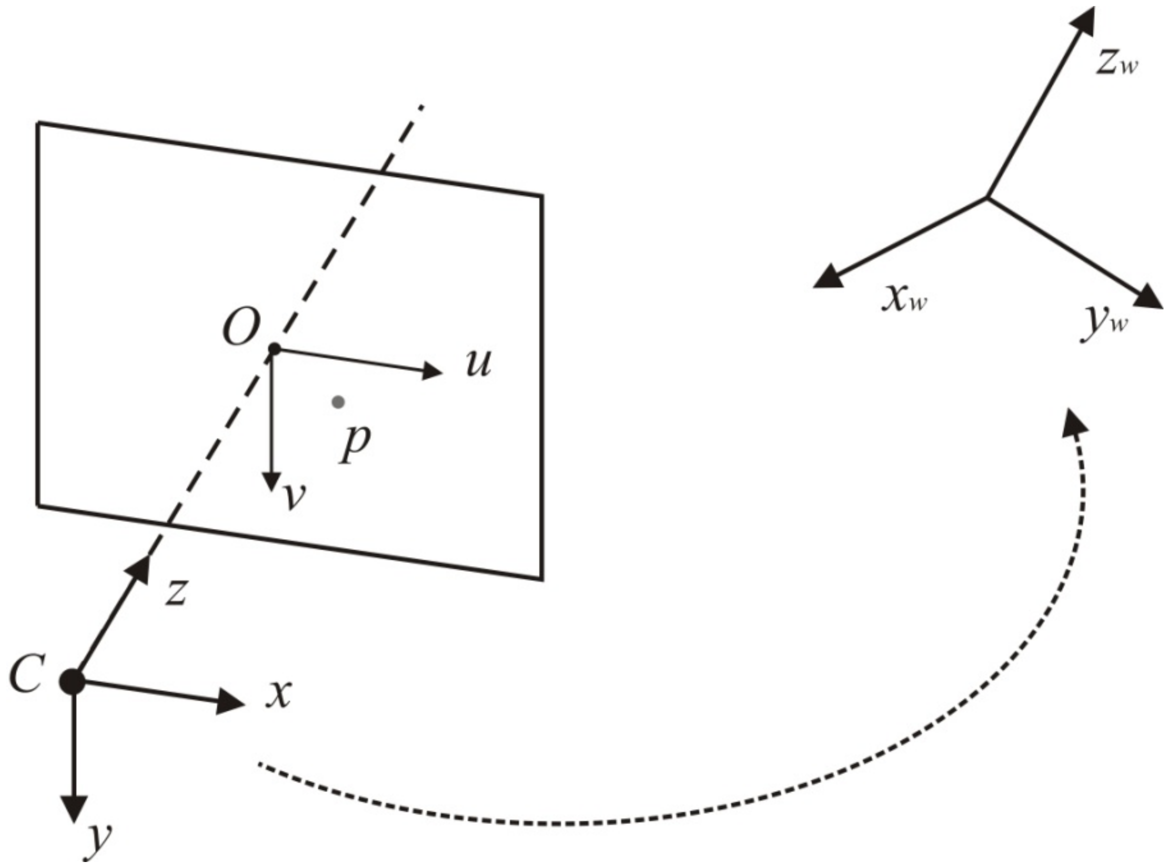
Projection onto the image plane

- Rigid body transformation from the World to the Camera reference frame

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T$$

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A \cdot R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T$$

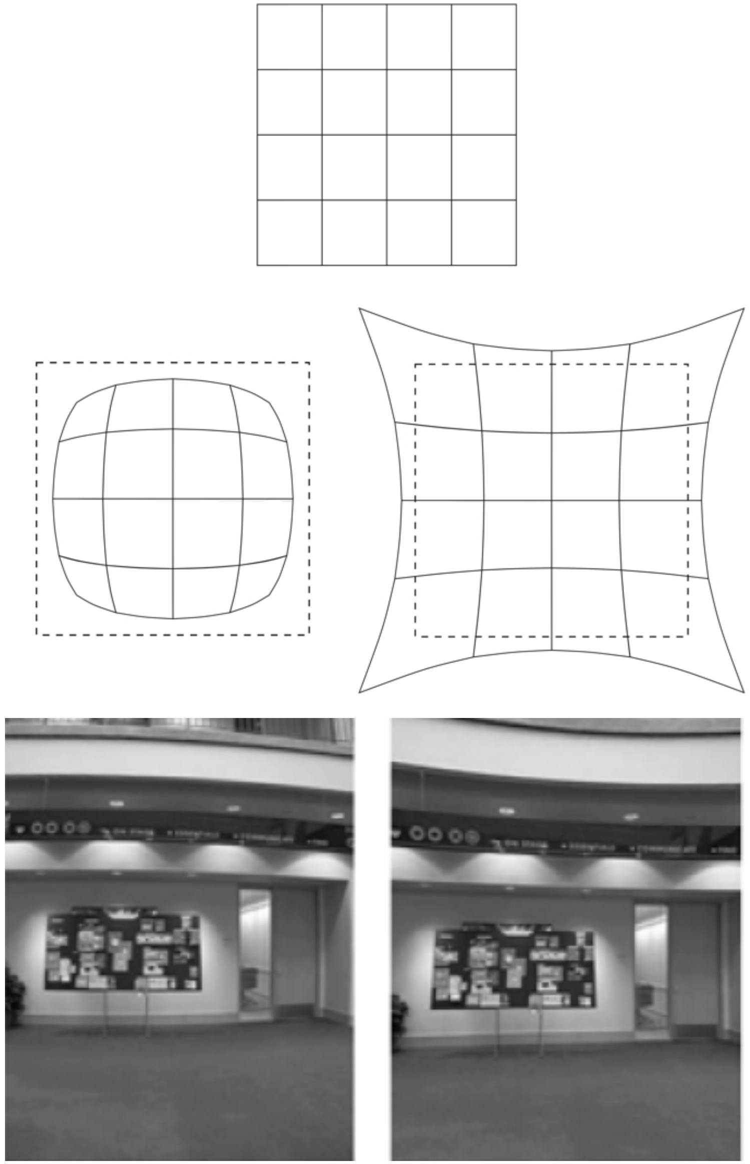


Radial distortion

$$\begin{bmatrix} u_d \\ v_d \end{bmatrix} = (1 + k_1 r^2) \begin{bmatrix} u - u_0 \\ v - v_0 \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$$

where

$$r^2 = (u - u_0)^2 + (v - v_0)^2 .$$



Barrel distortion Pincushion distortion

Camera Calibration

- How many parameters do we need to model a camera?

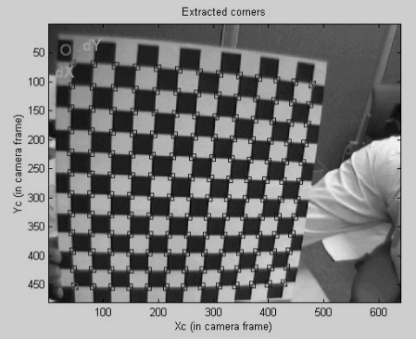
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T \qquad \begin{bmatrix} u_d \\ v_d \end{bmatrix} = (1 + k_1 \rho^2) \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

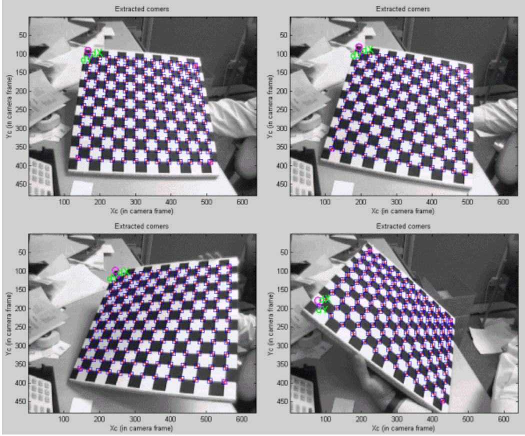
- 5 “intrinsic” parameters: α_u, α_v, u_0, v_0, k_1
- Camera pose?
- 6 “extrinsic” parameters (or 0 if the world and the camera frames coincide)

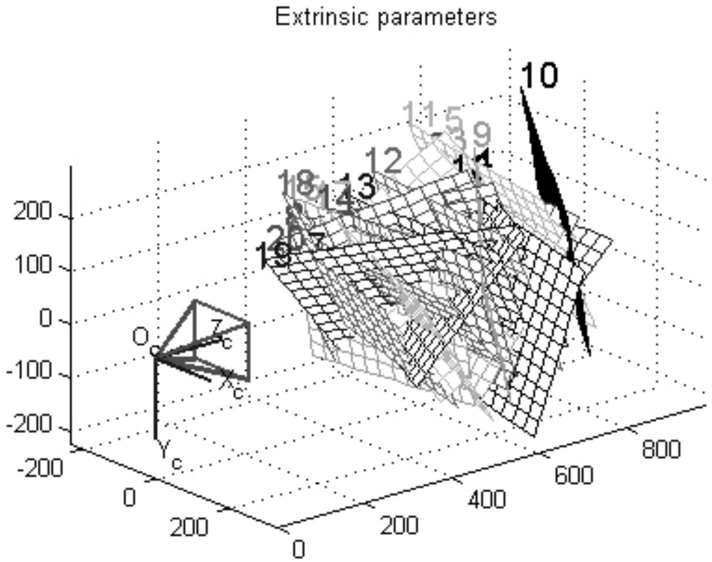
Camera Calibration: how does it work?

- Calibration: measuring accurately **intrinsic** + **extrinsic** parameters of the camera model.
- Parameters: govern mapping from scene points to image points
- Idea: known:
  - pixel coordinates of image points *p*
  - 3D coordinates of the corresponding scene points *P*
  - => compute the unknown parameters A, R, T by solving the perspective projection equation

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot R \begin{bmatrix} x_w \\ y_w \\ z \end{bmatrix} + T$$

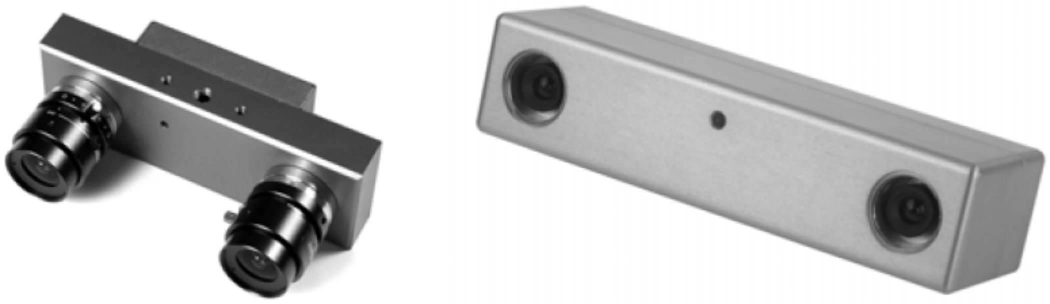






How do we measure distances with cameras?

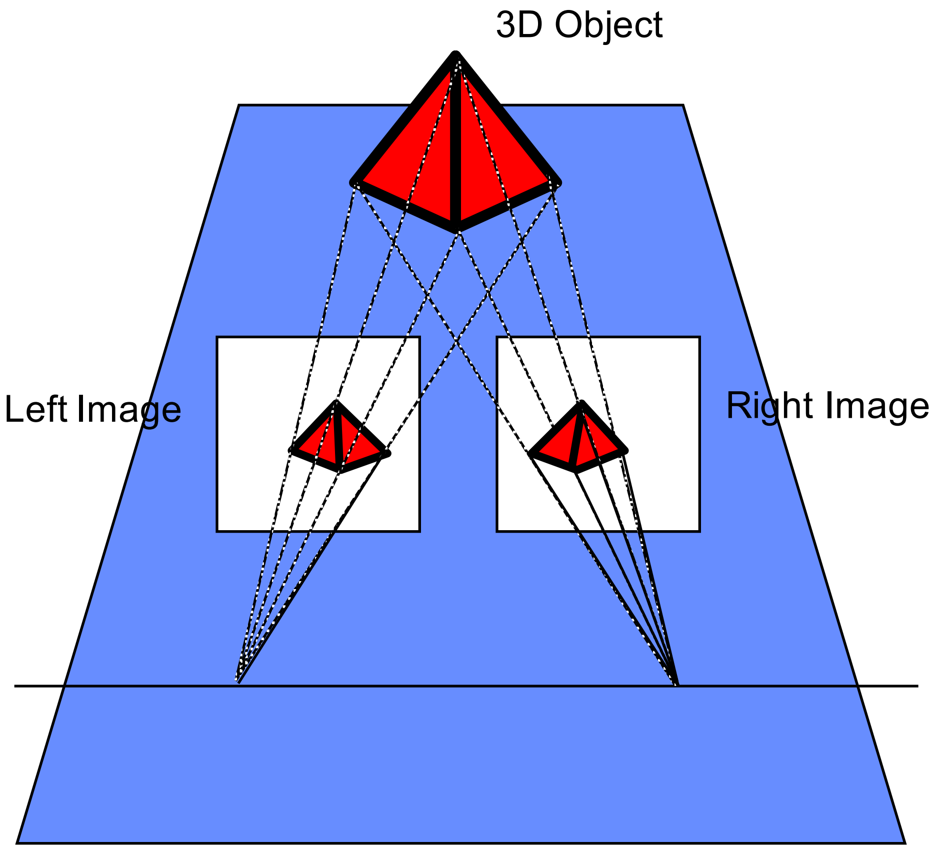
- Structure from stereo (Stereo-vision):
  - use two cameras with known relative position and orientation



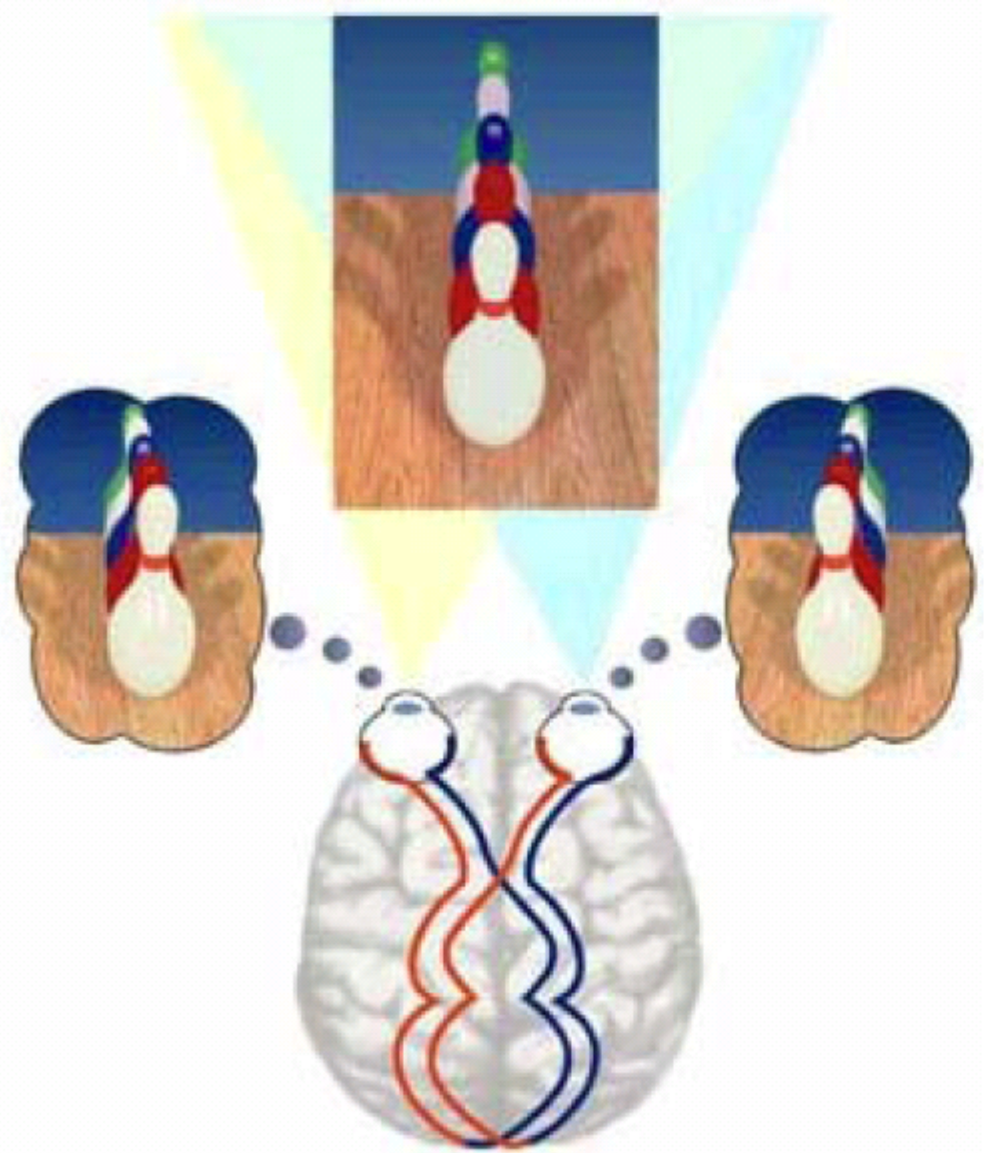
- Structure from motion:
  - use a single moving camera: both 3D structure and camera motion can be estimated up to a scale

Stereo Vision

- Allows to reconstruct a 3D object from two images taken at different locations

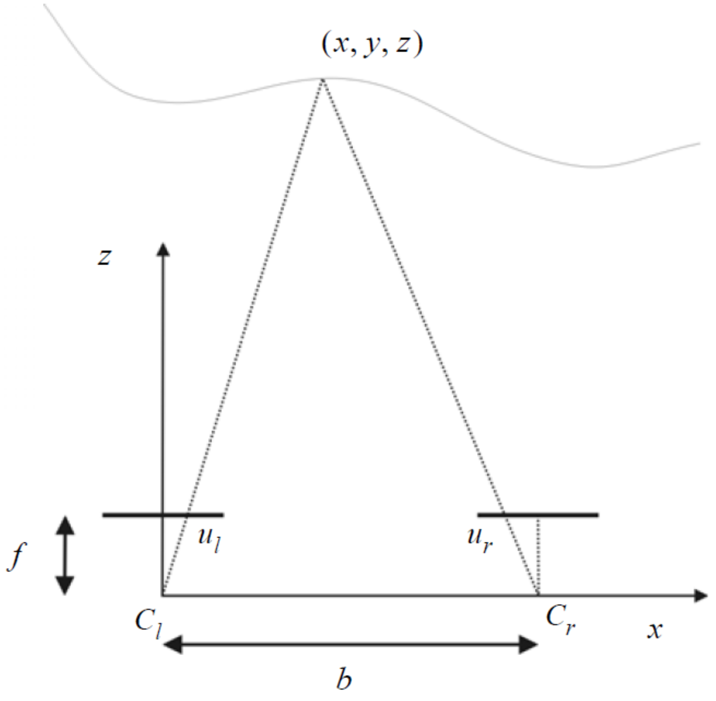


Disparity in the human retina



Stereo Vision - The simplified case

- The simplified case is an ideal case. It assumes that both cameras are identical and are aligned on a horizontal axis



$$\frac{f}{z} = \frac{u_l}{x}, \qquad \frac{f}{z} = \frac{u_r}{b - x}$$

$$\Rightarrow \quad z = b\frac{f}{u_l - u_r}$$

Distance

- $\boldsymbol{b}$ = baseline, distance between the optical centers of the two cameras
- $\boldsymbol{f}$ = focal length
- $\boldsymbol{u_l}$-$\boldsymbol{u_r}$ = disparity

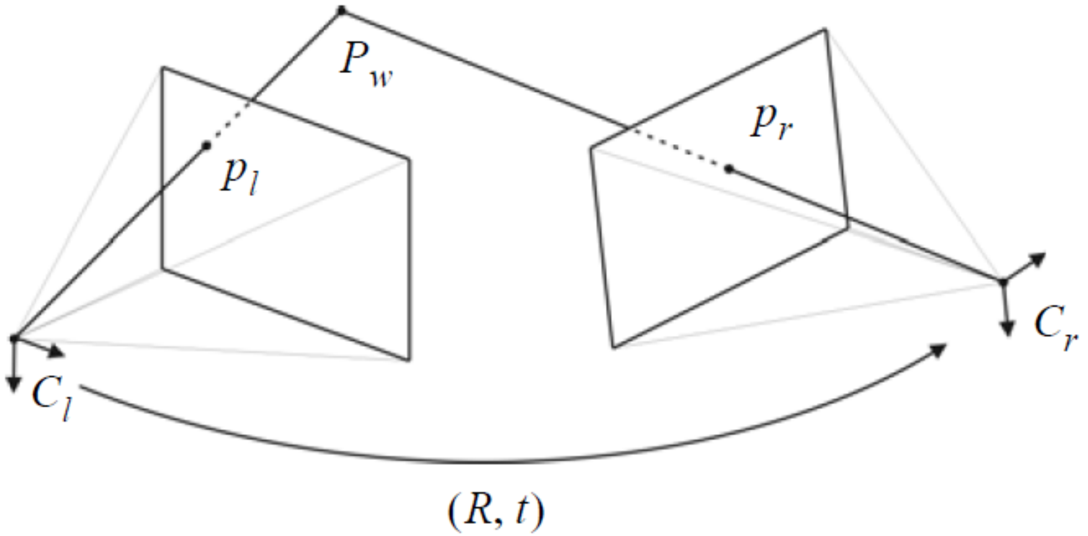
Stereo Vision: how to improve accuracy?

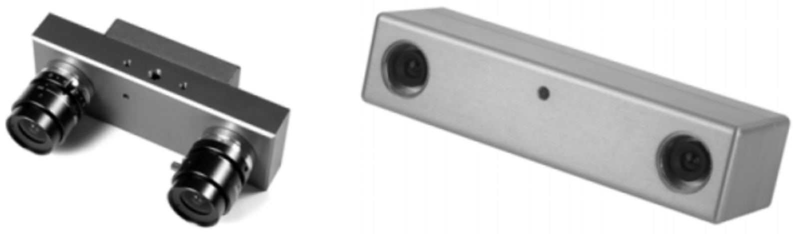
$$z = b\frac{f}{u_l - u_r}$$

1. Distance is inversely proportional to disparity (**u_l-u_r**)
   - closer objects can be measured more accurately
2. Disparity is proportional to ***b***
   - For a given disparity error, the accuracy of the depth estimate increases with increasing baseline ***b***.
   - However, as ***b*** is increased, some objects may appear in one camera, but not in the other.
3. Increasing image resolution improves accuracy

Stereo Vision – the general case

- Two identical cameras do not exist in nature!
- Aligning both cameras on a horizontal axis is very hard, also with the most expensive stereo cameras!

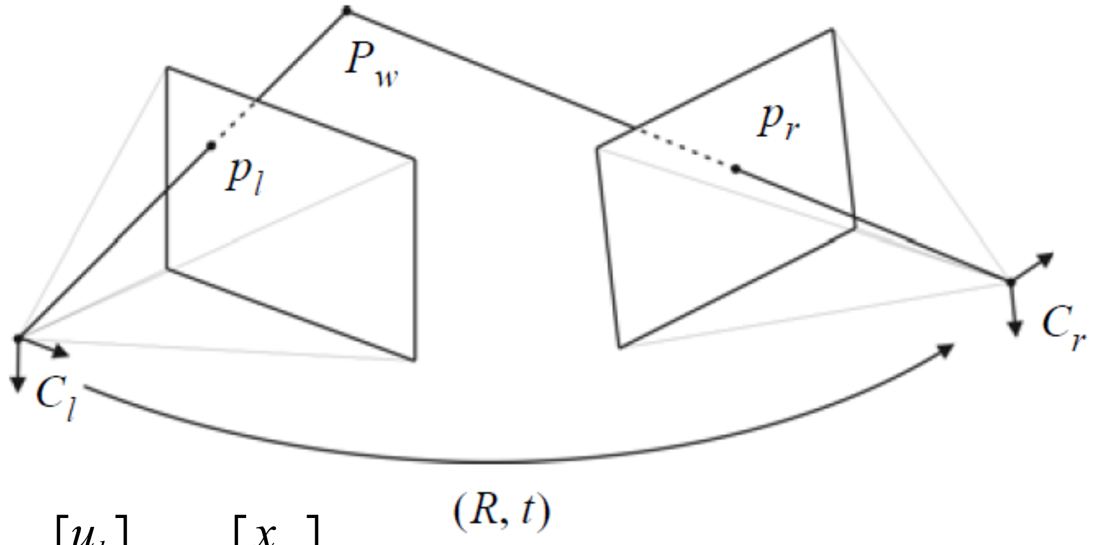




- In order to be able to use a stereo camera, we need first to estimate the relative pose between the cameras, that is, **Rotation** and **Translation**
- However, as the two cameras are not identical, we need to estimate:
  focal length, **image center**, **radial distortion**

Stereo Vision – the general case

- To estimate the 3D position we just construct the system of equations of the left and right camera



$$\lambda_l \begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = A_l \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

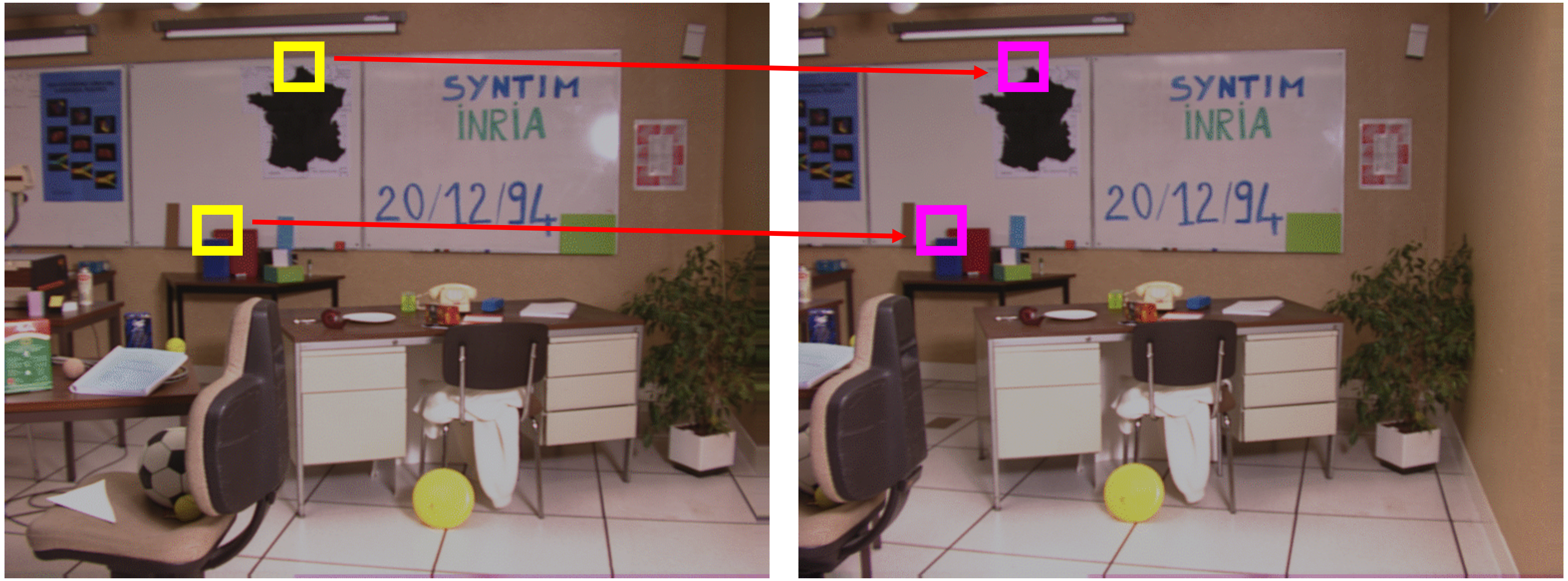
Left camera; world frame coincides with the left camera frame

$$\lambda_r \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = A_r \cdot R \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + T$$

Right camera

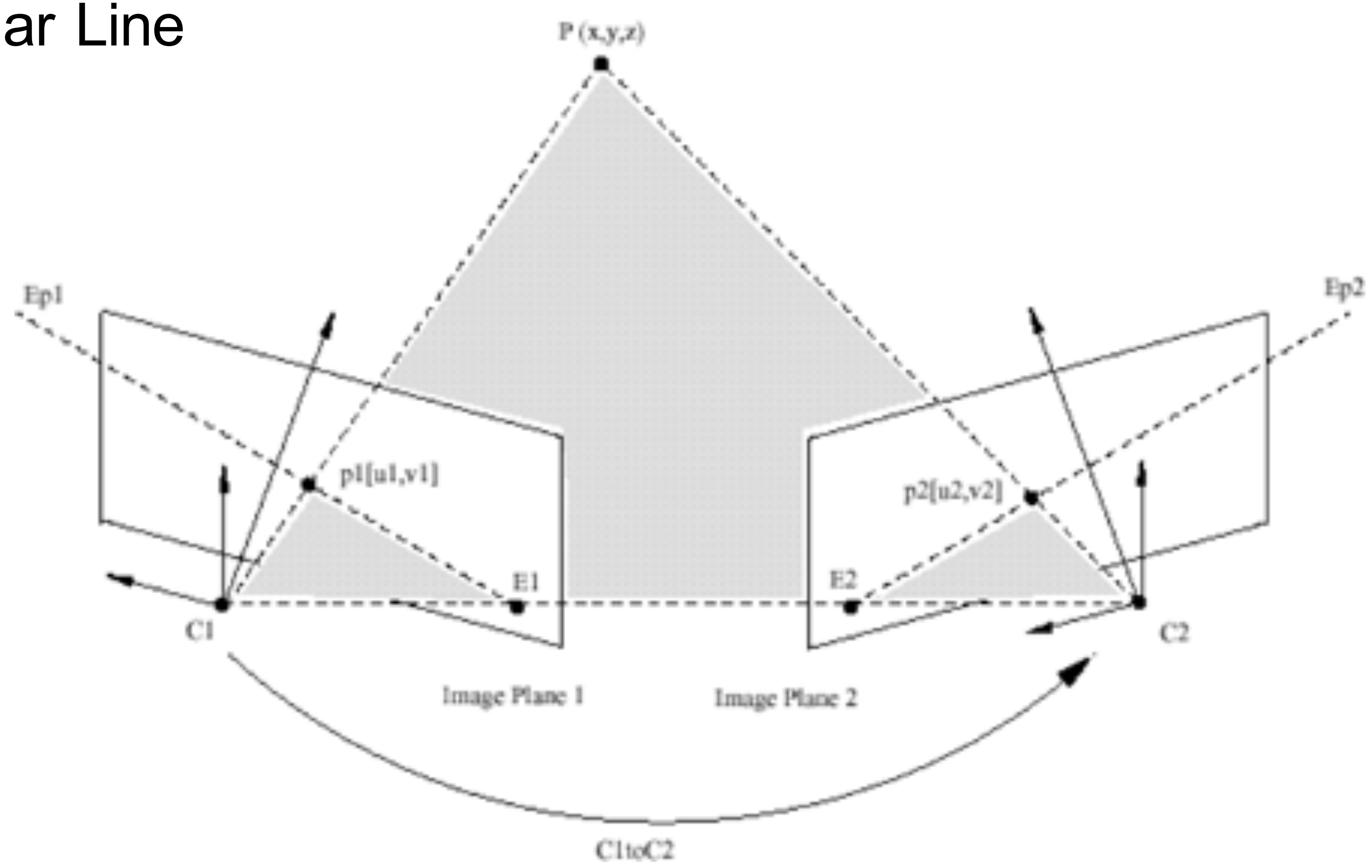
Stereo Vision: Correspondence Problem

- Matching between points in the two images which are projection of the same 3D real point
- Correspondence search could be done by comparing the observed points with all other points in the other image. Typical similarity measures are the Correlation and image Difference.
- This image search can be computationally very expensive! Is there a way to make the correspondence search 1 dimensional?



Correspondence Problem: Epipolar Constraint

- The correspondent of a point in an image must lie on a line in the other image, called Epipolar Line

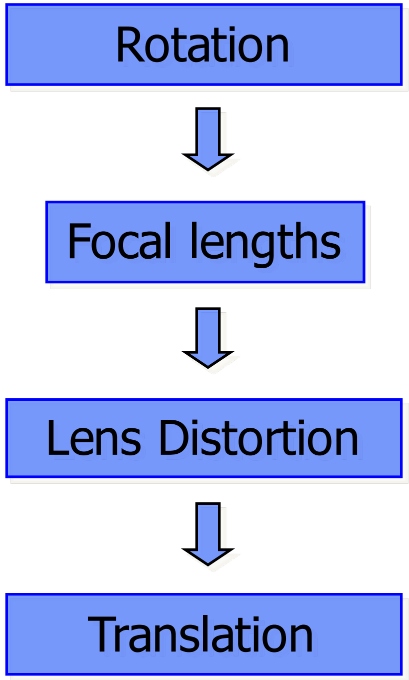


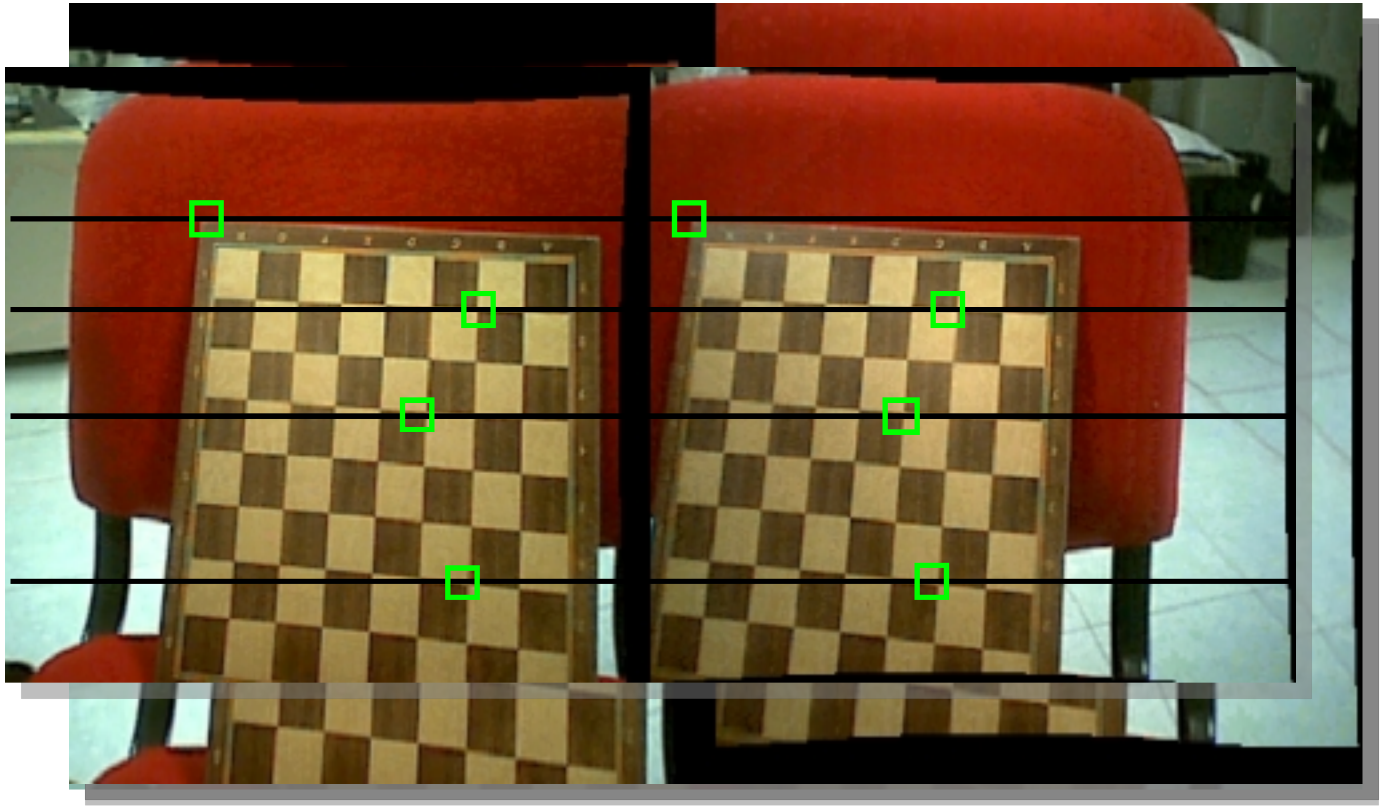
Correspondence Problem: Epipolar Constraint

- Thanks to the epipolar constraint, conjugate points can be searched along epipolar lines: this reduces the computational cost to 1 dimension!

Epipolar Rectification

- Determines a transformation of each image plane so that pairs of conjugate epipolar lines become collinear and parallel to one of the image axes (usually the horizontal one)





Stereo Vision Output 1 – Disparity map

- Find the correspondent points of all image pixels of the original images
- For each pair of conjugate points compute the disparity ***d* = *v-v'***
- ***d(x,y)*** is called Disparity map.

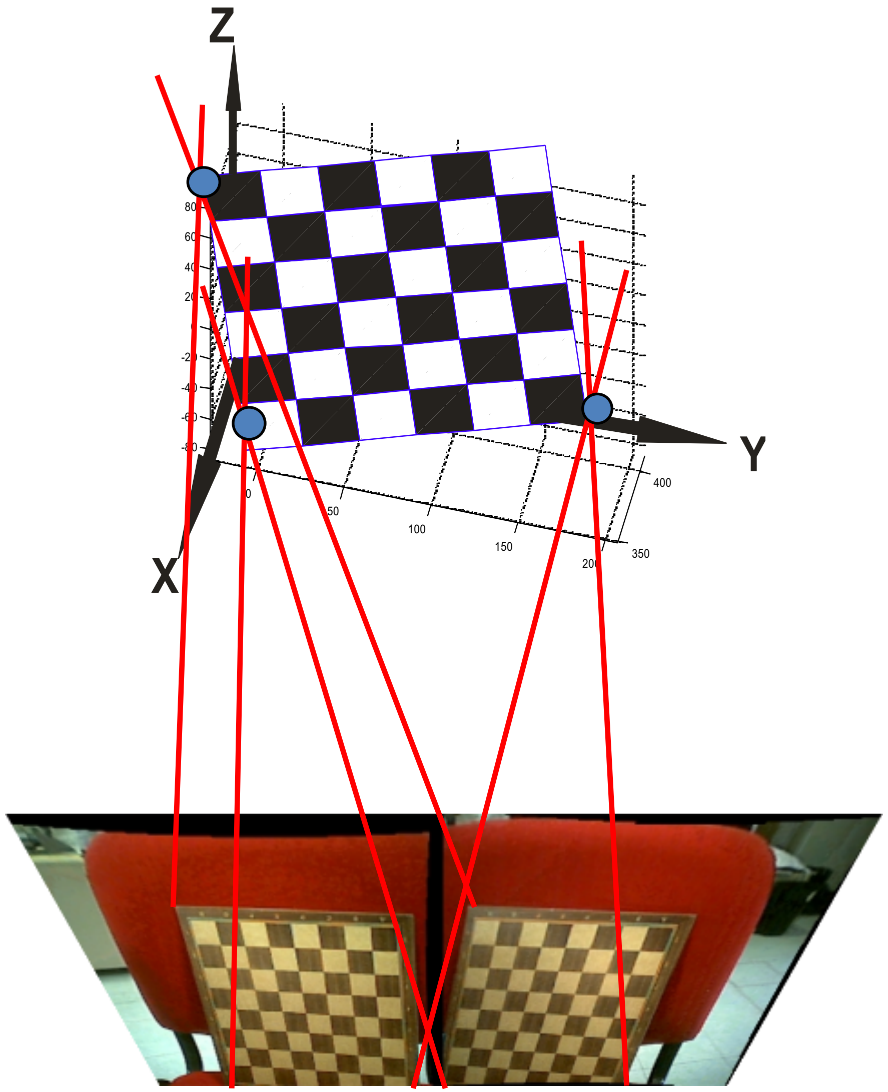
Left image

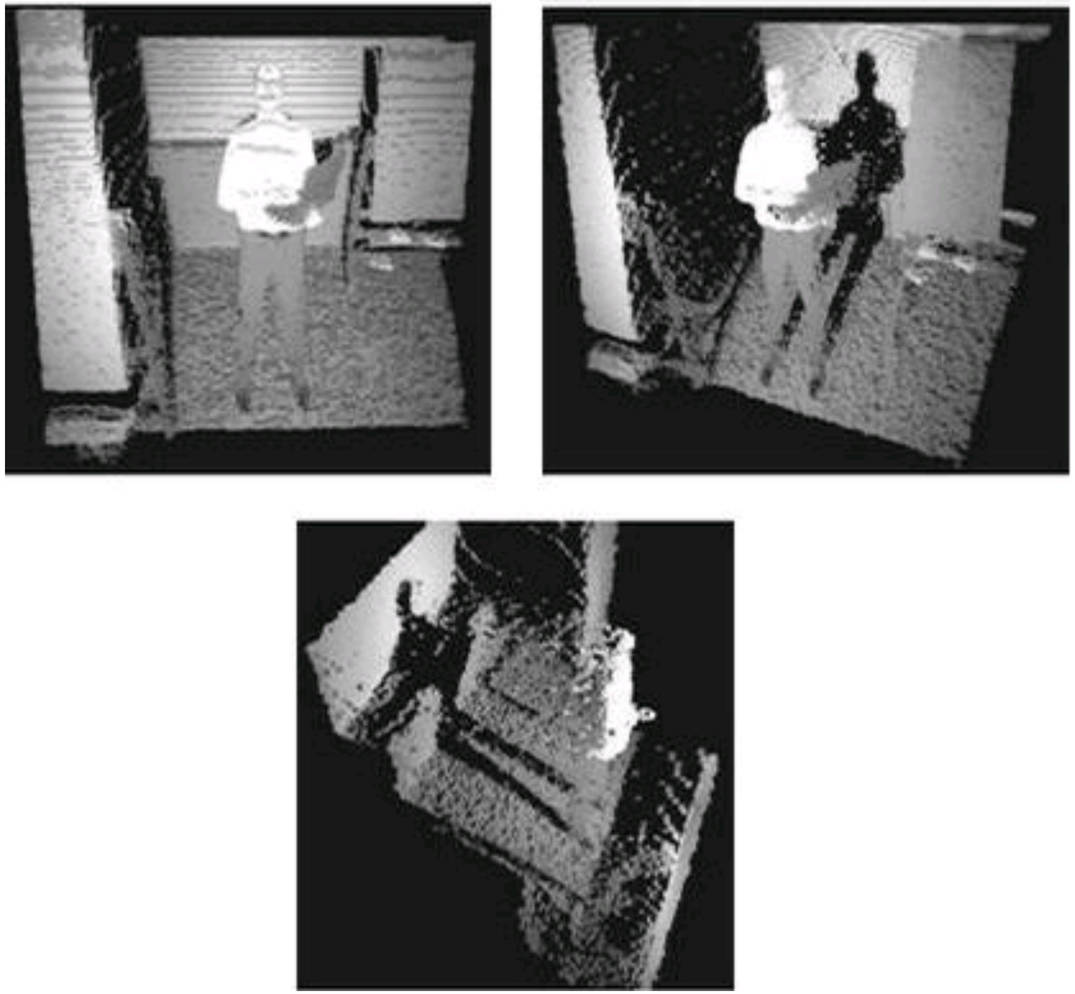
Right image

Disparity map

- Disparity maps are usually visualized as grey-scale images. Objects that are closer to the camera appear lighter, those who are further appear darker.

Stereo Vision Output 2 - 3D Reconstruction via triangulation





Stereo Camera Calibration

Totals:

Estimate the parameters for each camera in order to compute the 3D→2D transformation

10 parameters for each camera need to be estimated

PLUS:

5 parameters for each camera image to compensate for lens distortion (radial & tangential distortion)

$$\begin{cases} \begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix} = \begin{bmatrix} f_{u,L} & 0 & u_{0,L} \\ 0 & f_{v,L} & v_{0,L} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \\ \begin{bmatrix} u_R \\ v_R \\ 1 \end{bmatrix} = \begin{bmatrix} f_{u,R} & 0 & u_{0,R} \\ 0 & f_{v,R} & v_{0,R} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \end{cases}$$

$$\begin{cases} \begin{bmatrix} u_{d,L} \\ v_{d,L} \end{bmatrix} = (1 + kc_{1,L}\rho^2 + kc_{2,L}\rho^4 + kc_{5,L}\rho^6) \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 2kc_{3,L} \cdot u \cdot v + kc_{4,L}(\rho^2 + 2u^2) \\ kc_{3,L}(\rho^2 + 2v^2) + 2kc_{4,L} \cdot u \cdot v \end{bmatrix} \\ \begin{bmatrix} u_{d,R} \\ v_{d,R} \end{bmatrix} = (1 + kc_{1,R}\rho^2 + kc_{2,R}\rho^4 + kc_{5,R}\rho^6) \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 2kc_{3,R} \cdot u \cdot v + kc_{4,R}(\rho^2 + 2u^2) \\ kc_{3,R}(\rho^2 + 2v^2) + 2kc_{4,R} \cdot u \cdot v \end{bmatrix} \end{cases}$$

Stereo Camera Calibration

Estimates the parameters that manage the 3D – 2D transformation

$$\begin{cases} \begin{bmatrix} u_L \\ v_L \end{bmatrix} = f\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \\ \begin{bmatrix} u_R \\ v_R \end{bmatrix} = f\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \end{cases}$$

Stereo Camera Calibration

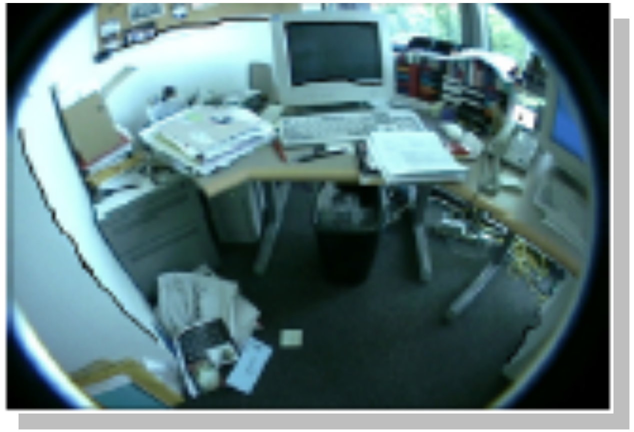
Totally:
Estimates the parameters that manage the 3D – 2D transformation
10 parameters for each camera need to be estimated

$$
\begin{cases}
\begin{bmatrix} u_L \\ v_L \\ 1 \end{bmatrix} = \begin{bmatrix} f_{u,L} & 0 & u_{0,L} \\ 0 & f_{v,L} & v_{0,L} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \\
\begin{bmatrix} u_R \\ v_R \\ 1 \end{bmatrix} = \begin{bmatrix} f_{u,R} & 0 & u_{0,R} \\ 0 & f_{v,R} & v_{0,R} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}
\end{cases}
$$

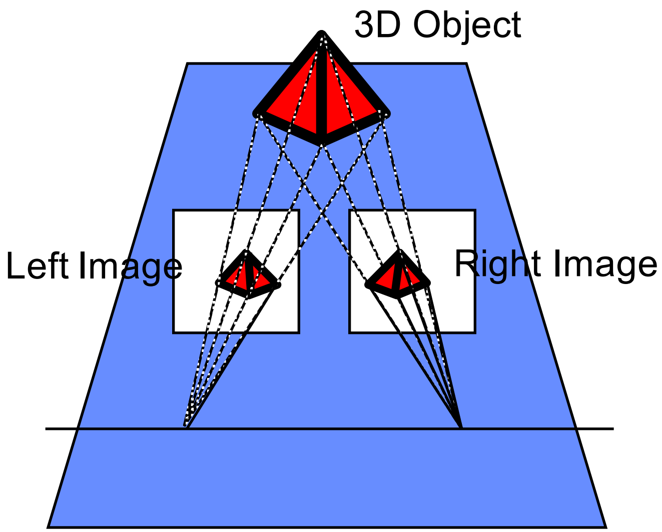
Stereo Camera Calibration

PLUS
5 parameters for each camera in order to compensate for lens distortion (radial & tangential distortion)

$$\begin{cases} \begin{bmatrix} u_{d,L} \\ v_{d,L} \end{bmatrix} = (1 + kc_{1,L}\rho^2 + kc_{2,L}\rho^4 + kc_{5,L}\rho^6) \cdot \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 2kc_{3,L} \cdot u \cdot v + kc_{4,L}(\rho^2 + 2u^2) \\ kc_{3,L}(\rho^2 + 2v^2) + 2kc_{4,L} \cdot u \cdot v \end{bmatrix} \\ \begin{bmatrix} u_{d,R} \\ v_{d,R} \end{bmatrix} = (1 + kc_{1,R}\rho^2 + kc_{2,R}\rho^4 + kc_{5,R}\rho^6) \cdot \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 2kc_{3,R} \cdot u \cdot v + kc_{4,R}(\rho^2 + 2u^2) \\ kc_{3,R}(\rho^2 + 2v^2) + 2kc_{4,R} \cdot u \cdot v \end{bmatrix} \end{cases}$$



Stereo Vision - summary



1. Stereo camera calibration -> compute camera relative pose
2. Epipolar rectification -> align images
3. Search correspondences
4. Output: compute stereo triangulation or disparity map
5. Consider baseline and image resolution to compute accuracy!