上海科技大学
ShanghaiTech University

CS283: **Robotics Spring 2023: Sensors & Perception**

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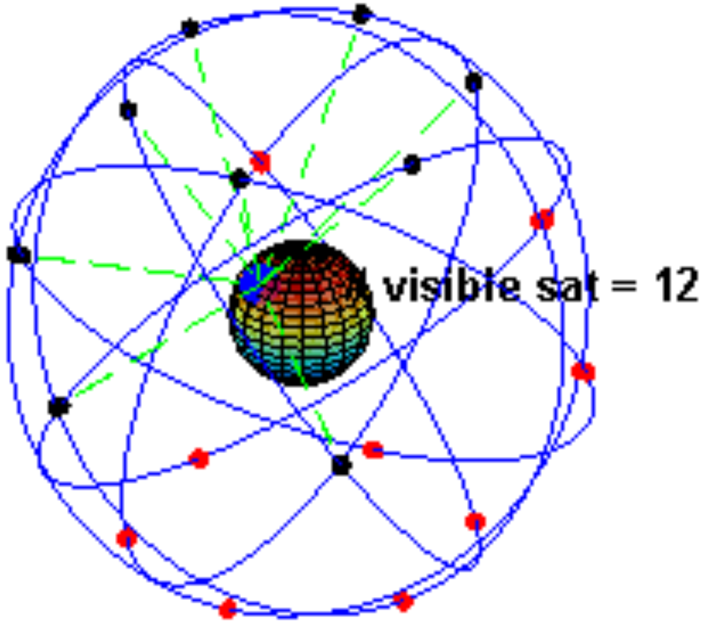
ShanghaiTech University

SENSORS CONTINUED

Global Positioning System (GPS)

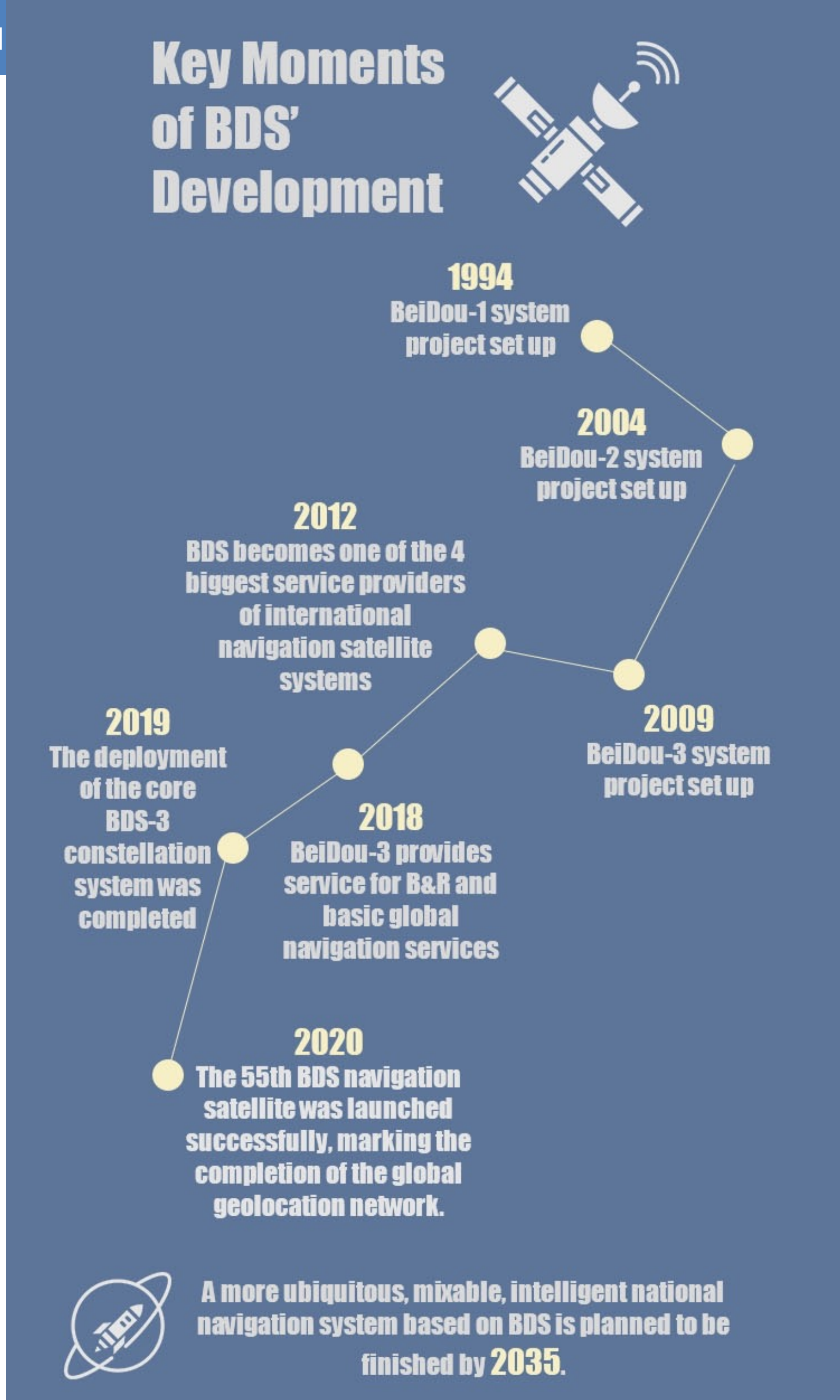
- Developed for military use
- 1995 it became accessible for commercial applications
- 24 satellites orbiting the earth every 12 hours at a height of 20.190 km.
- 4 satellites are located in each of 6 orbits with 60 degrees orientation between each other. The orbital planes do not rotate with respect to stars. Orbits arranged so that at least 6 satellites are always within line of sight from any point on Earth's surface.
- From 2008: 32 satellites to improve localization accuracy through redundancy
- Location of any GPS receiver is determined through a time of flight measurement (satellites send orbital location (*ephemeris*) plus time; the receiver computes its location through **trilateration** and **time correction**)



- Technical challenges:
  - Time synchronization between the individual satellites and the GPS receiver
  - Real time update of the exact location of the satellites
  - Precise measurement of the time of flight
  - Interferences with other signals

Other Global Positioning Systems

- GLONASS
  - Russian GPS – developed since 1976
  - Full global coverage as of 2011 (24 sattelites)
- Galileo
  - European GPS – initiated 2003
  - 22 operational satellites in orbit
- IRNSS (Indian Regional Navigation Satellite System)
  - Initiated 2010
  - 8 satellites for Indian Coverage in orbit
  - Full operation
- BeiDou Navigation Satellite System 北斗卫星导航系统
  - Chinese GPS – developed since 1994
  - BeiDou Satellite Navigation System (BDS)
  - 2011 full China coverage – 2019 global coverage
  - 55 satellites system



GPS coordinate errors

- HD maps: save locations in coordinates (e.g. location of traffic light)
- Many sources of errors – e.g.:
  - Wrong survey (input data is wrong)
    - Human error and instrument error
  - Continental drift! (Australia: 7 cm per year!)

- China: GCJ-02: WGS 84 plus random offsets (about 300-500m) (for national safety) makes autonomous driving development difficult :/
  地形图非线性保密处理算法
  - Called “Mars coordinate system”
  - Need special license to calculate correct position
  - afaik: 7 companies in China have this license

- Baidu: BD-09 further offsets (so competitors don’t copy)

Australia Is Drifting So Fast GPS Can't Keep Up

A significant correction must be made by the end of the year for navigation technology to keep working smoothly.

BY BRIAN CLARK HOWARD

NATIONAL GEOGRAPHIC

PUBLISHED SEPTEMBER 24, 2016 • 2 MIN READ

Australia is not quite where you think it is. The continent has shifted by 4.9 feet since the last adjustment was made to GPS coordinates in 1994, reports the *New York Times*.

All of the Earth’s continents float on tectonic plates, which glide slowly over a plastic-like layer of the upper mantle. And the plate that Australia sits on has been moving relatively fast, about 2.7 inches a year (northward and with a slight clockwise rotation).

https://www.nationalgeographic.com/science/article/australia-moves-gps-coordinates-adjusted-continental-drift


ZHONGJIAZHAI
中家宅
Shunqing Sichuan
顺庆川菜馆
Chuanyanghe
Chuanyang River
科苑路
WENJIAZHAI
文家宅
X620
HUIZHOU BRANCH
徽州店
XILIUJIA ZHAI
西刘家宅
Luoshan Rd
Sanba River
Jinke Rd
611 County Rd
YUMINCUN
渔民村
Zhongke Rd
Haiqu Road
海趣路
DONGPANJIA ZHAI
东潘家宅
LOUJIAZHAI
娄家宅
Huanke Road
环科路
中环路
NANPANJIA ZHAI
南潘家宅
LUJIAZHAI
陆家宅
Convenience
金栋便利
Middle Ring Rd
QIAN ZHAOJIAZHAI
钱赵家宅
X611
DONGQIANJIA ZHAI
东钱家宅
Middle Ring Rd
Google

RANGE SENSING

Range Sensing

- Color/ gray scale cameras: do NOT measure the distance to the object
- Range sensing: get the distance to the object
- Basic principles:
  - Time of flight
    - Sound/ Ultrasound (in air, underwater)
    - Light (Based on Phase or based on time)
      - Single rotating laser beam (LRF; e.g. Sick)
      - Multiple rotating laser beams (3D LRF; e.g. Velodyne)
      - Solid state laser (e.g. Intel RealSense L515)
      - LED light & imager (ToF camera, e.g. Kinect 2)
    - Radio Waves (Radar)
- Projected Pattern
  - Single laser (Triangulation)
  - 2D pattern (e.g. Kinect 1)
- Stereo Vision
  - Passive
  - Active with pattern (e.g. Intel RealSense D435)

RANGE SENSING: TIME OF FLIGHT

Range Sensors (time of flight) (1)

- Large range distance measurement -> called range sensors
- Range information:
  - key element for localization and environment modeling
- Ultrasonic sensors as well as laser range sensors make use of propagation speed of sound or electromagnetic waves respectively. The traveled distance of a sound or electromagnetic wave is given by

$$d = c \cdot t$$

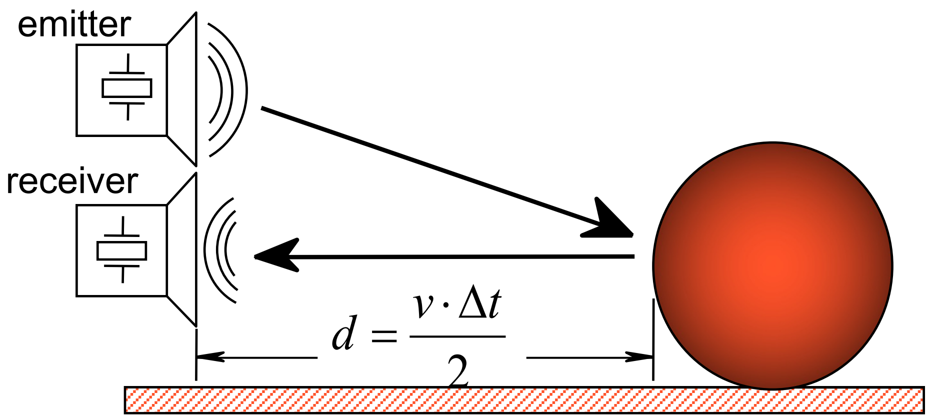
- Where
  - d = distance traveled (usually round-trip)
  - c = speed of wave propagation
  - t = time of flight.

Range Sensors (time of flight) (2)

- It is important to point out
  - Propagation speed v of sound in air: 0.3 m/ms - 300 m/s
  - Propagation speed v of sound in water: 1.5 m/ms - 1,500 m/s
  - Propagation speed v of of electromagnetic signals: 0.3 m/ns,
    - one million times faster.
  - 3 meters
    - is 10 ms for an ultrasonic system
    - only 10 ns for a laser range sensor
    - time of flight with electromagnetic signals is not an easy task
    - laser range sensors expensive and delicate

- The quality of time of flight range sensors mainly depends on:
  - Inaccuracies in the time of fight measure (laser range sensors)
  - Opening angle of transmitted beam (especially ultrasonic range sensors)
  - Interaction with the target (surface, specular reflections)
  - Variation of propagation speed (sound)
  - Speed of mobile robot and target (if not at stand still)

Factsheet: Ultrasonic Range Sensor (1)



1. Operational Principle

An ultrasonic pulse is generated by a piezo-electric emitter, reflected by an object in its path, and sensed by a piezo-electric receiver. Based on the speed of sound in air and the elapsed time from emission to reception, the distance between the sensor and the object is easily calculated.

2. Main Characteristics

- Precision influenced by angle to object (as illustrated on the next slide)
- Useful in ranges from several cm to several meters
- Typically relatively inexpensive

3. Applications

- Distance measurement (also for transparent surfaces)
- Collision detection

Ultrasonic Sensor (time of flight, sound) (1)

- transmit a packet of (ultrasonic) pressure waves
- distance *d* of the echoing object can be calculated based on the propagation speed of sound *c* and the time of flight *t*.

$$d = \frac{c \cdot t}{2}$$

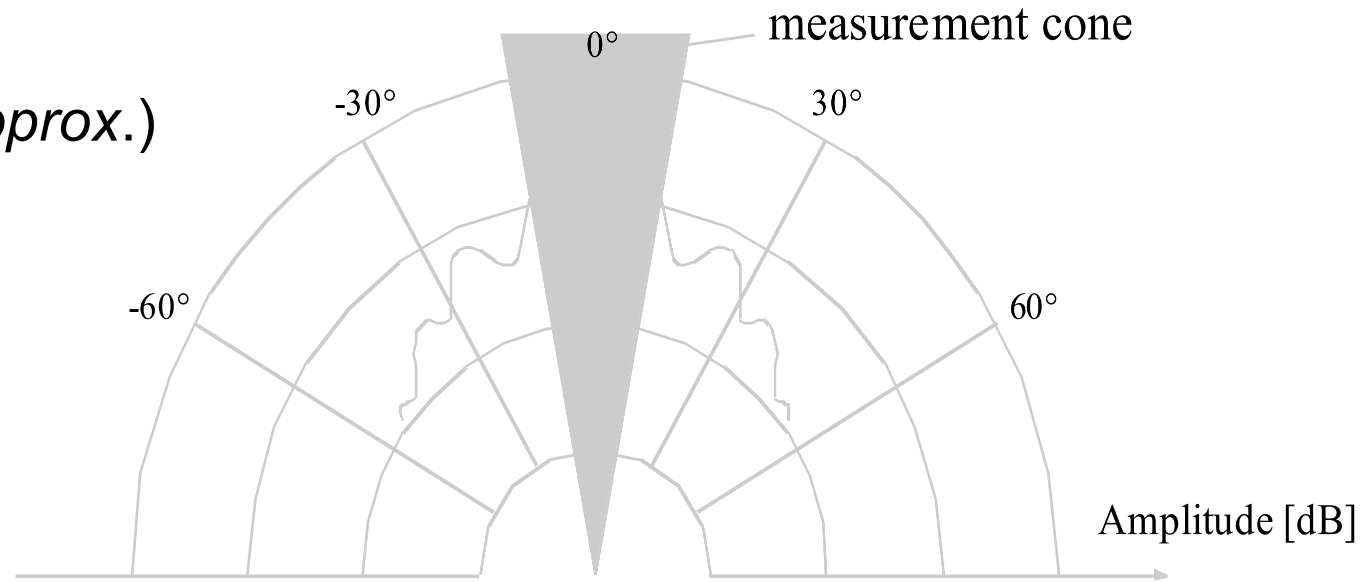
- The speed of sound *c* (340 m/s) in air is given by $\sqrt{\gamma \cdot R \cdot T}$

Where

γ : adiabatic index (isentropic expansion factor) - ratio of specific heats of a gas
R: gas constant
T: temperature in degree Kelvin

Ultrasonic Sensor (time of flight, sound) (2)

- typical frequency: 40kHz - 180 kHz
  - Lower frequencies correspond to longer range
- generation of sound wave: piezo transducer
  - transmitter and receiver separated or not separated
- Range between 12 cm up to 5 m
- Resolution of ~ 2 cm
- Accuracy 98% => relative error 2%
- sound beam propagates in a cone (*approx.*)
  - opening angles around 20 to 40 degrees
  - regions of constant depth
  - segments of an arc (sphere for 3D)



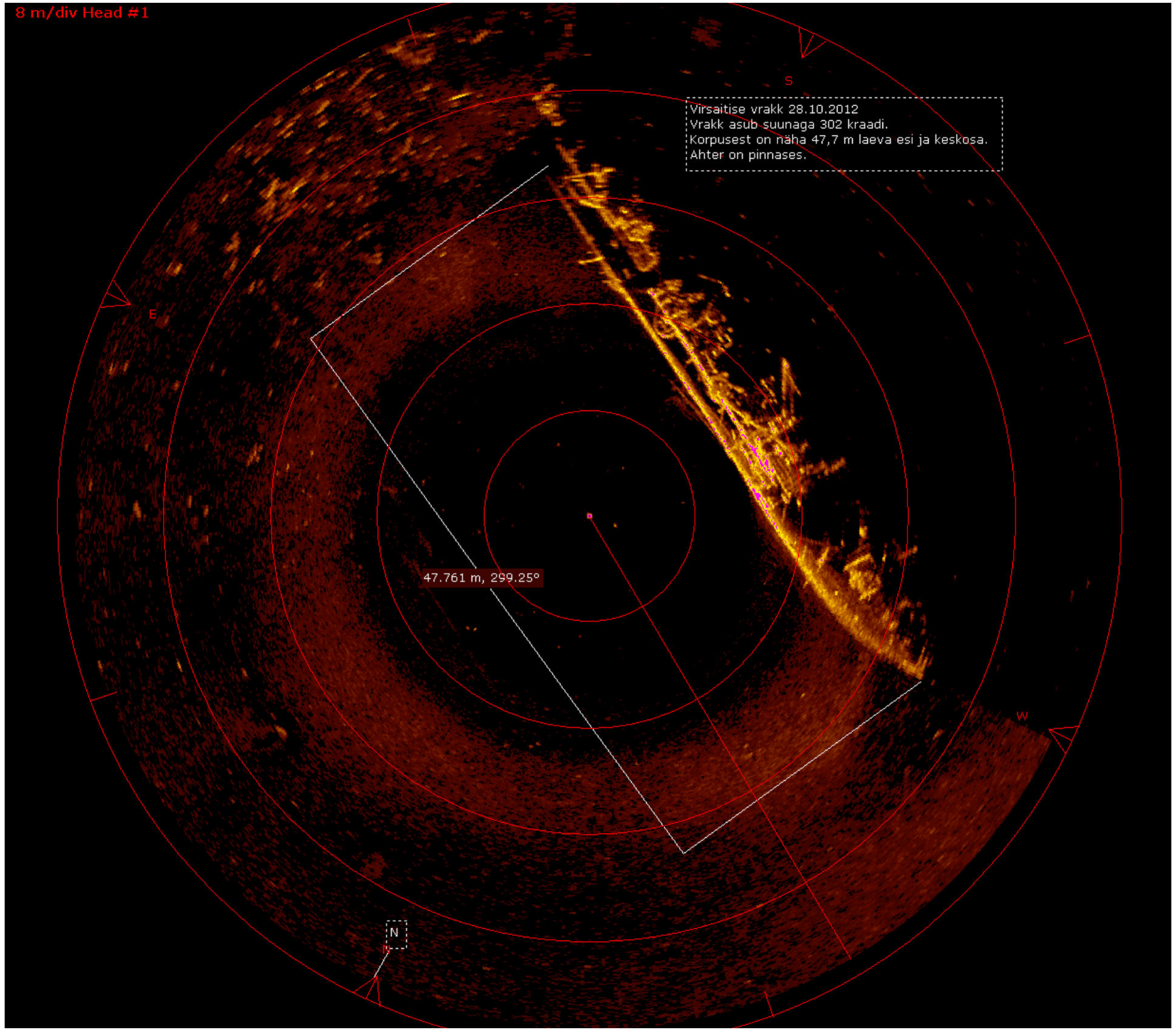
Typical intensity distribution of a ultrasonic sensor

Ultrasonic Sensor (time of flight, sound) (4)

- Bandwidth
  - measuring the distance to an object that is 3 m away will take such a sensor 20 ms, limiting its operating speed to 50 Hz. But if the robot has a ring of 20 ultrasonic sensors, each firing sequentially and measuring to minimize interference between the sensors, then the ring's cycle time becomes 0.4 seconds => frequency of each one sensor = 2.5 Hz.
  - This update rate can have a measurable impact on the maximum speed possible while still sensing and avoiding obstacles safely.

Underwater Sonar

- Light visibility very low => often sonar the only/ best sensor available
- Types:
  - Sonar
  - Side-scanning
  - Synthetic aperture sonar
- Problems:
  - Absorption
  - Reflections:
    - Layers of different water temperature
    - Layers of different salinity
    - …



Laser Range Sensor (time of flight, electromagnetic)

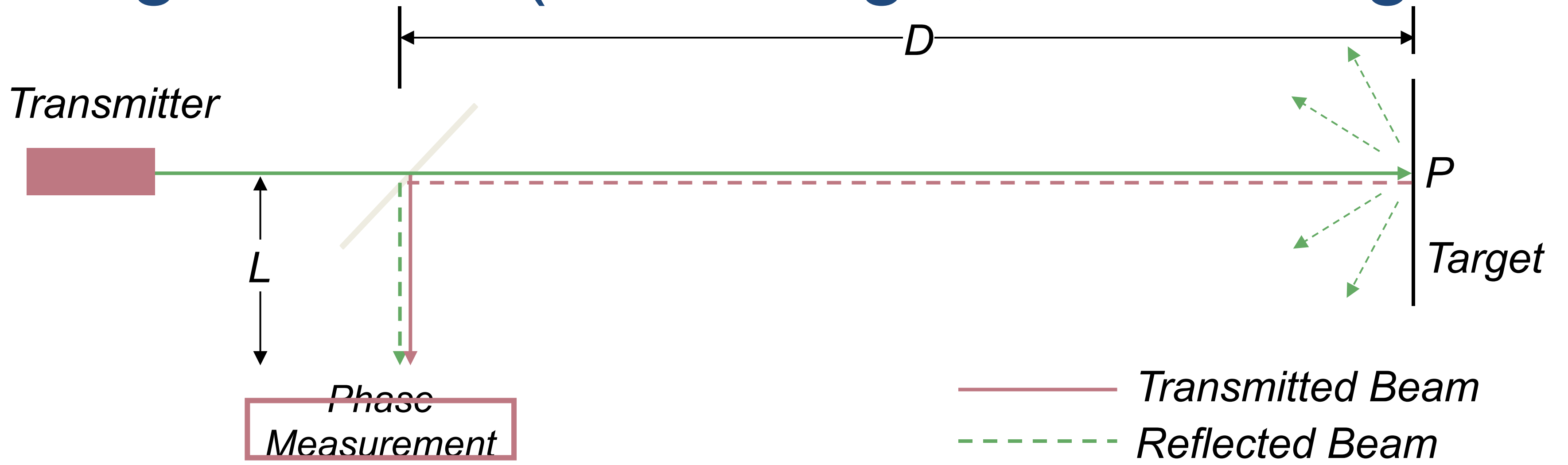
- Is called Laser range finder or Lidar (LIght Detection And Ranging)



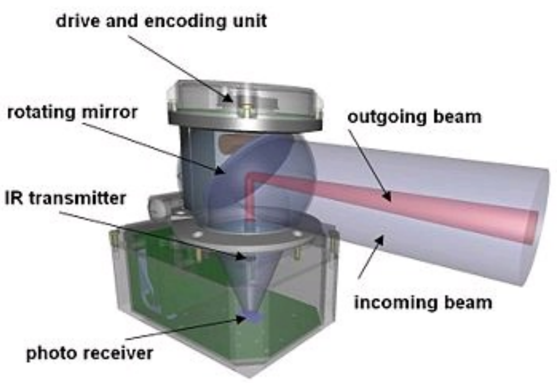
Many 3D range sensing companies are emerging!

E.g. RoboSense (China), Livox (China)

Laser Range Sensor (time of flight, electromagnetic) (1)



- Transmitted and received beams coaxial
- Transmitter illuminates a target with a collimated laser beam
- Received detects the time needed for round-trip
- A mechanical mechanism with a mirror sweeps
  - 2D or 3D measurement



Laser Range Sensor (time of flight, electromagnetic) (2)

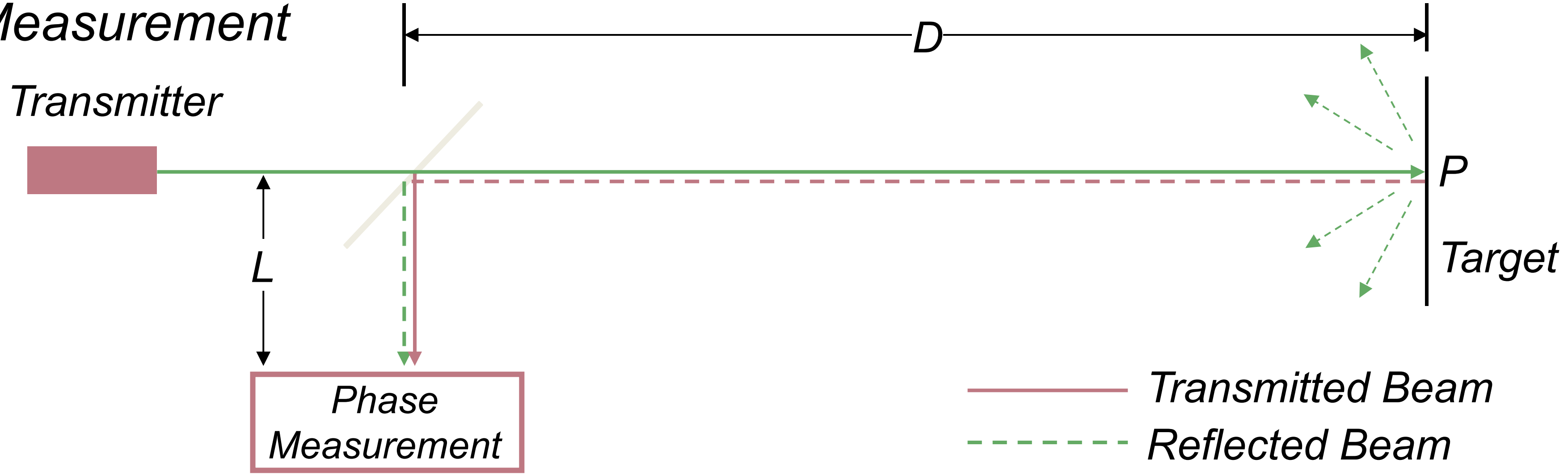
Time of flight measurement

- Pulsed laser (today the standard)
  - measurement of elapsed time directly
  - resolving picoseconds
- Phase shift measurement to produce range estimation
  - technically easier than the above method

- (3D) Laser Scanner == Lidar (<u>Li</u>ght <u>d</u>etection <u>a</u>nd <u>r</u>anging)

Laser Range Sensor (time of flight, electromagnetic) (3)

- *Phase-Shift Measurement*



$$D' = L + 2D = L + \frac{\theta}{2\pi}\lambda \qquad \lambda = \frac{c}{f}$$

Where:

c: is the speed of light; *f* the modulating frequency; *D'* the distance covered by the emitted light.

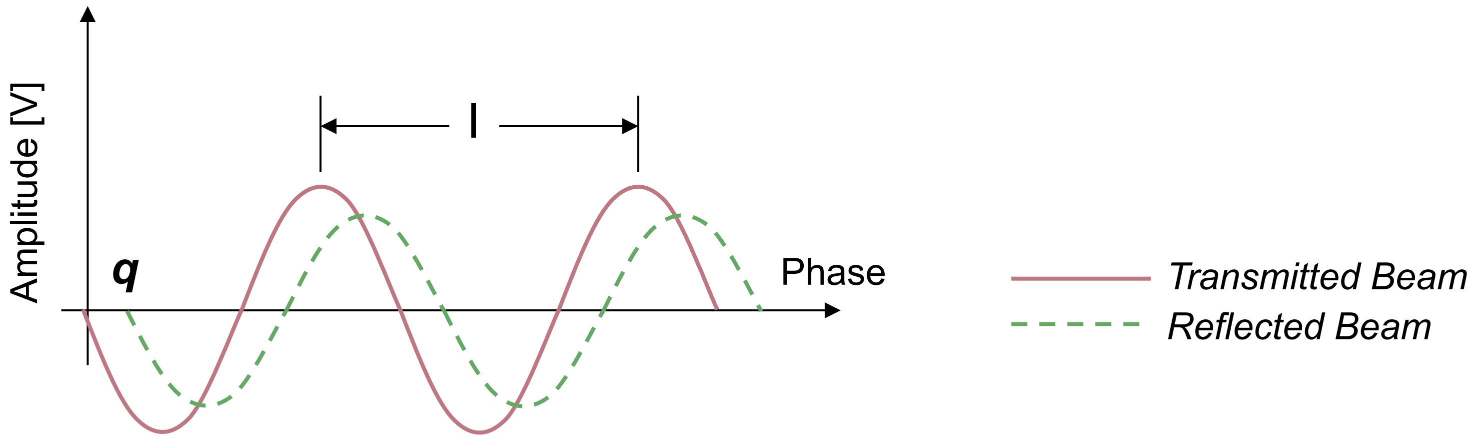
- for f = 5 MHz λ = 60 meters

Laser Range Sensor (time of flight, electromagnetic) (4)

- Distance D, between the beam splitter and the target

$$D = \frac{\lambda}{4\pi}\theta$$

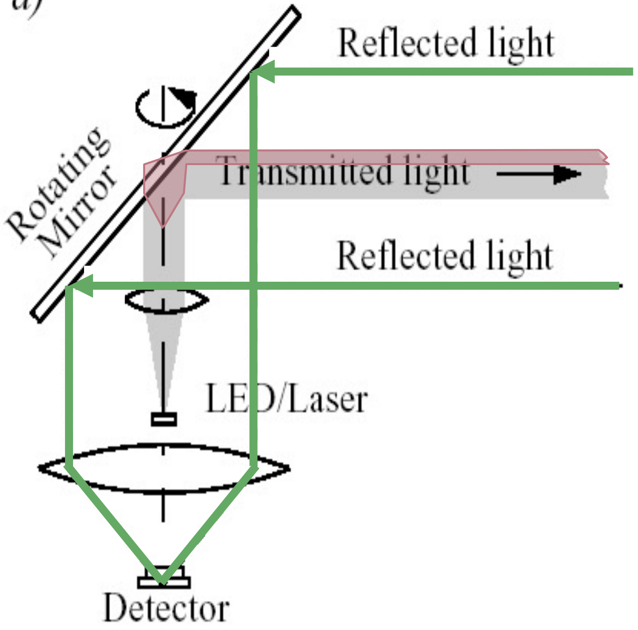
- where
  - θ: phase difference between transmitted and reflected beam
- Theoretically ambiguous range estimates
  - since for example if λ = 60 meters, a target at a range of 5 meters = target at 35 meters



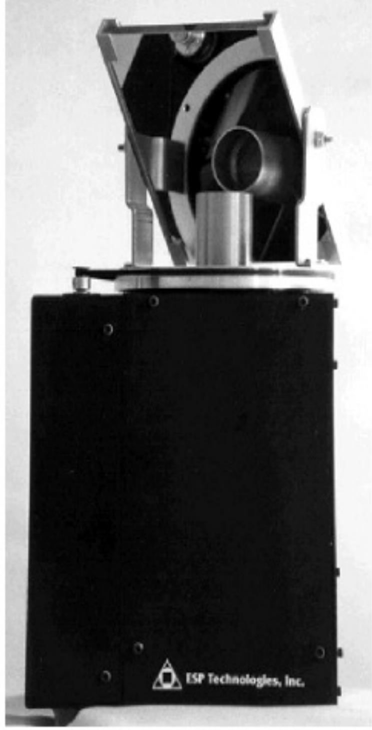
Laser Range Sensor (time of flight, electromagnetic) (5)

- Uncertainty of the range (phase/time estimate) is inversely proportional to the square of the received signal amplitude.
  - Hence dark, distant objects will not produce such good range estimated as closer brighter objects …

a)

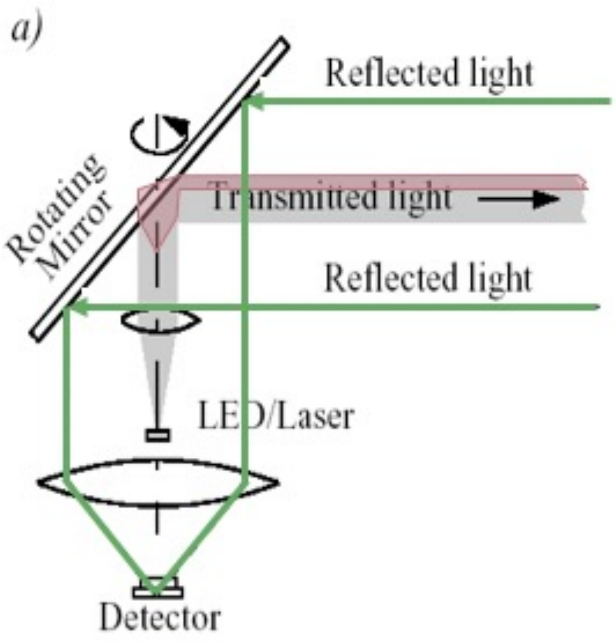


b)



c)

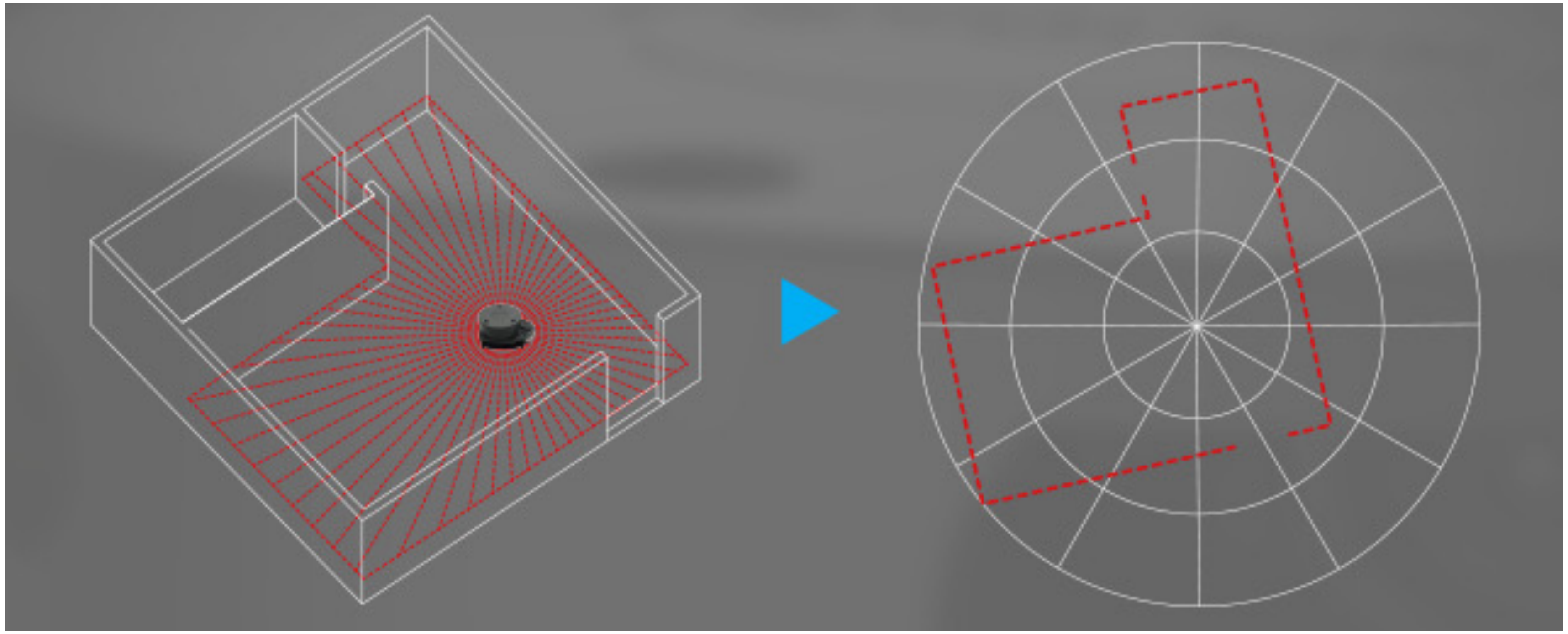

SICK


a)
Reflected light
Rotating Mirror
Transmitted light
Reflected light
LED/Laser
Detector


SICK

Laser Range Sensor (time of flight, electromagnetic)

- Typical range image of a 2D laser range sensor with a rotating mirror



The SICK LMS 200 Laser Scanner

- Angular resolution 0.25 deg
- Depth resolution ranges between 10 and 15 mm and the typical accuracy is 35 mm, over a range from 5 cm up to 20 m or more (up to 80 m)
  - depending on the reflectivity of the object being ranged.
- This device performs seventy five 180-degrees per sec.
- Dimensions: W155 x D155 x H210mm,
- Weight: 1,2 kg

- Cost: about RMB 22,000

Hokuyo UTM-30LX

- Long Detection range: 30m
- 0.1 to 10m: ± 30mm, 1
- 0 to 30m: ± 50mm*1
- Field of View: 270°
- 40Hz
- Outdoor Environment
- Dimensions: 60 x 60 x 87 mm
- Weight: 370g

- Cost: about 35,000 RMB

URG-04LX-UG01

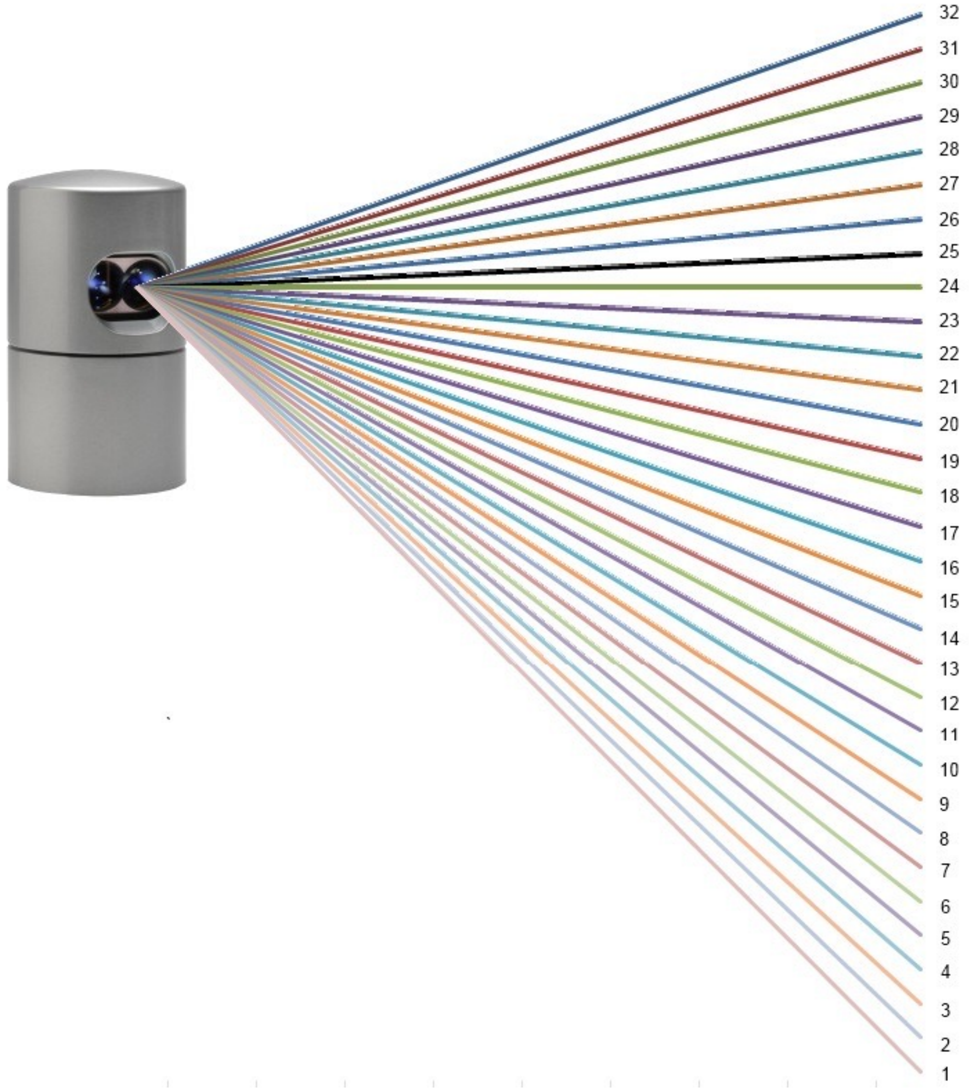
- Low-power consumption (2.5W)
- Wide-range (5600mm×240°).
- 60 to 1,000mm : ±30mm,
- 1,000 to 4,095mm : ±3% of measurement
- 10Hz
- Dimensions: 50 x 50 x 70 mm
- Weight: 160g

- Cost: about 6,500 RMB

Velodyne hdl-32e

- 32 beams
- Range: up to 80 – 100 m
- +10.67 to -30.67 degrees field of view (vertical)
- 360° field of view (horizontal)
- 10 Hz frame rate
- Accuracy: <2 cm (one sigma at 25 m)
- Angular resolution (vertical) 1.33°
- 700,000 points per second
- internal MEMS accelerometers and gyros for six-axis motion correction

- Dimensions:
  - Diameter: 85mm,
  - Height: 144 mm
- Weight: 1kg
- Cost: about 220,000 RMB

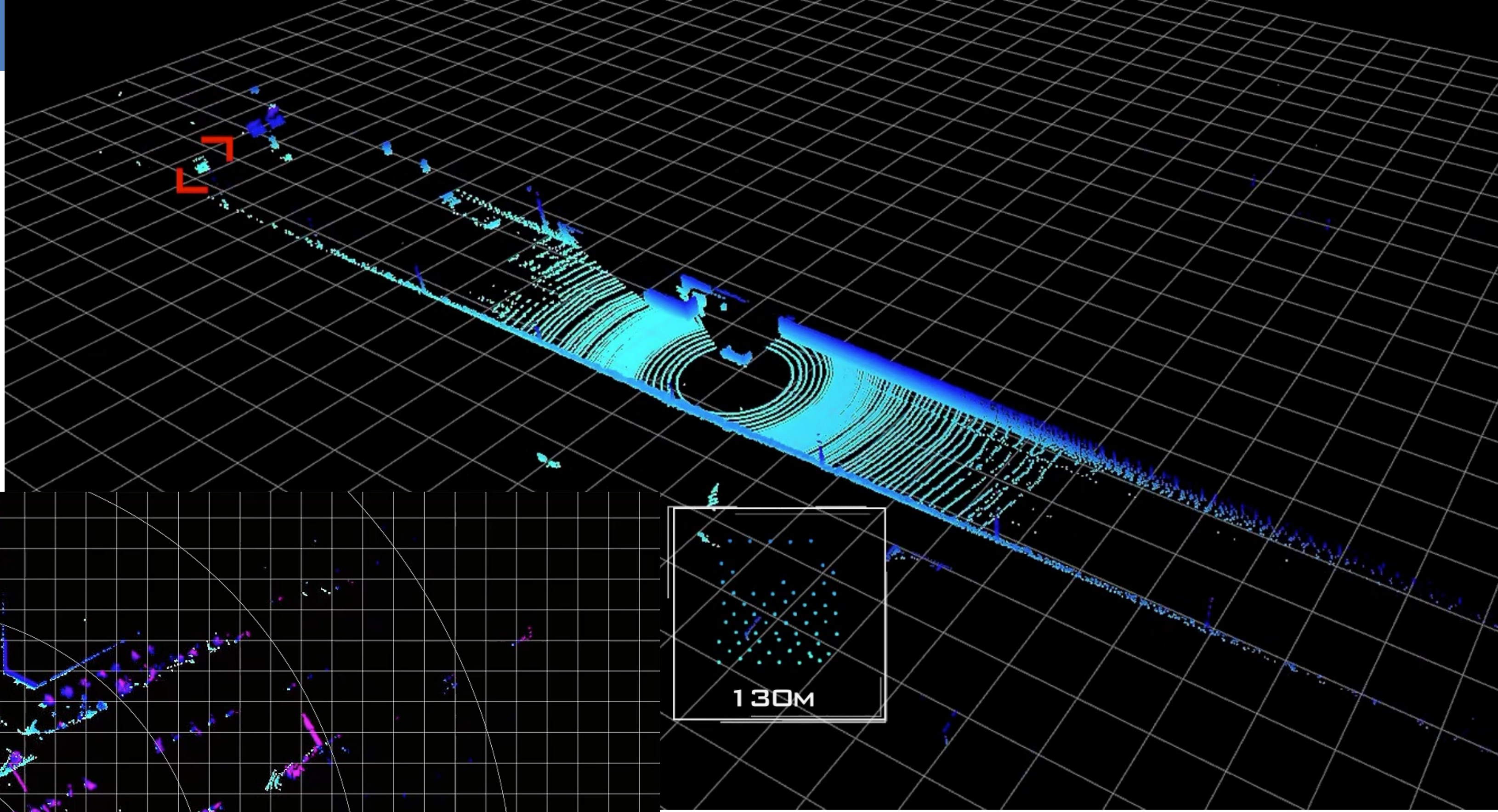


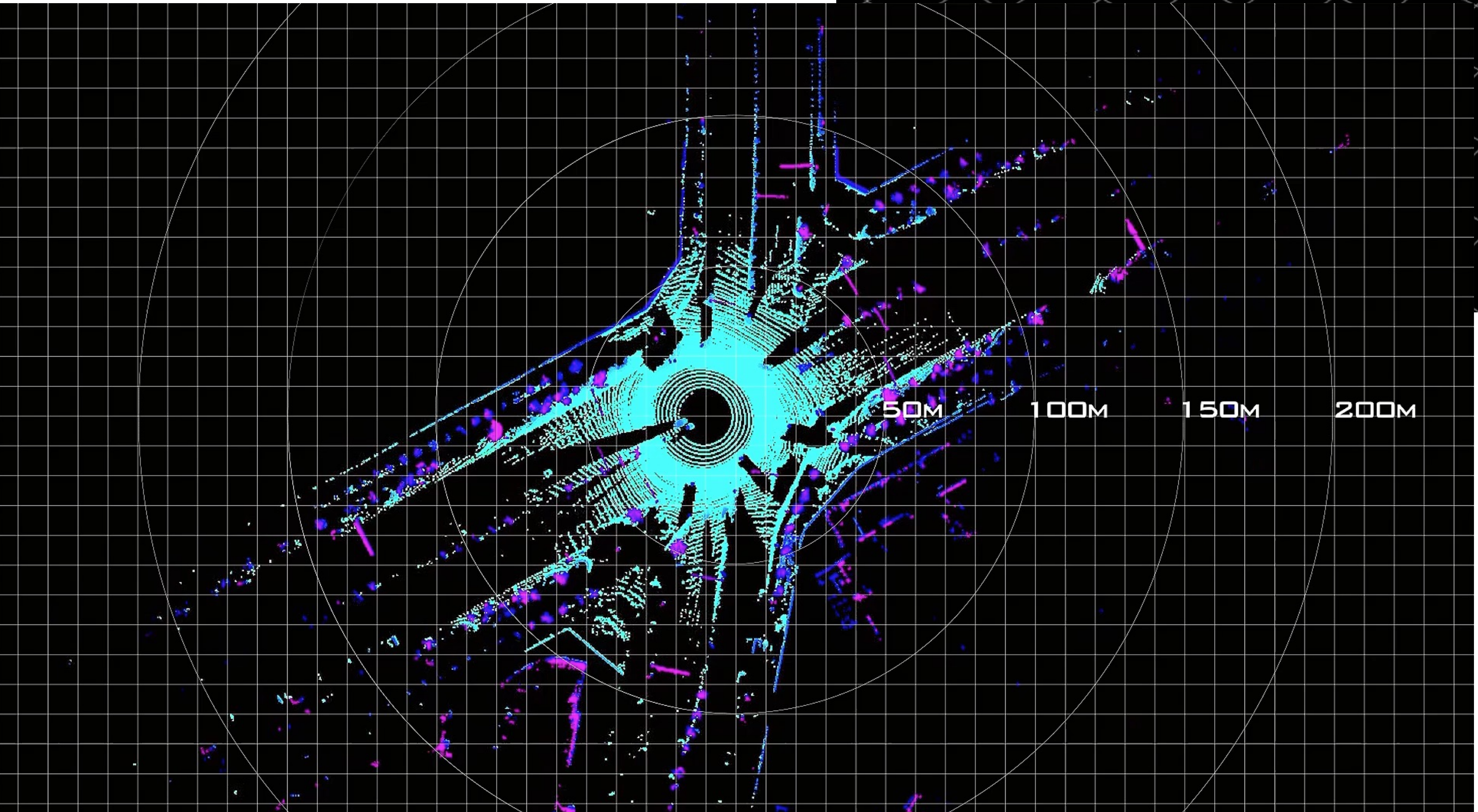
RoboSense Ruby

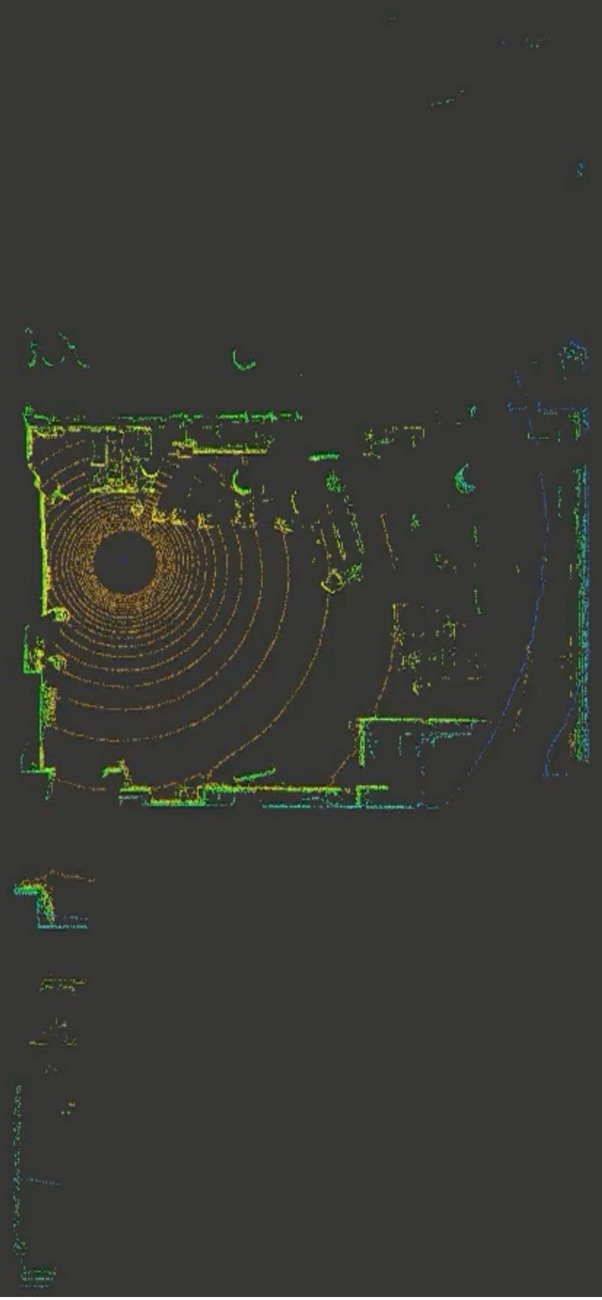
- 128 beams
- Range: 250m (200m@10% NIST)
- Range Accuracy (Typical): Up to ±3cm
- Vertical FOV: 40°
- Horizontal Resolution: 0.1°/ 0.2°/ 0.4°
- Vertical Resolution: Up to 0.1°
- Frame Rate: 5Hz/10Hz/20Hz
- Points Per Second: 2,304,000pts/s (Single return Mode)
- Points Per Second: 4,608,000pts/s (Dual return Mode)
- Operating Voltage: 19V - 32V
- Power Consumption: 45W
- Weight (without cabling): ~3.75 kg

- Cost: about RMB 500,000

RoboSense Ruby on Road



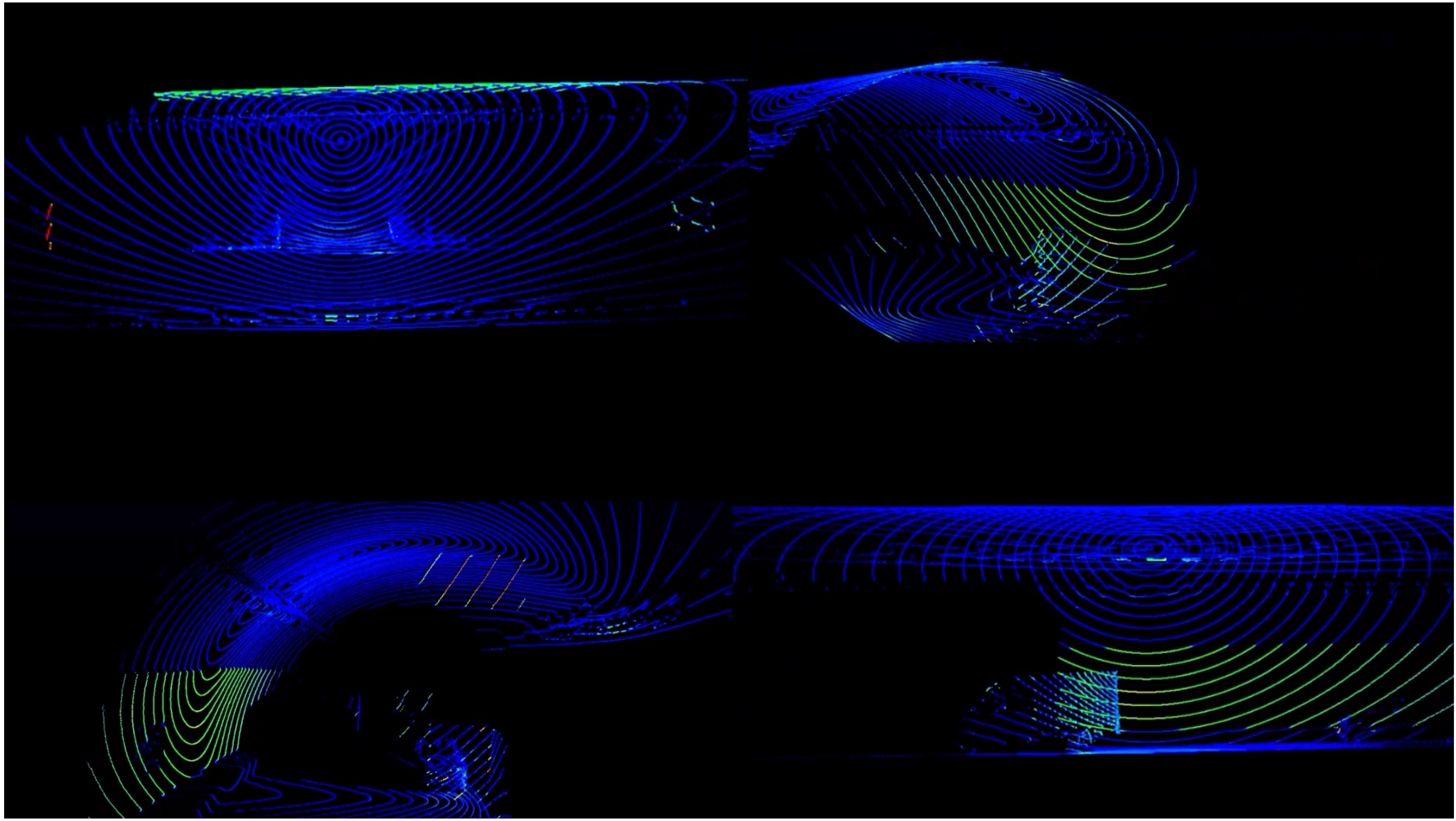




RoboSense Bpearl

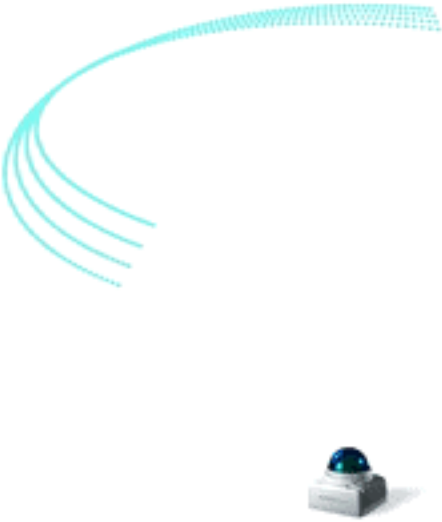
- Hemispherical Lidar
- 32 beams

- Range: 100m (30m@10% NIST)
- Range Accuracy (Typical): Up to ±3cm
- Frame Rate: 10Hz/20Hz
- Points Per Second: 576,000pts/s (Single return Mode)
- Points Per Second: 1,152,000pts/s (Dual return Mode)
- Operating Voltage: 9V - 32V
- Power Consumption: 13W
- Weight: ~0.92 kg



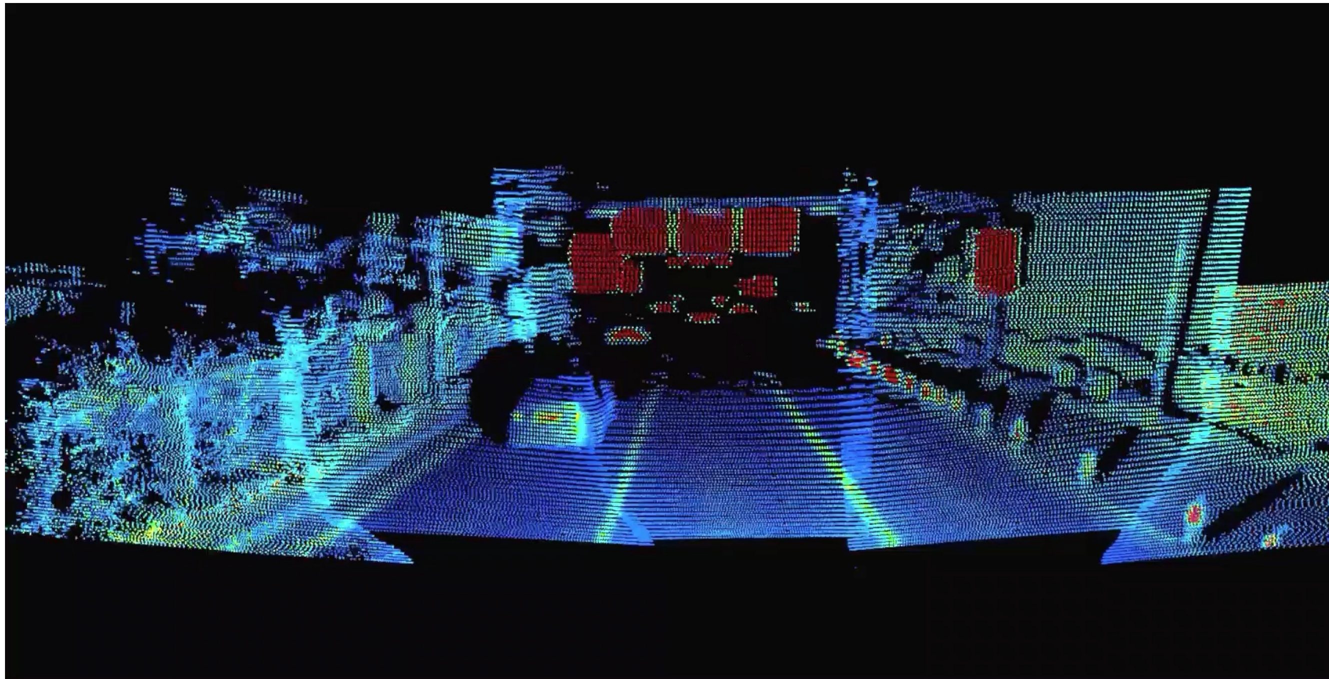
Livox Mid-360

- hybrid solid-state LiDAR
- 40 lines
- Introduced Jan 2023
- 70m max range, 4m at 10%
- Weight: 265 g



RoboSense Lidar M1

- Solid state Lidar (300 x 125 pixel)
- Accuracy: Up to ±5cm
- Range: 150m on 10% NIST
- Vertical FOV: 25° (-12.5° ~ +12.5°)
- Vertical angular resolution: 0.2º
- Horizontal FOV: 120° (-60.0° ~ +60.0°)
- Horizontal angular resolution: 0.2º
- Refresh Rate: 15 Hz
- Data rate: 1,125,000pts/s (single return)
- Power consumption: 25w
- Weight: ~ 800g



Intel RealSense L515

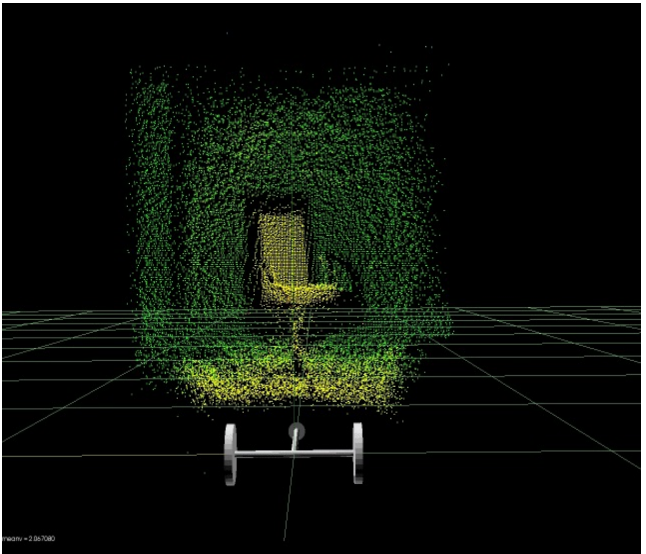
- 9m distance
- Depth: 1024 x 786 pixel @ 30Hz => 23mill pts per second
- RGB: 1920 × 1080 @ 30Hz
- Depth FOV: 70° × 55° (±2°)
- Weight: 100g
- With IMU

- Solid state laser with RGB camera

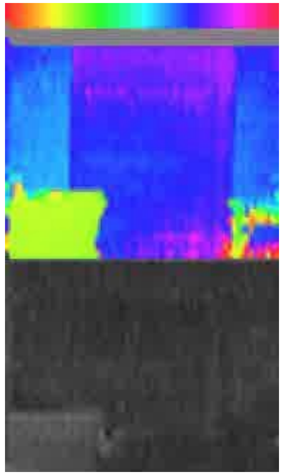
- Sensitive to ambient infrared light =>
  Does NOT work outdoors!

3D Range Sensor: Time Of Flight (TOF) camera

- 3D information with high data rate (100 Hz)
- Compact and easy to manage
- High, non-uniform measurement noise
- High outlier rate at jump edges
- Wrap-around error (phase ranging)
- Sensitive to ambient infrared light => Does NOT work outdoors!



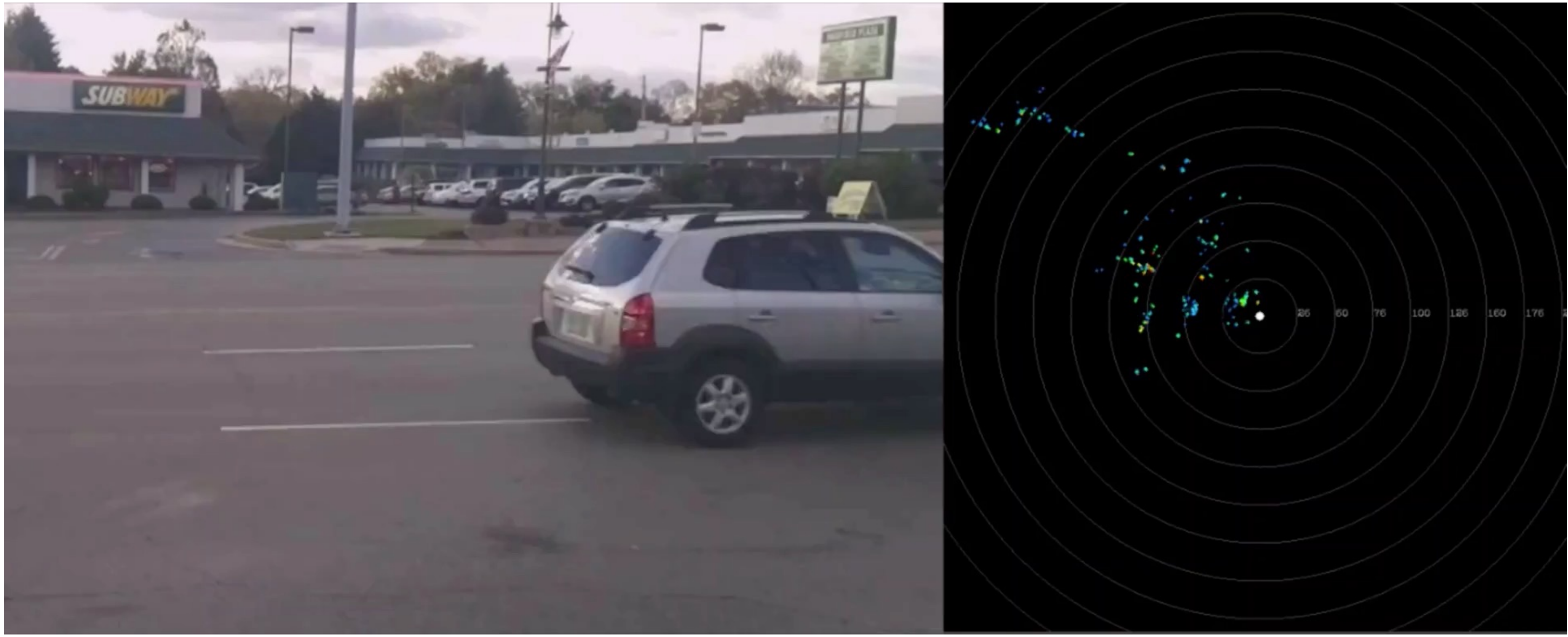
Swiss Ranger 3000
(produced by MESA)



- Kinect 2
- Resolution 1920x1080 pixels
- Field of view: 70 deg (H), 60 deg (V)
- Claimed accuracy: 1 mm
- Claimed max range: 6 meters

Radar, 4D imaging radar (e.g. Oculii)

- Works in various weather and environment conditions:
  - fog, heavy rain, pitch darkness, air pollution
- High range (300+ meters)
- Capture doppler shifts (speed of other objects in a single scan) – this is the 4th dimension
- 250M+
- <1º Resolution
- 120º FOV

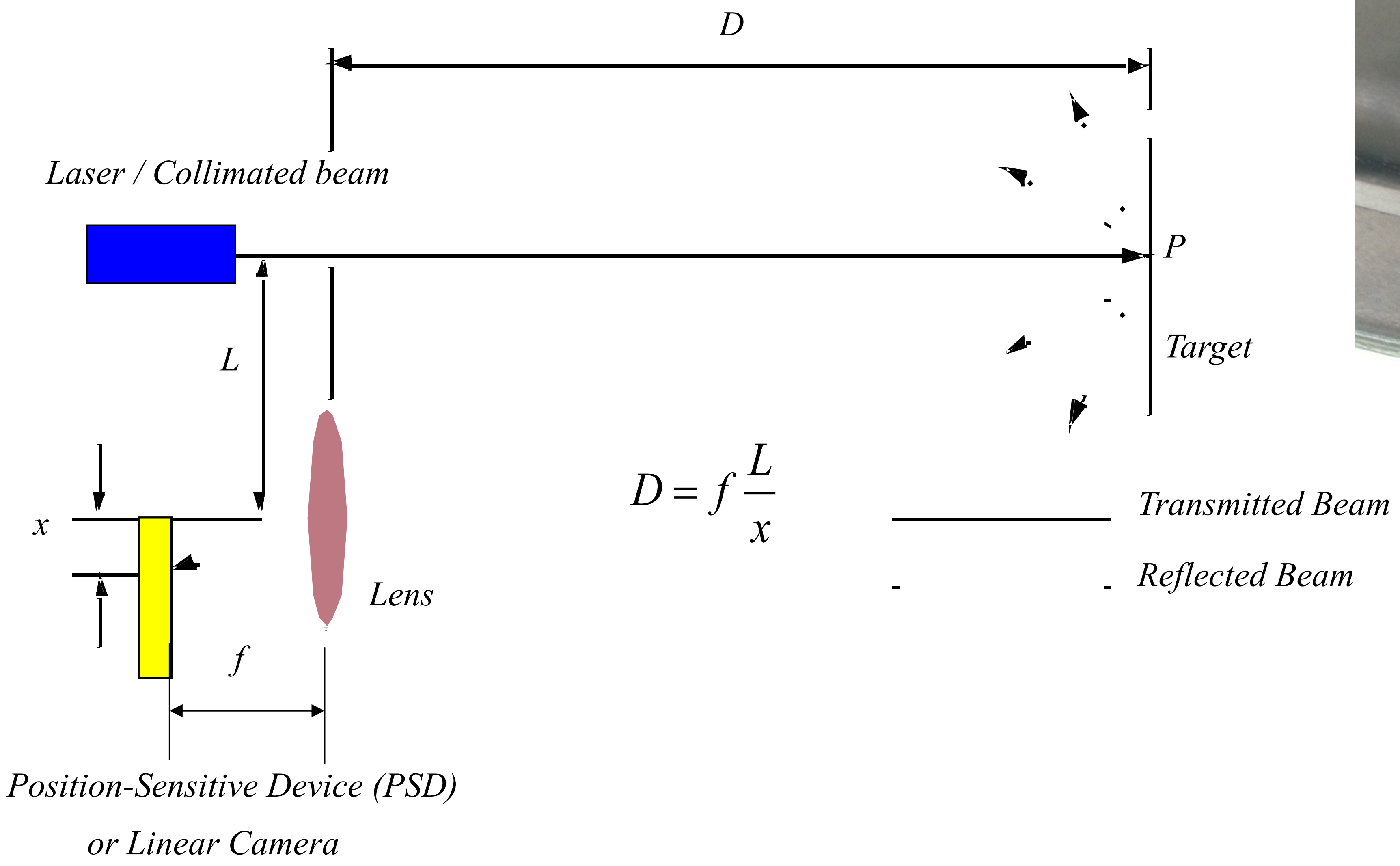


RANGE SENSING: PROJECTED PATTERN

Triangulation Ranging

- Geometrical properties of the image to establish a distance measurement
- e.g. project a well defined light pattern (e.g. point, line) onto the environment.
  - reflected light is than captured by a photo-sensitive line or matrix (camera) sensor device
  - simple triangulation allows to establish a distance.
- e.g. size of an captured object is precisely known
  - triangulation without light projecting

Laser Triangulation (1D)

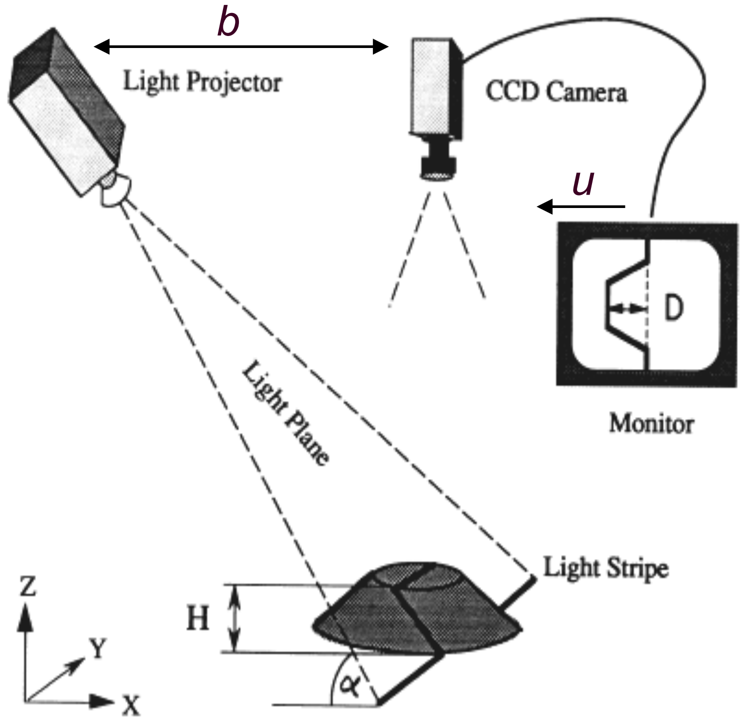


$$D = f\frac{L}{x}$$

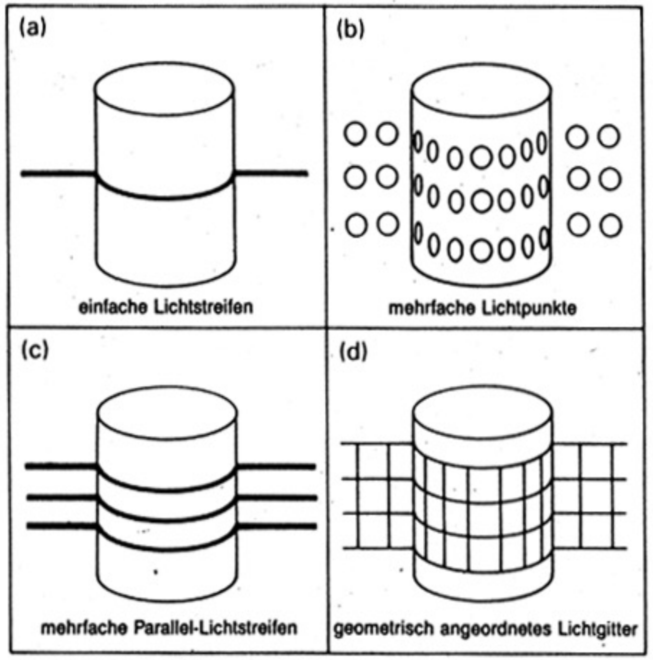
Cliff sensor on Turtle-bot

Structured Light (vision, 2 or 3D): Structured Light

a



b



- Eliminate the correspondence problem by projecting structured light on the scene.
- Slits of light or emit collimated light (possibly laser) by means of a rotating mirror.
- Light perceived by camera
- Range to an illuminated point can then be determined from simple geometry.

Structured Light (vision, 2 or 3D)

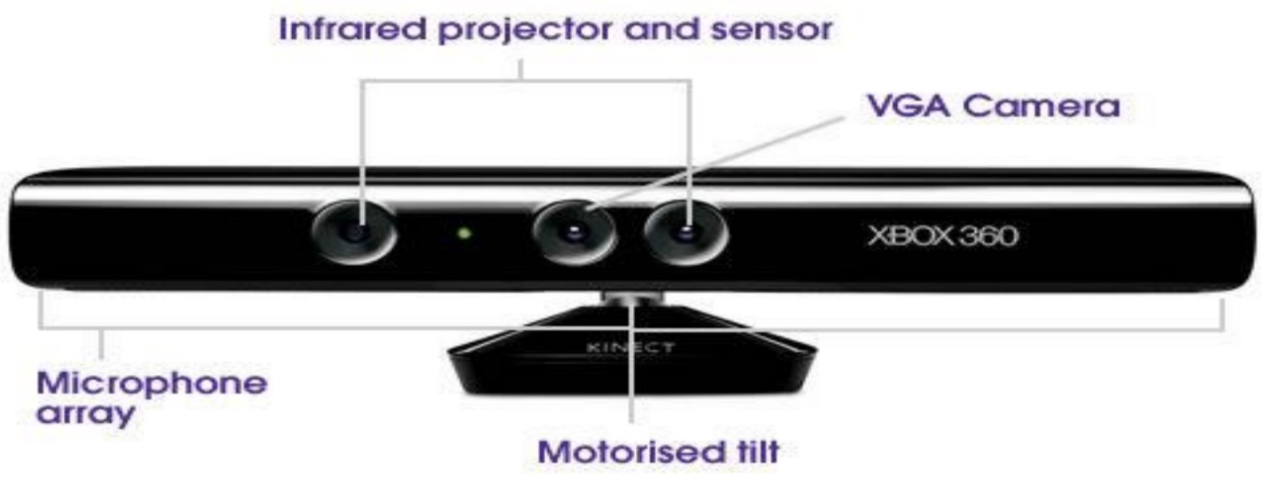
- Baseline length *b*:
  - the smaller b is the more compact the sensor can be.
  - the larger b is the better the range resolution is.

Note: for large b, the chance that an illuminated point is not visible to the receiver increases.

- Focal length f:
  - larger focal length f can provide
    - either a larger field of view
    - or an improved range resolution
  - however, large focal length means a larger sensor head

PrimeSense Cameras

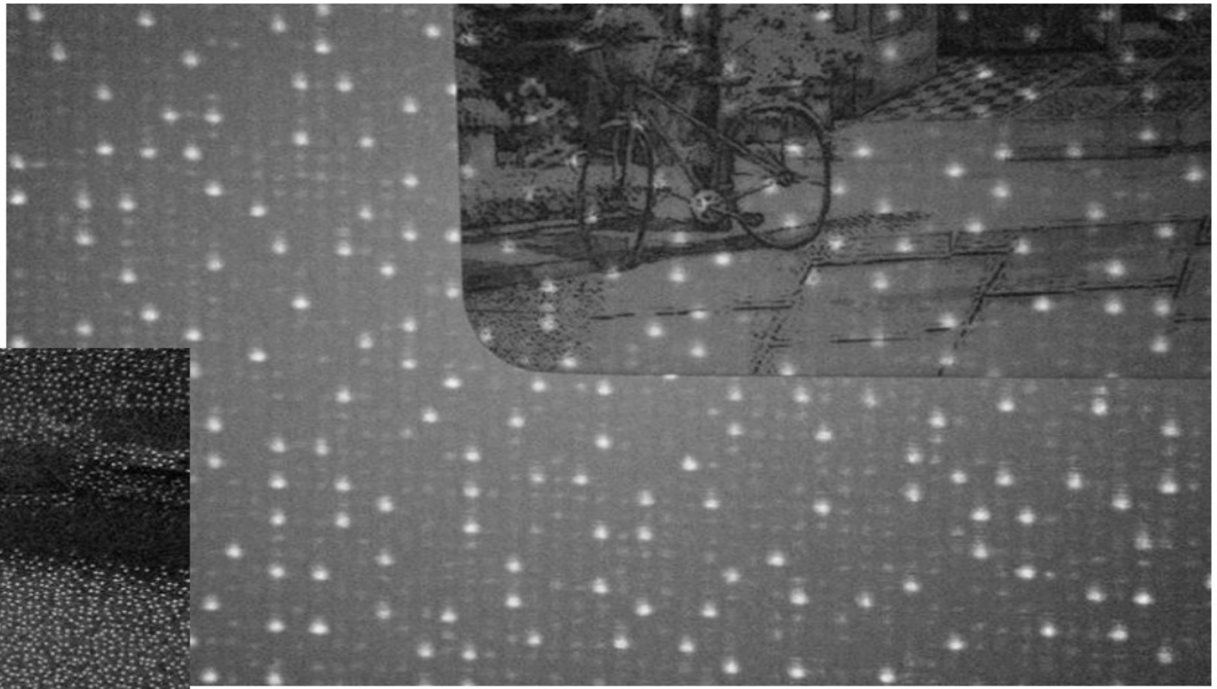
- Devices: Microsoft Kinect and Asus Xtion
- Developed by Israeli company PrimeSense in 2010



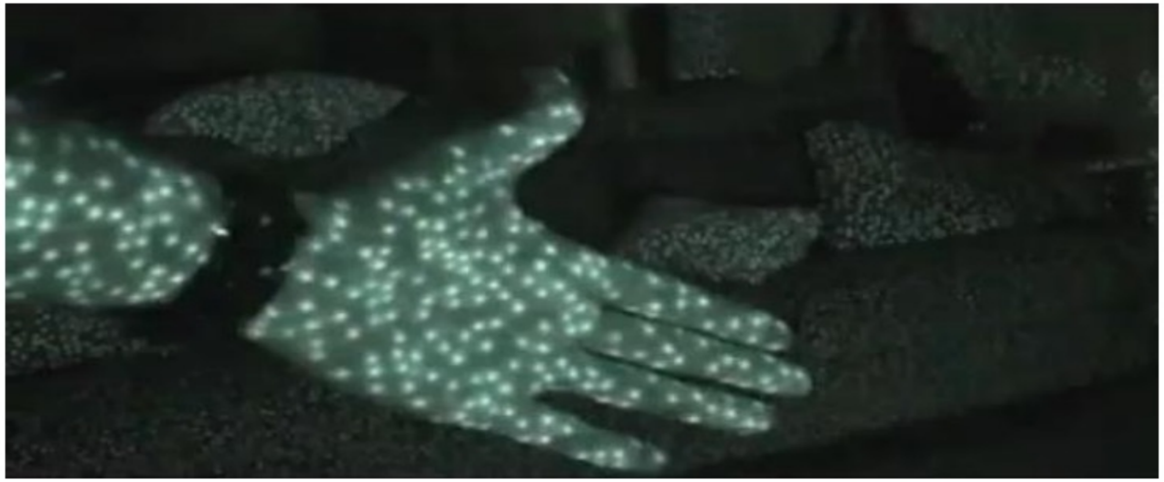
- Components:
  - IR camera (640 x 480 pixel)
  - IR Laser projector
  - RGB camera (640 x 480 or 1280 x 1024)
  - Field of View (FoV):
    - 57.5 degrees horizontally,
    - 43.5 degrees vertically

IR Pattern

Sensitive to infrared light
=>
Does NOT work outdoors!



Depth Map



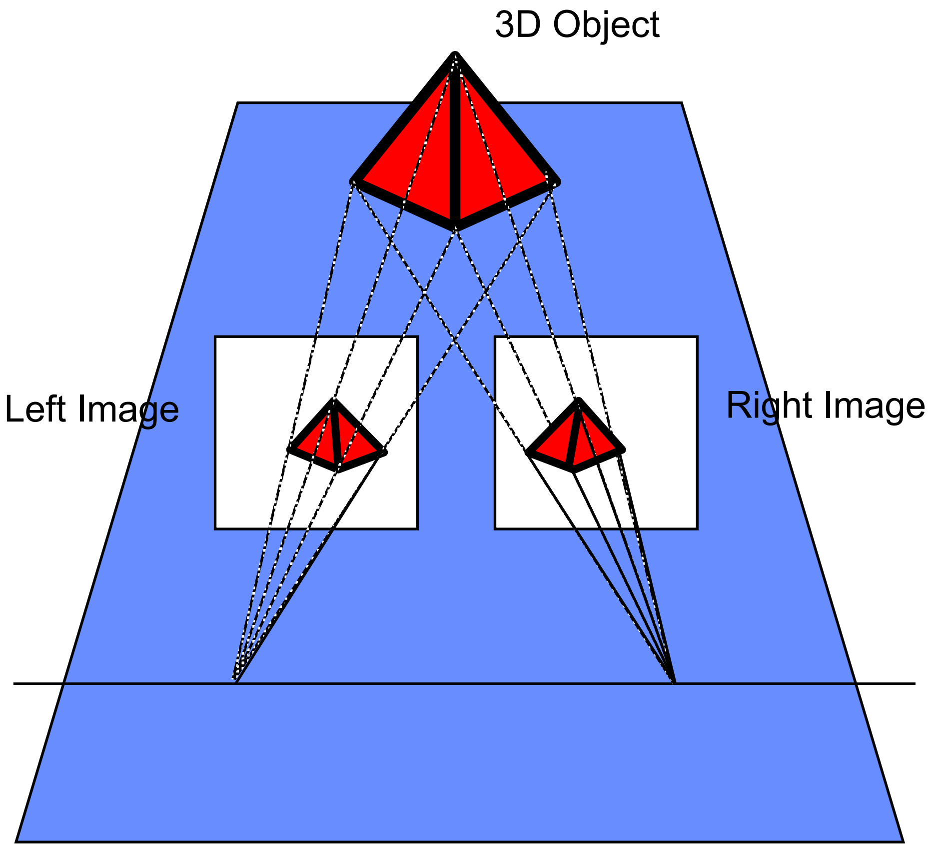
Microsoft Kinect: Depth Computation (1)

- **Depth from Stereo**
  - The Kinect uses an infrared projector and an infrared sensor; it does not use its RGB camera for depth computation
  - The technique of analyzing a known pattern is structured light
  - The IR projector projects a pseudo-random pattern across the surface of the room.
  - The direction of each speckle of the patter is known (from pre calibration during manufacturing) and is hardcoded into the memory of the Kinect
  - By measuring the position of each speckle in the IR image, its depth can be computed

RANGE SENSING: STEREO VISION

Stereo Cameras

- Theory will be covered in detail in Vision Lecture
- Estimate depth by using 2 cameras

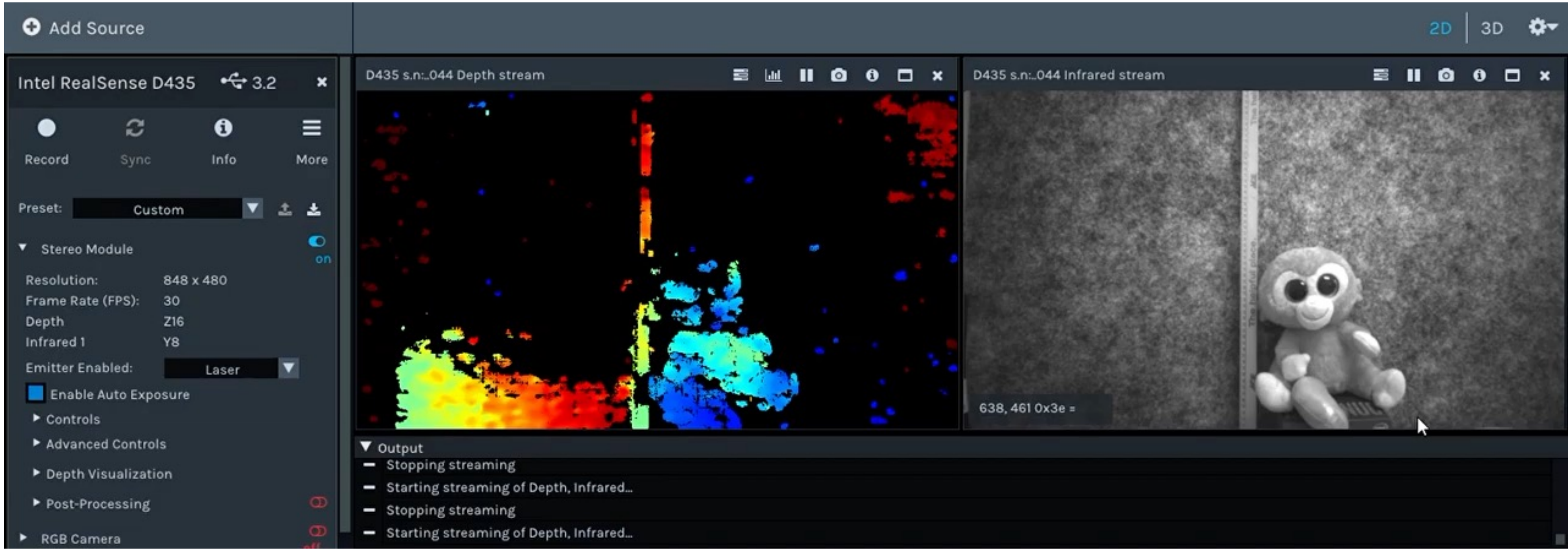


Stereo example: ZED camera

- Dual 4MP Camera @15Hz (lower resolution => higher fps)
- Up to 20m distance
- Passive Sensor
- Doesn’t work on single color surfaces (e.g. white wall)!

Intel RealSense D435

- Stereo Infrared – works indoors and outdoors
- Active pattern (e.g. for white wall) – only works indoors!
- Depth resolution: 1280 × 720
- Depth Field of View (FOV): 86° × 57° (±3°)
- RGB: 1920 × 1080
- Small and lightweight
- With IMU



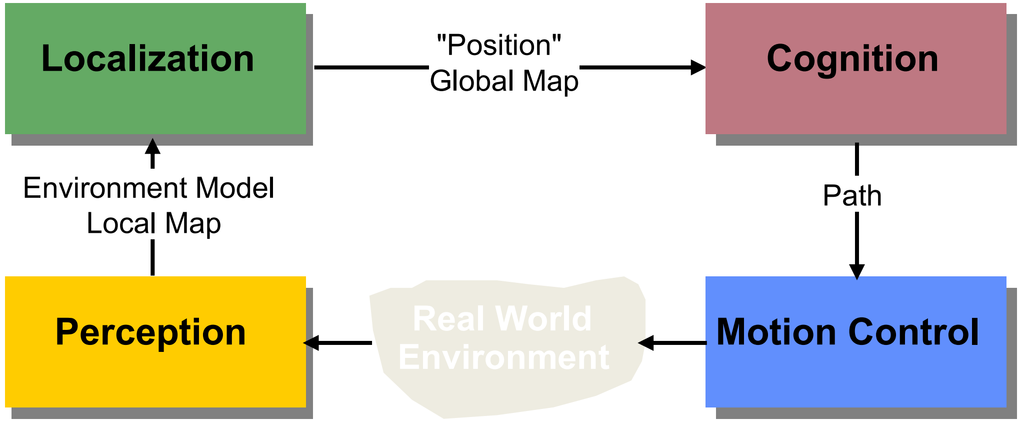
ADMIN

Admin

- HW 1 is due today.
- HW 2 will be published today

- Project:
  - Selection due today!
  - Make an appointment with the supervising MARS Lab graduate and Prof. Schwertfeger:
    - Next week
    - Discuss the details

  - You will need to write a proposal (details follow soon).
    - You will need to do a literature research about your topic.
    - You will need to present one of those papers “as if they were your own”. 10 minutes!



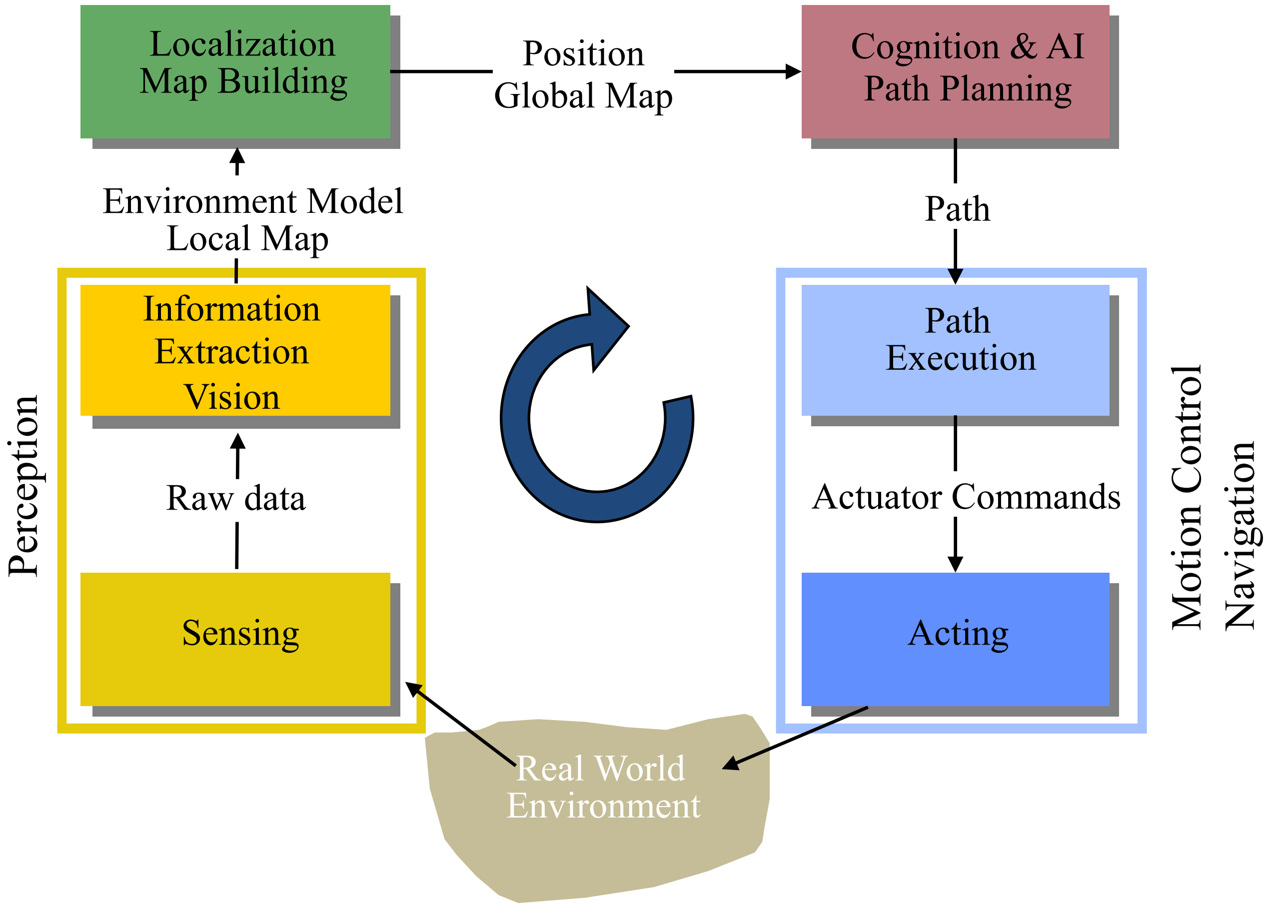
PERCEPTION

Line extraction from laser scans

Vision

Sildes from Roland Siegwart and Davide Scaramuzza, ETH Zurich

General Control Scheme for Mobile Robot Systems



LINE EXTRACTION

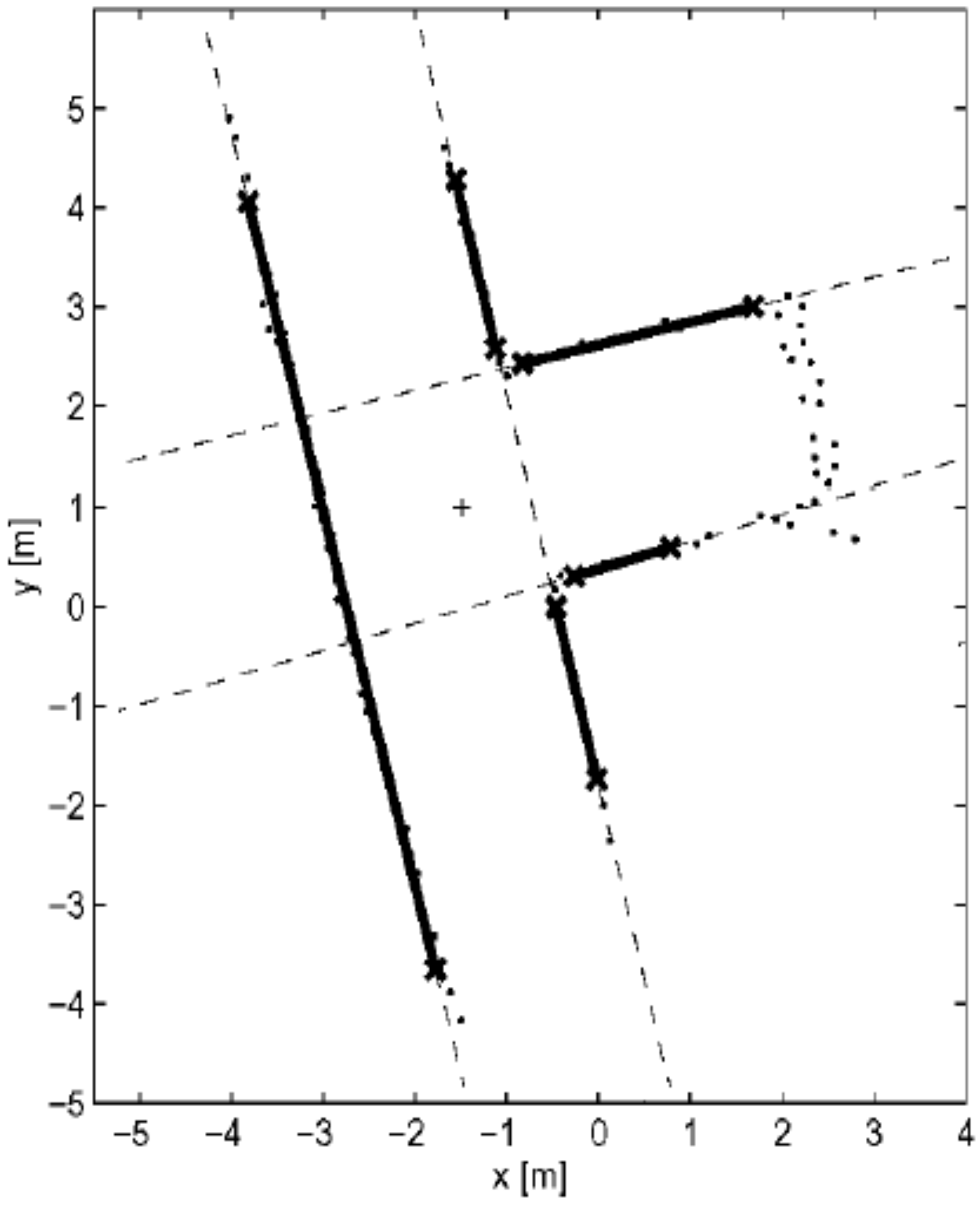
Split and merge
Linear regression
RANSAC
Hough-Transform

Line Extraction: Motivation

- Laser Range Scan
  - Example: 360 deg – black points
  - Example: dashed lines: desired line extractions

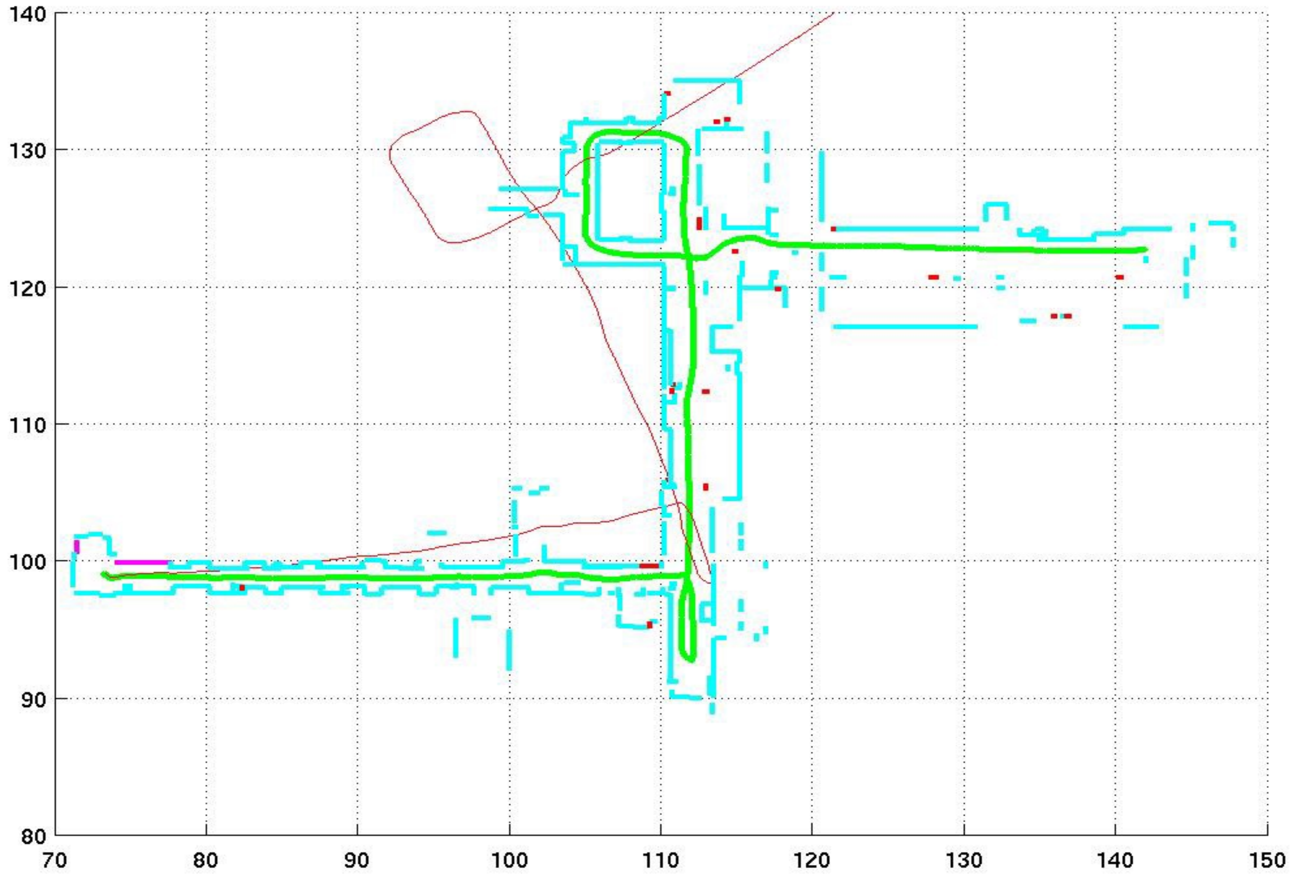
- Use detected lines for:
  - Scan registration (find out transform between frames of two consecutive LRF scans – change due to robot motion)

  OR
  - Mapping using line representation



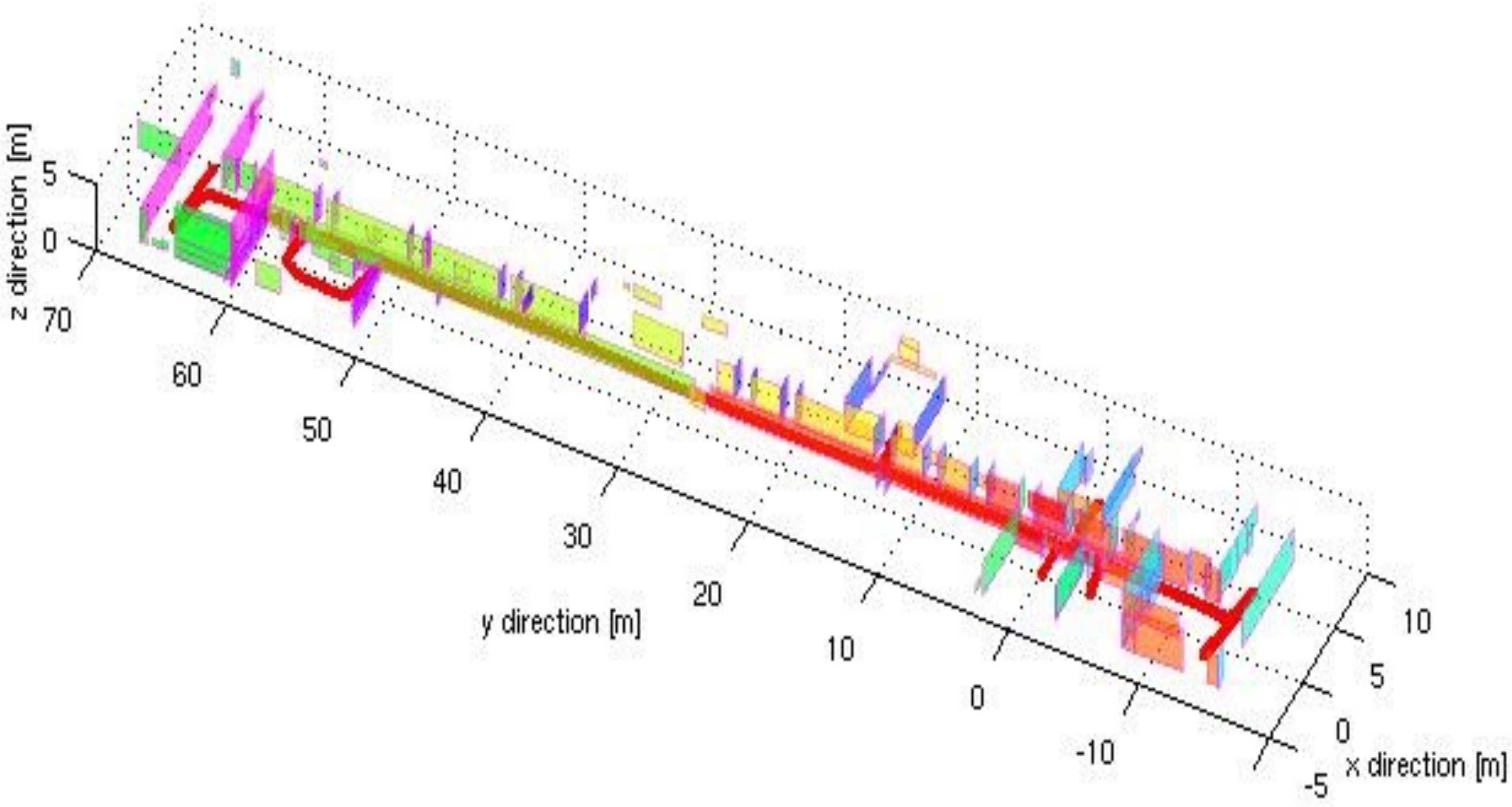
Line Extraction: Motivation

- Map of hallway built using line segments



Line Extraction: Motivation

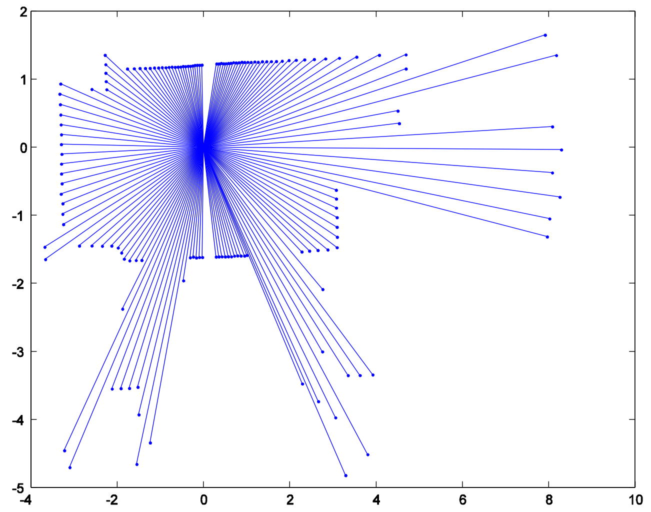
- Map of the hallway built using orthogonal planes constructed from line segments



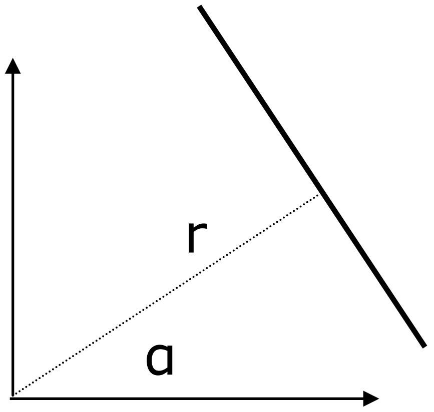
Line Extraction: Motivation

- Why laser scanner:
  - Dense and accurate range measurements
  - High sampling rate, high angular resolution
  - Good range distance and resolution.
- Why line segment:
  - The simplest geometric primitive
  - Compact, requires less storage
  - Provides rich and accurate information
  - Represents most office-like environment.

Line Extraction: The Problem



- Scan point in polar form: (ρ_i, θ_i)
- Assumptions:
  - Gaussian noise
  - Negligible angular uncertainty
- Line model in polar form:
  - $x \cos \alpha + y \sin \alpha = r$
  - $-\pi < \alpha <= \pi$
  - $r >= 0$



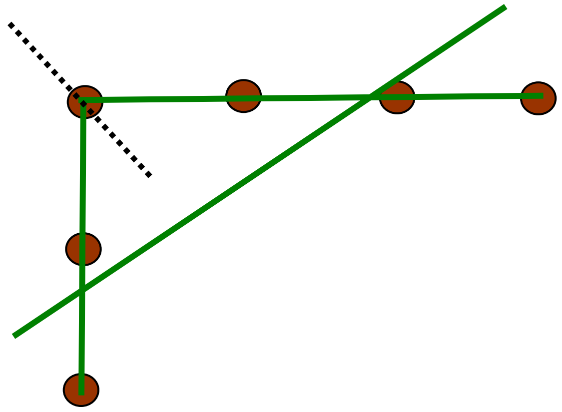
Line Extraction: The Problem (2)

- Three main problems:
  - How many lines ?
  - Which points belong to which line ?
    - This problem is called SEGMENTATION
  - Given points that belong to a line, how to estimate the line parameters ?
    - This problem is called LINE FITTING

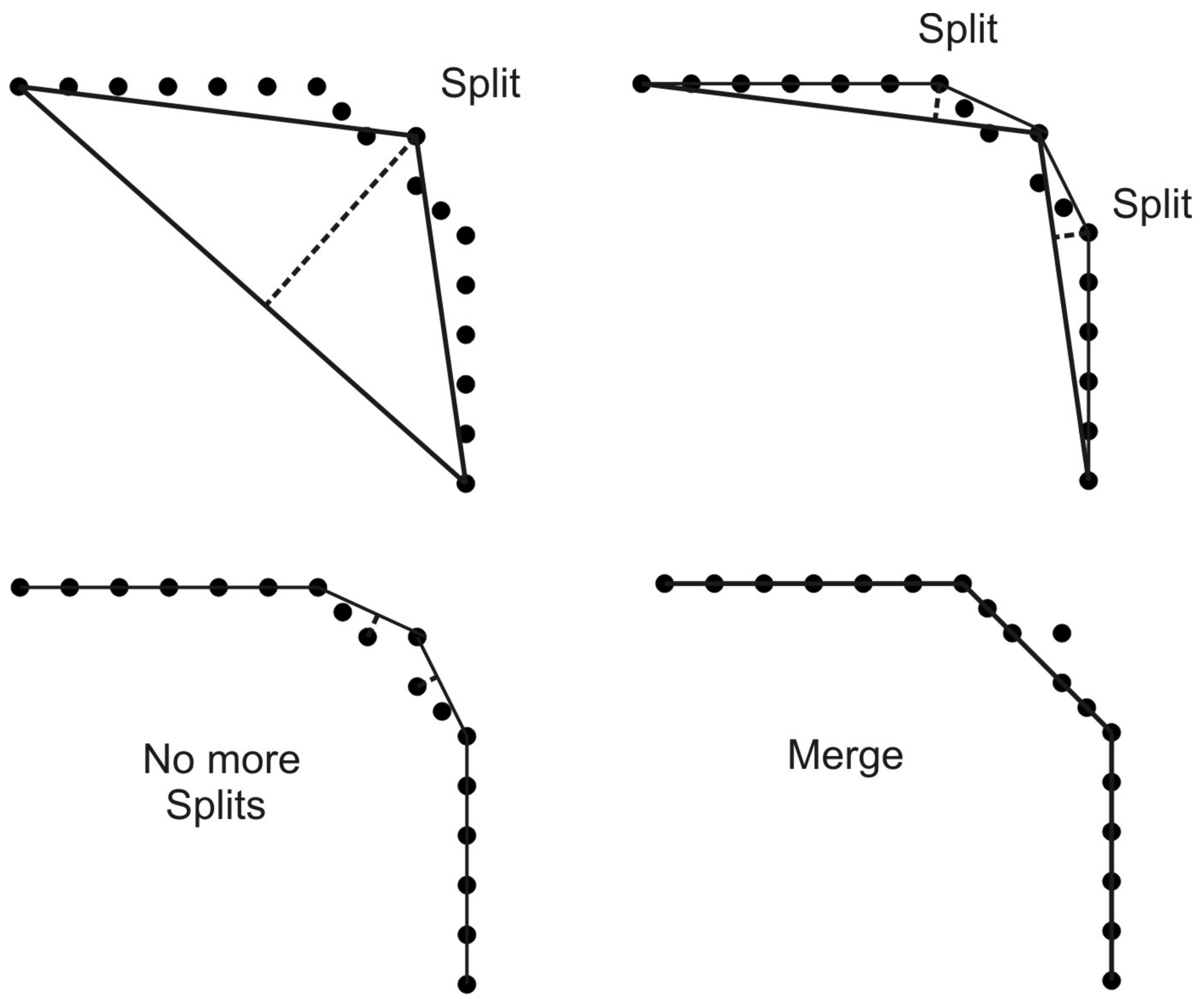
- The Algorithms we will see:
  1. Split and merge
  2. Linear regression
  3. RANSAC
  4. Hough-Transform

Algorithm 1: Split-and-Merge (standard)

- The most popular algorithm which is originated from computer vision.
- A recursive procedure of fitting and splitting.
- A slightly different version, called Iterative-End-Point-Fit, simply connects the end points for line fitting.



Algorithm 1: Split-and-Merge (Iterative-End-Point-Fit)

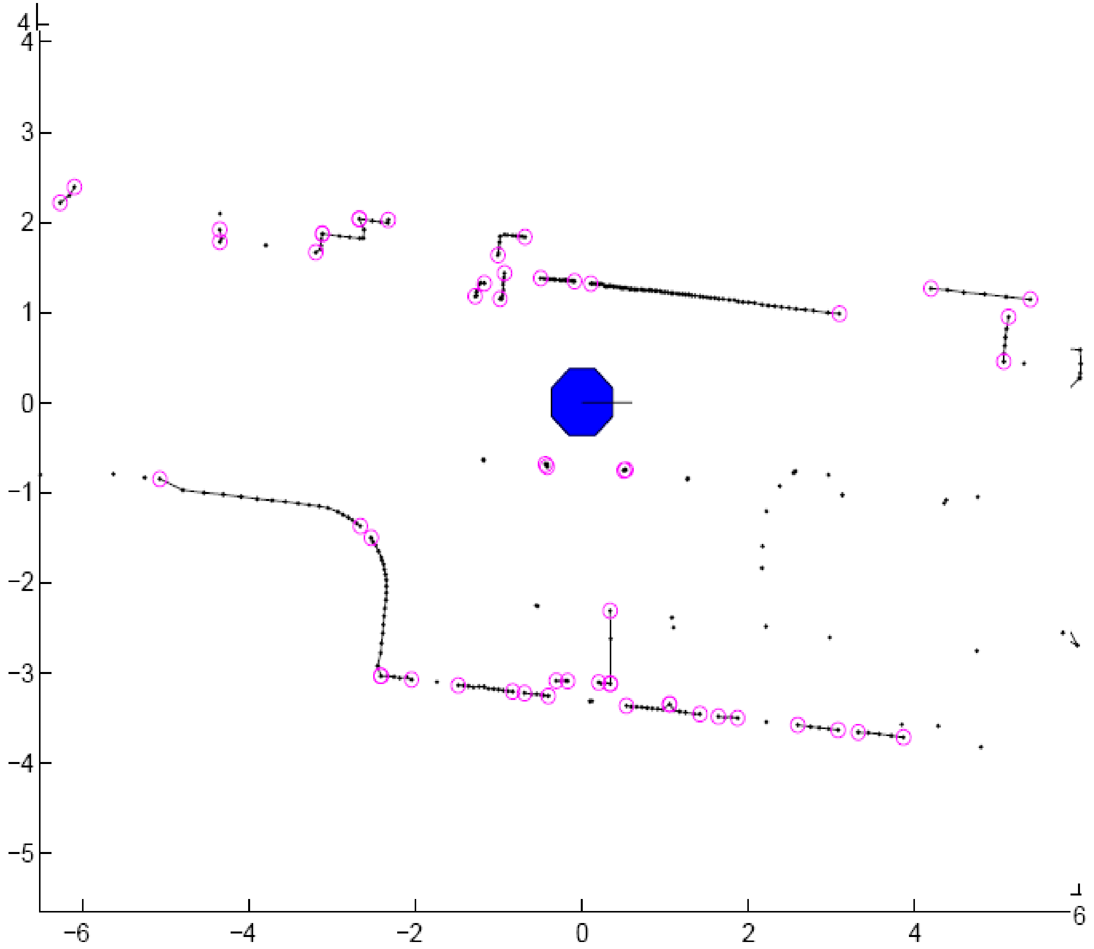


Algorithm 1: Split-and-Merge

Algorithm 1: *Split-and-Merge*

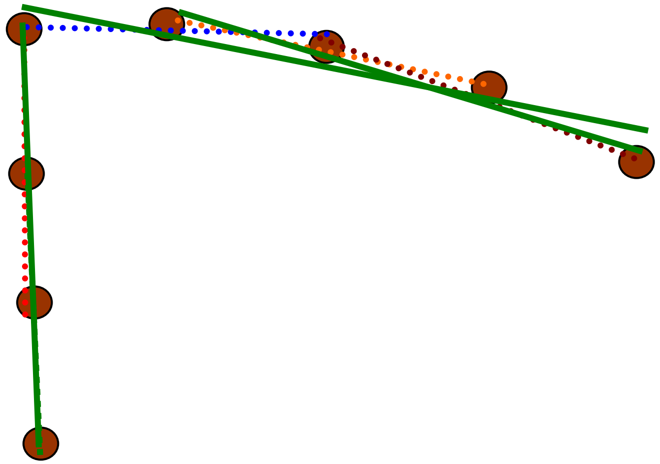
1. Initial: set s_1 consists of N points. Put s_1 in a list L
2. Fit a line to the next set s_i in L
3. Detect point P with maximum distance d_P to the line
4. If d_P is less than a threshold, continue (go to step 2)
5. Otherwise, split s_i at P into s_{i1} and s_{i2}, replace s_i in L by s_{i1} and s_{i2}, continue (go to 2)
6. When all sets (segments) in L have been checked, merge collinear segments.

Algorithm 1: Split-and-Merge: Example application



Algorithm 2: Line-Regression

- Uses a “sliding window” of size *Nf*
- The points within each “sliding window” are fitted by a segment
- Then adjacent segments are merged if their line parameters are close



$Nf = 3$

Algorithm 2: Line-Regression

Algorithm 2: *Line-Regression*

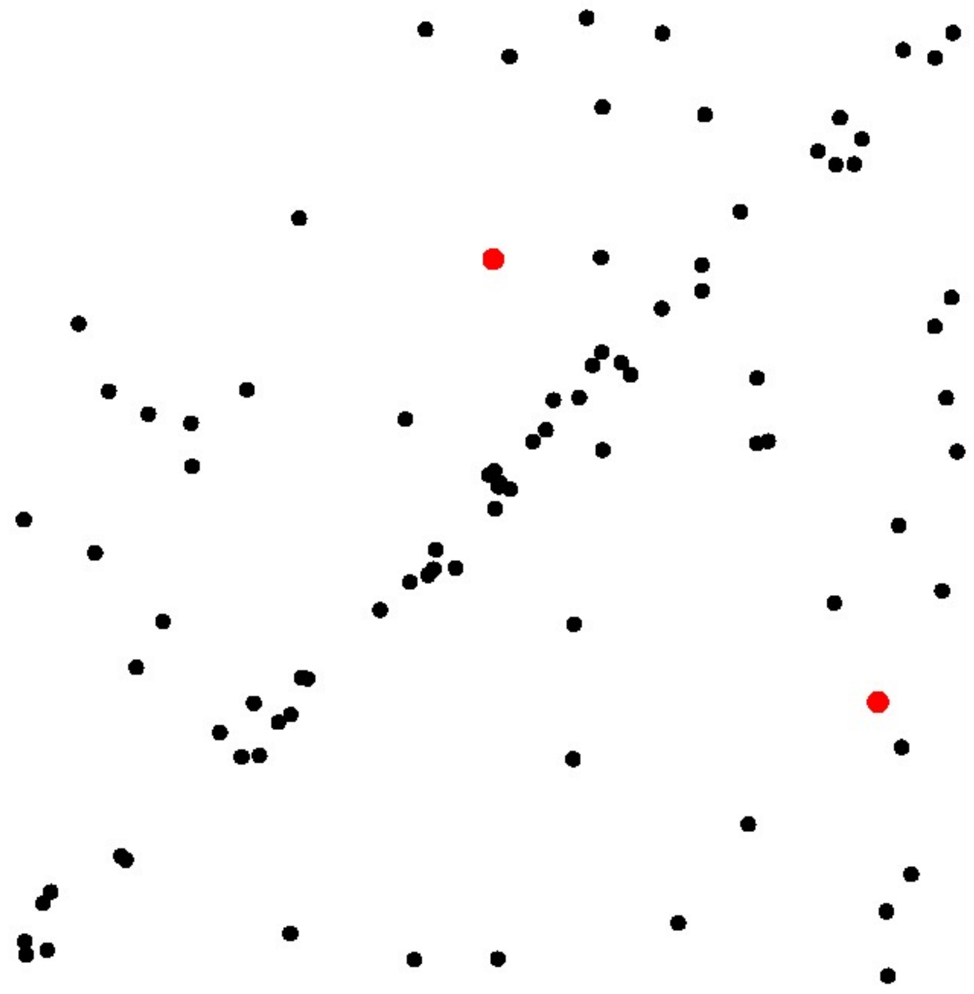
1. Initialize sliding window size N_f
2. Fit a line to every N_f consecutive points (a window)
3. Compute a line fidelity array, each is the sum of Mahalanobis distances between every three adjacent windows
4. Construct line segments by scanning the fidelity array for consecutive elements having values less than a threshold, using an AHC algorithm
5. Merge overlapped line segments and recompute line parameters for each segment

Algorithm 3: RANSAC

- Acronym: **Ran**dom **Sa**mple **C**onsensus.
- Generic & robust fitting algorithm of models with outliers
  - Outliers: points which do not satisfy a model
- RANSAC: apply to any problem where:
  - identify the inliers
  - which satisfy a predefined mathematical model.
- Typical robotics applications:
  - line extraction from 2D range data (sonar or laser);
  - plane extraction from 3D range data
  - structure from motion
- RANSAC:
  - iterative method & non-deterministic
- Drawback: A nondeterministic method, results are different between runs.

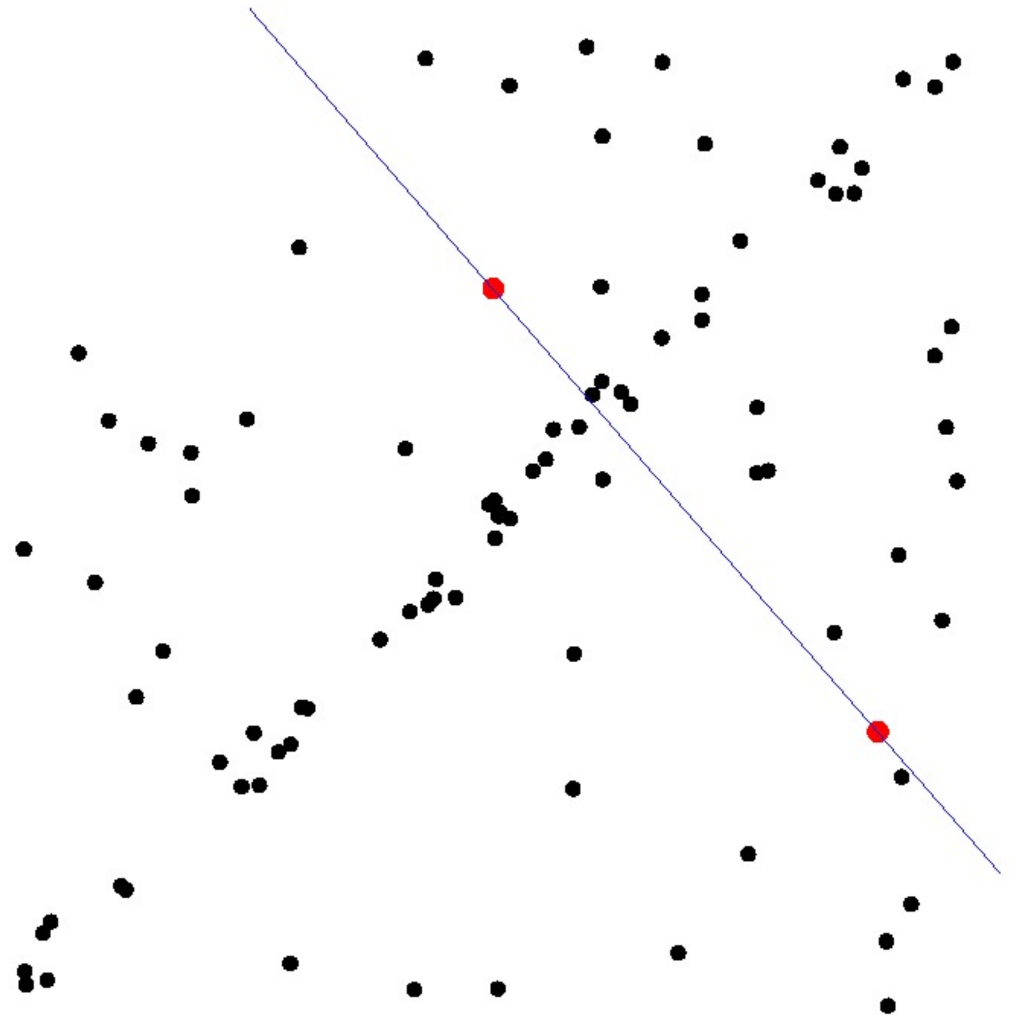
Algorithm 3: RANSAC

Algorithm 3: RANSAC



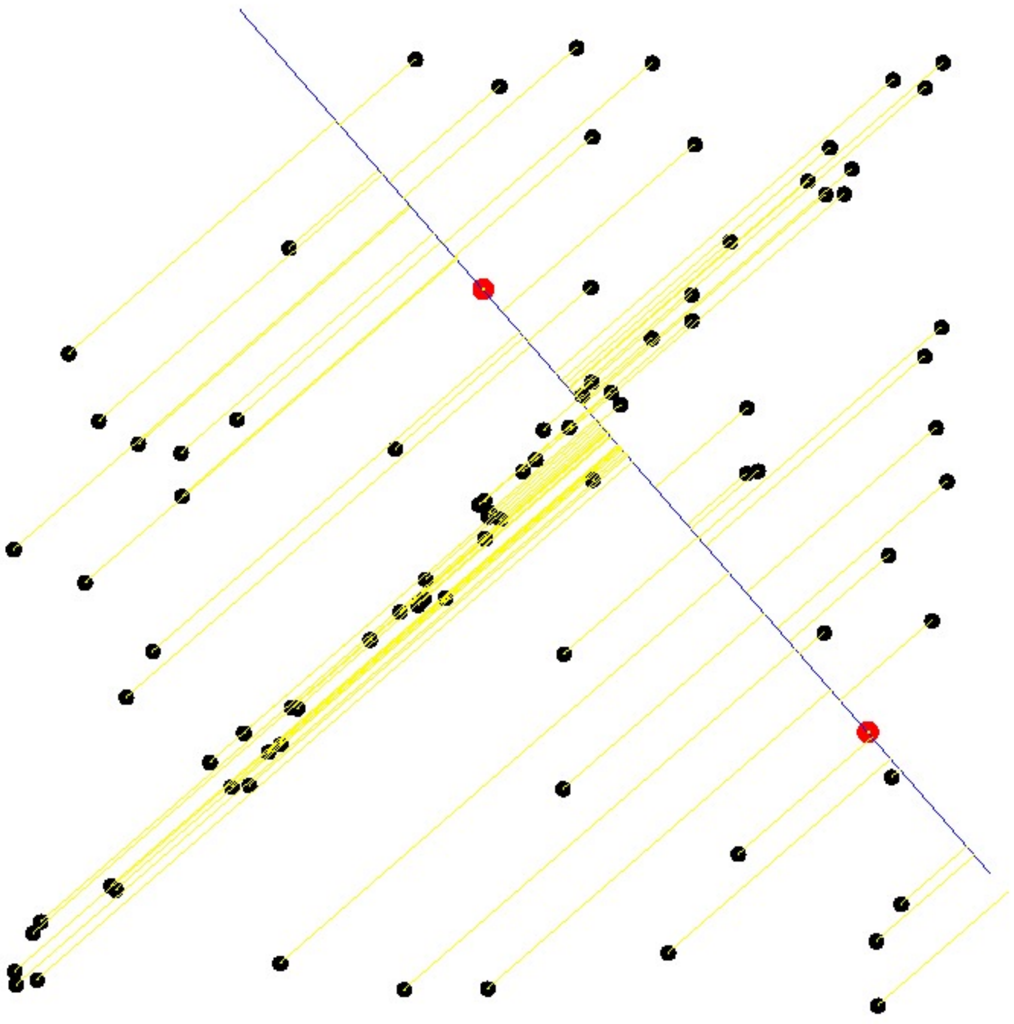
- **Select sample of 2 points at random**

Algorithm 3: RANSAC



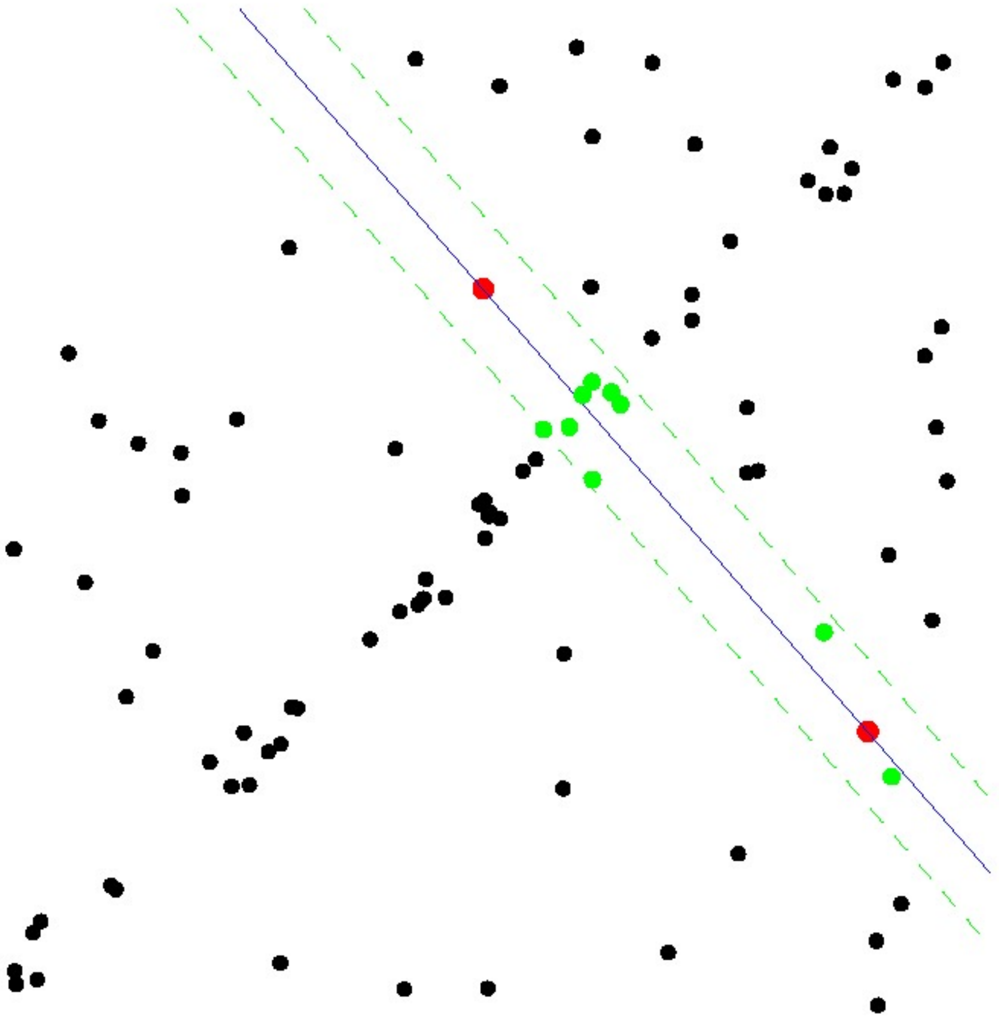
- Select sample of 2 points at random
- **Calculate model parameters that fit the data in the sample**

RANSAC



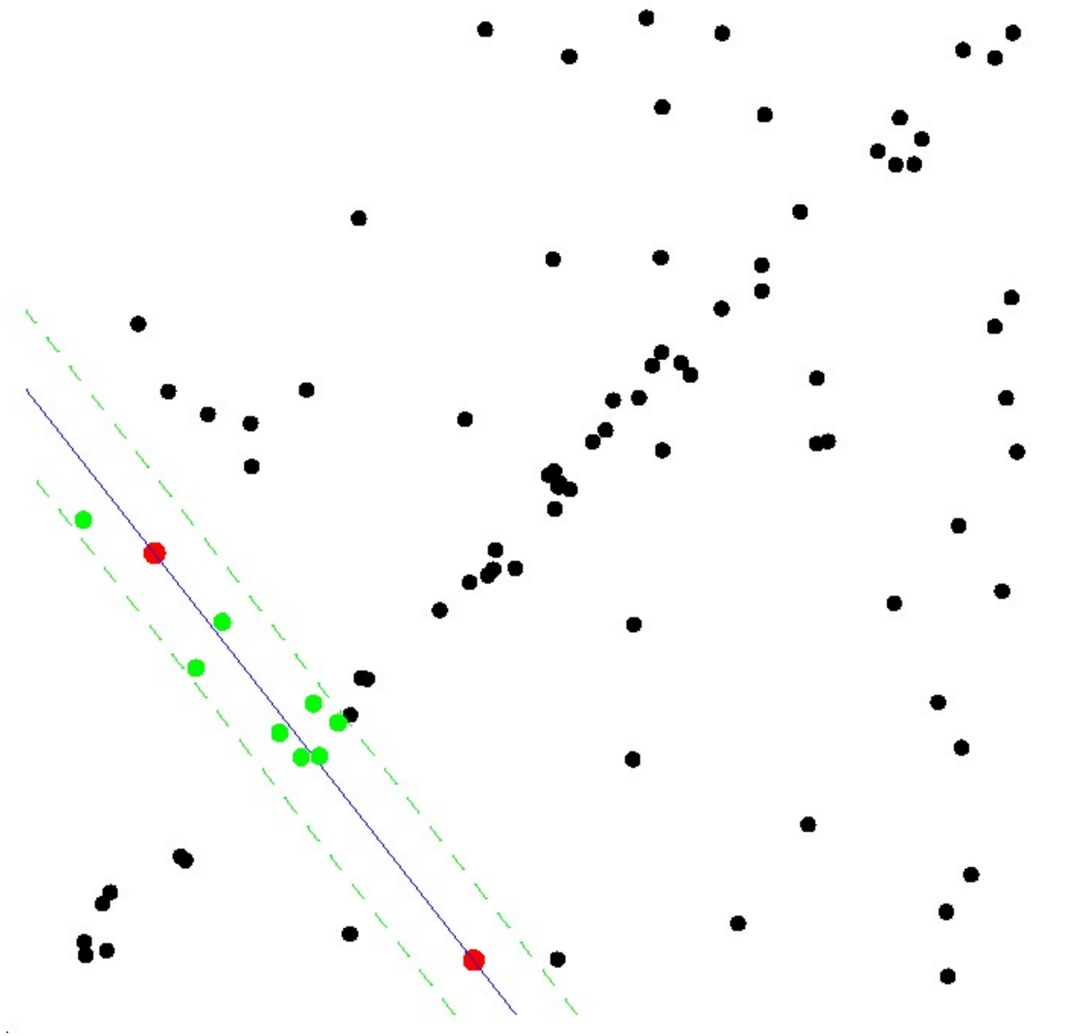
- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- **Calculate error function for each data point**

Algorithm 3: RANSAC



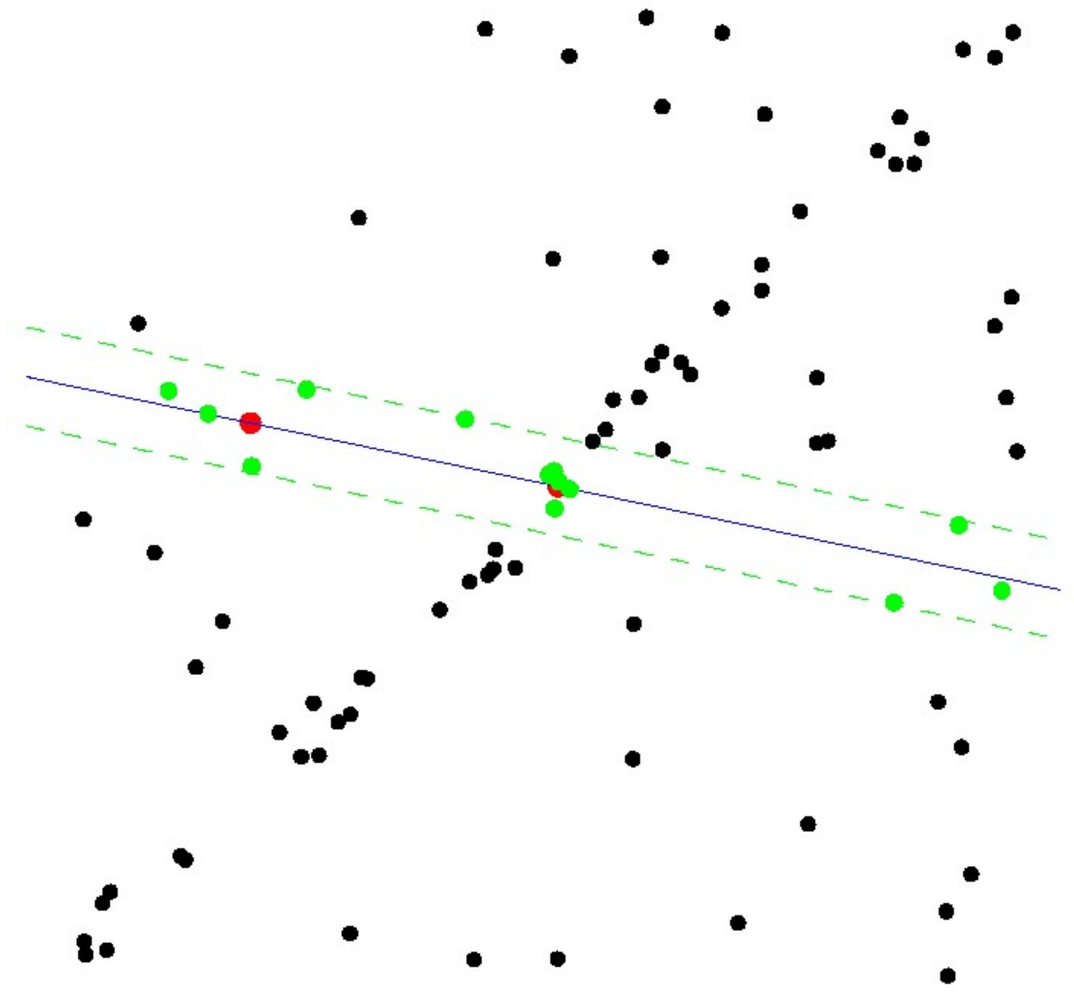
- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- **Select data that support current hypothesis**

Algorithm 3: RANSAC



- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

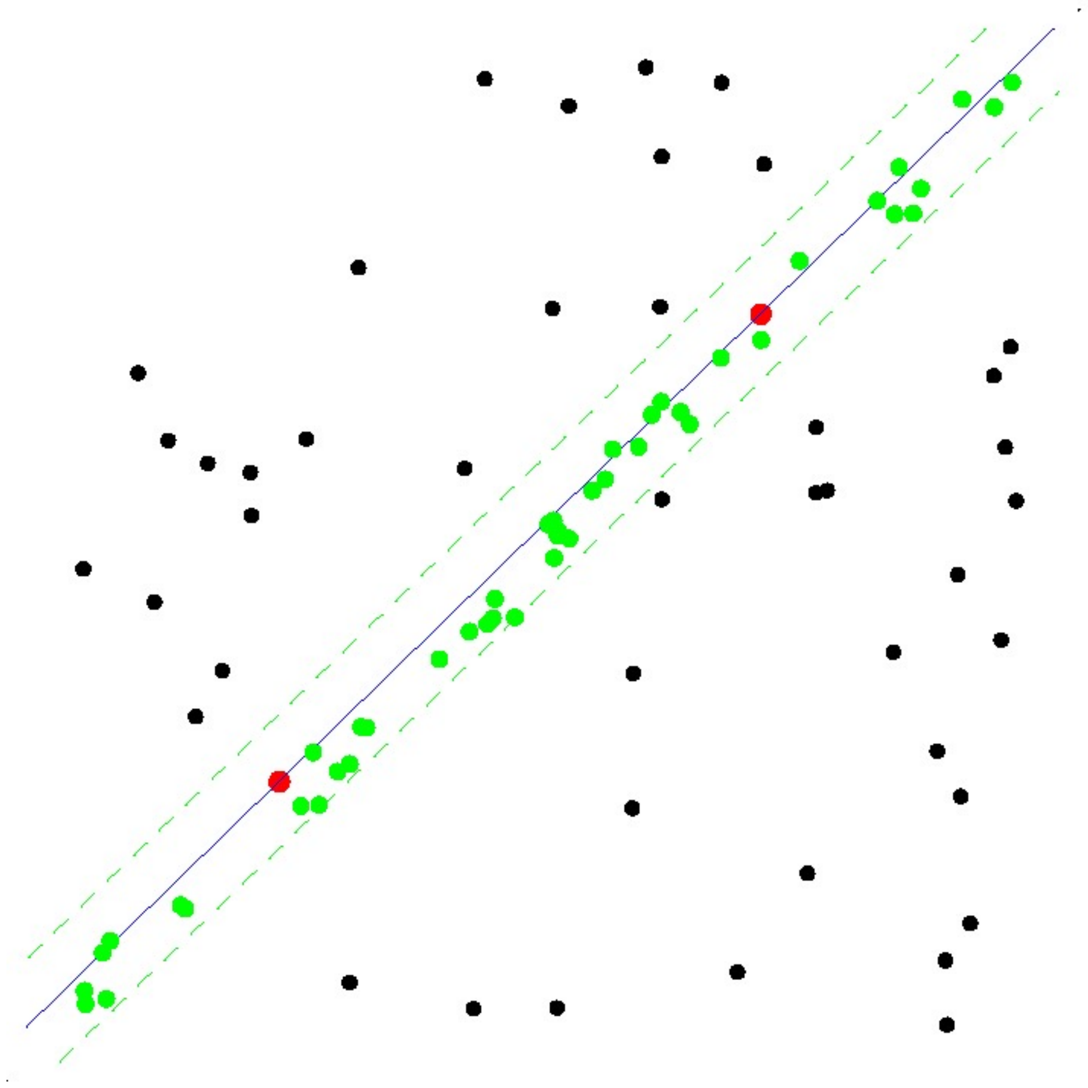
Algorithm 3: RANSAC



- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

Algorithm 3: RANSAC

ALL-INLIER SAMPLE



Algorithm 3: RANSAC

Algorithm 4: *RANSAC*

1. Initial: let A be a set of N points
2. **repeat**
3. Randomly select a sample of 2 points from A
4. Fit a line through the 2 points
5. Compute the distances of all other points to this line
6. Construct the inlier set (i.e. count the number of points with distance to the line $< d$)
7. Store these inliers
8. **until** Maximum number of iterations k reached
9. The set with the maximum number of inliers is chosen as a solution to the problem

Algorithm 3: RANSAC

How many iterations does RANSAC need?

- Because we cannot know in advance if the observed set contains the maximum number of inliers, the ideal would be to check all possible combinations of 2 points in a dataset of N points.
- The number of combinations is given by N(N-1)/2, which makes it computationally unfeasible if N is too large. For example, in a laser scan of 360 points we would need to check all 360*359/2= 64,620 possibilities!
- Do we really need to check all possibilities or can we stop RANSAC after iterations? The answer is that indeed we do not need to check all combinations but just a subset of them if we have a rough estimate of the percentage of inliers in our dataset
- This can be done in a probabilistic way