上海科技大学
ShanghaiTech University

CS283: Robotics Spring 2023: Path Planning

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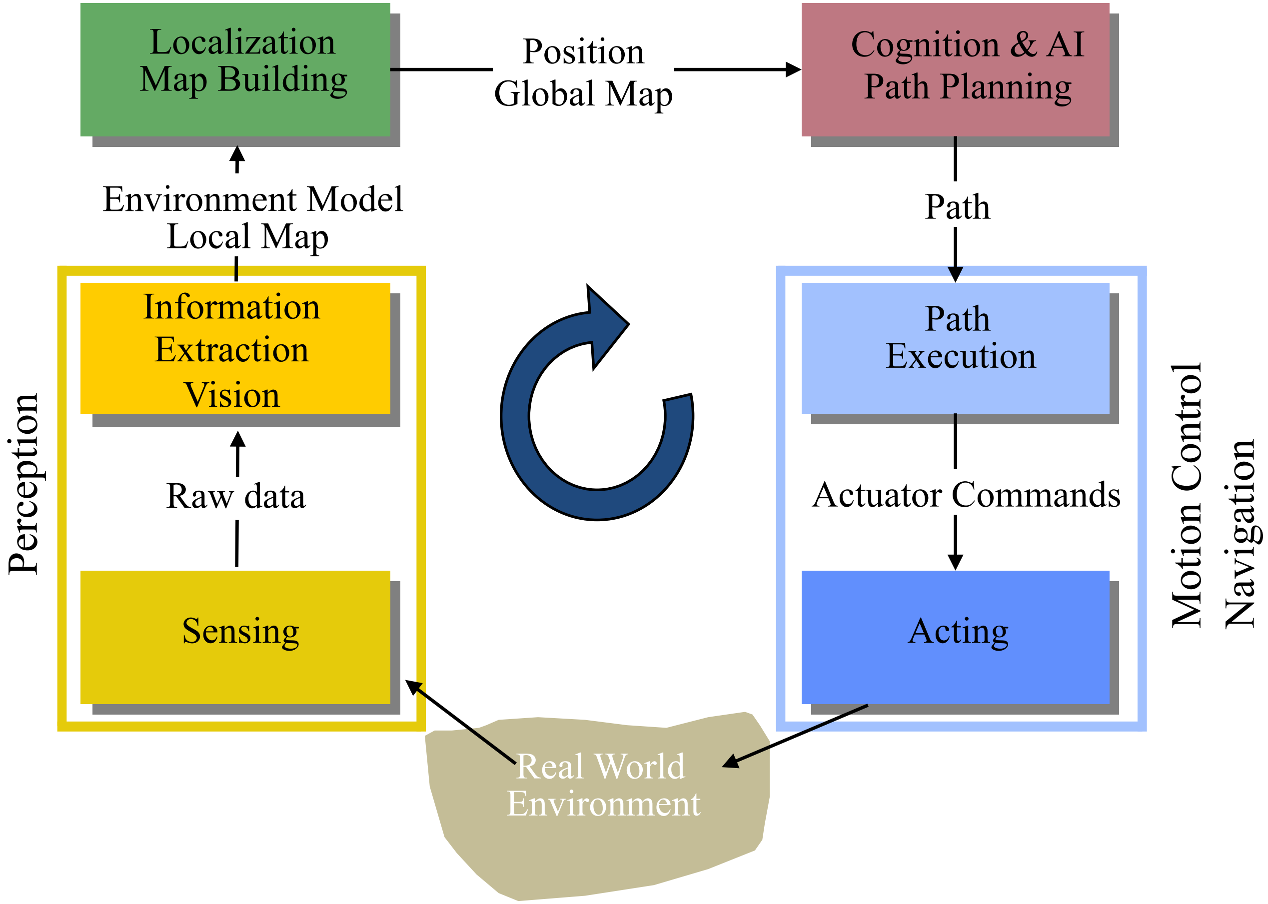
PLANNING

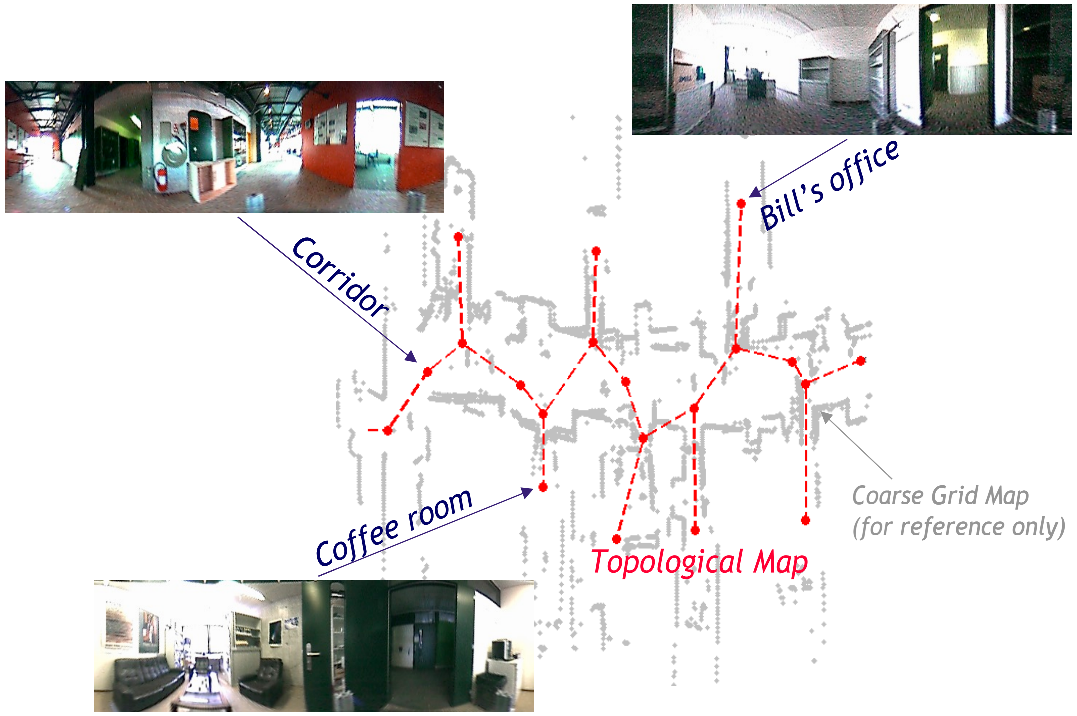
General Control Scheme for Mobile Robot Systems



With material from Roland Siegwart and Davide Scaramuzza, ETH Zurich

The Planning Problem

- The problem: find a path in the work space (physical space) from the initial position to the goal position avoiding all collisions with the obstacles
- Assumption: there exists a good enough map of the environment for navigation.

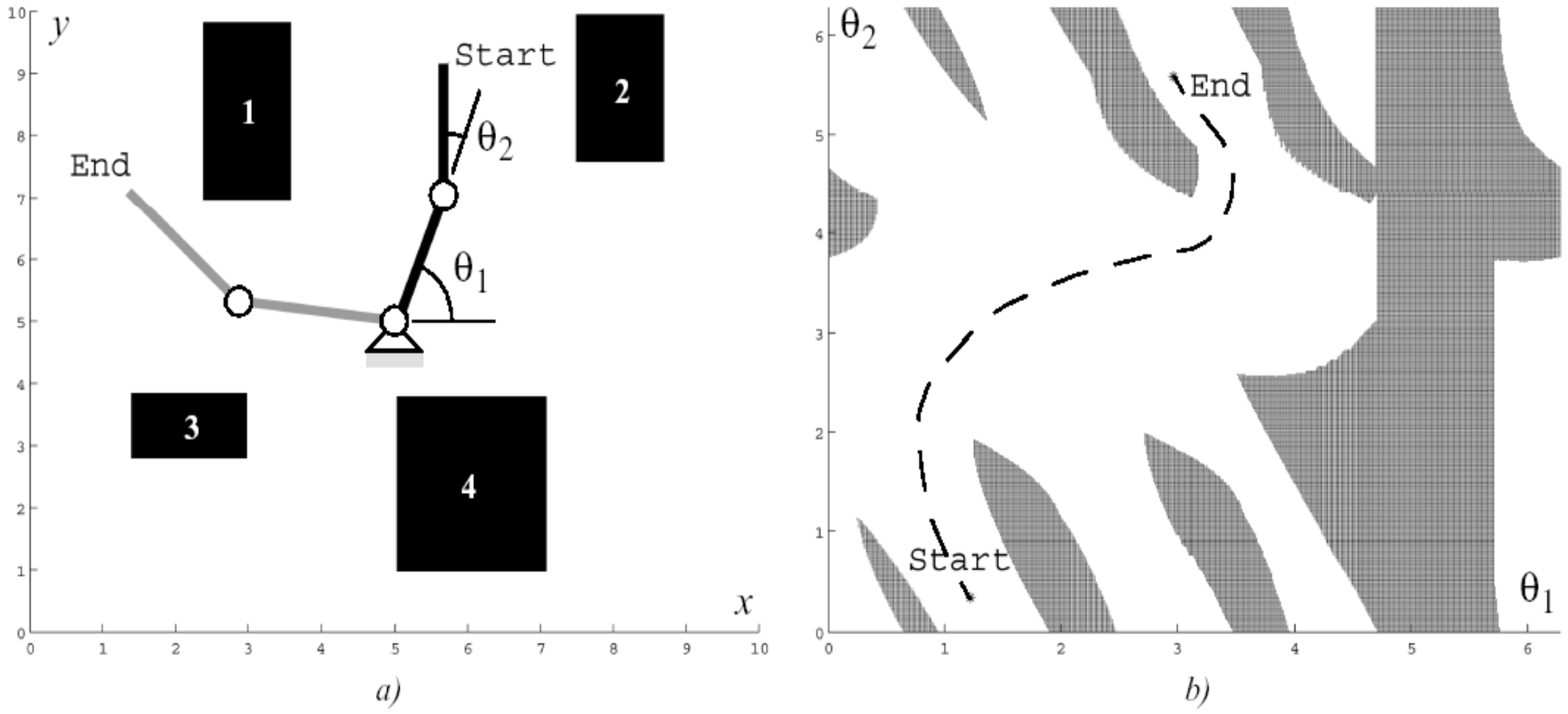


The Planning Problem

- We can generally distinguish between
  - (global) path planning and
  - (local) obstacle avoidance.
- First step:
  - Transformation of the map into a representation useful for planning
  - This step is planner-dependent
- Second step:
  - Plan a path on the transformed map
- Third step:
  - Send motion commands to controller
  - This step is planner-dependent (e.g. Model based feed forward, path following)

Work Space (Map) → Configuration Space

- State or configuration *q* can be described with *k* values q_i



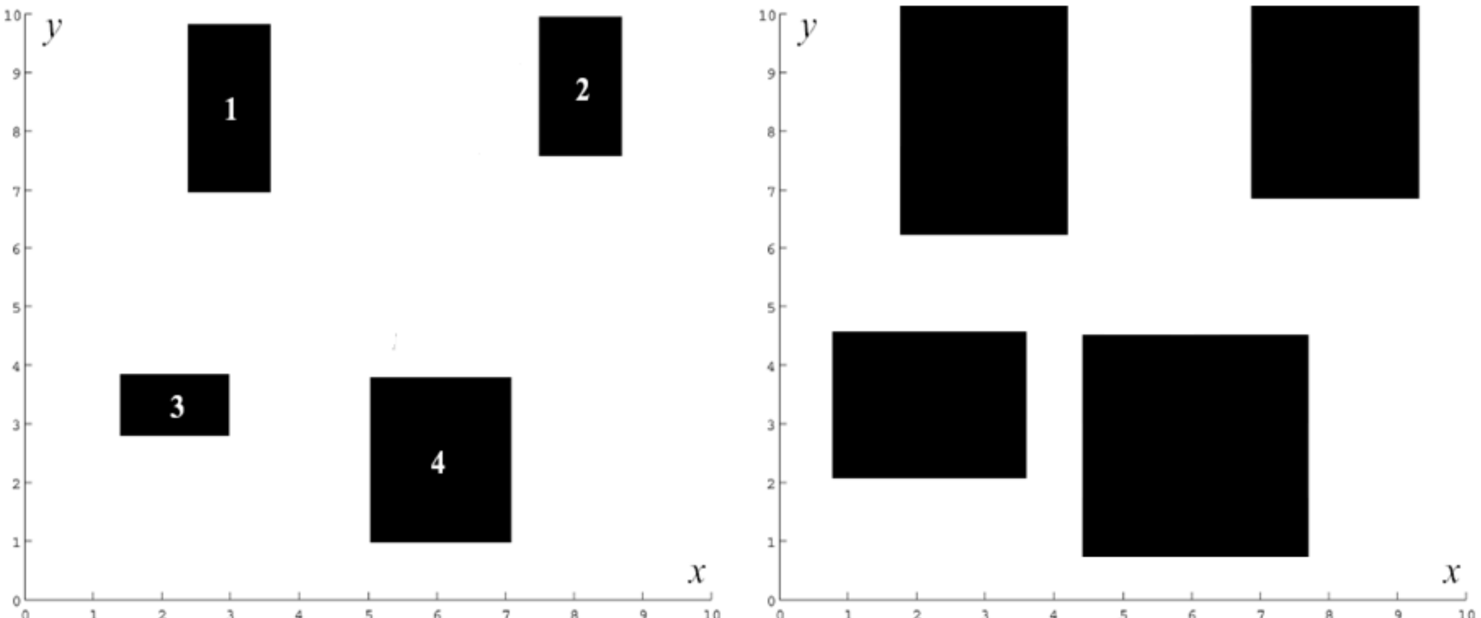
Work Space

Configuration Space:
the dimension of this
space is equal to the Degrees of Freedom (DoF)
of the robot

- What is the configuration space of a mobile robot?

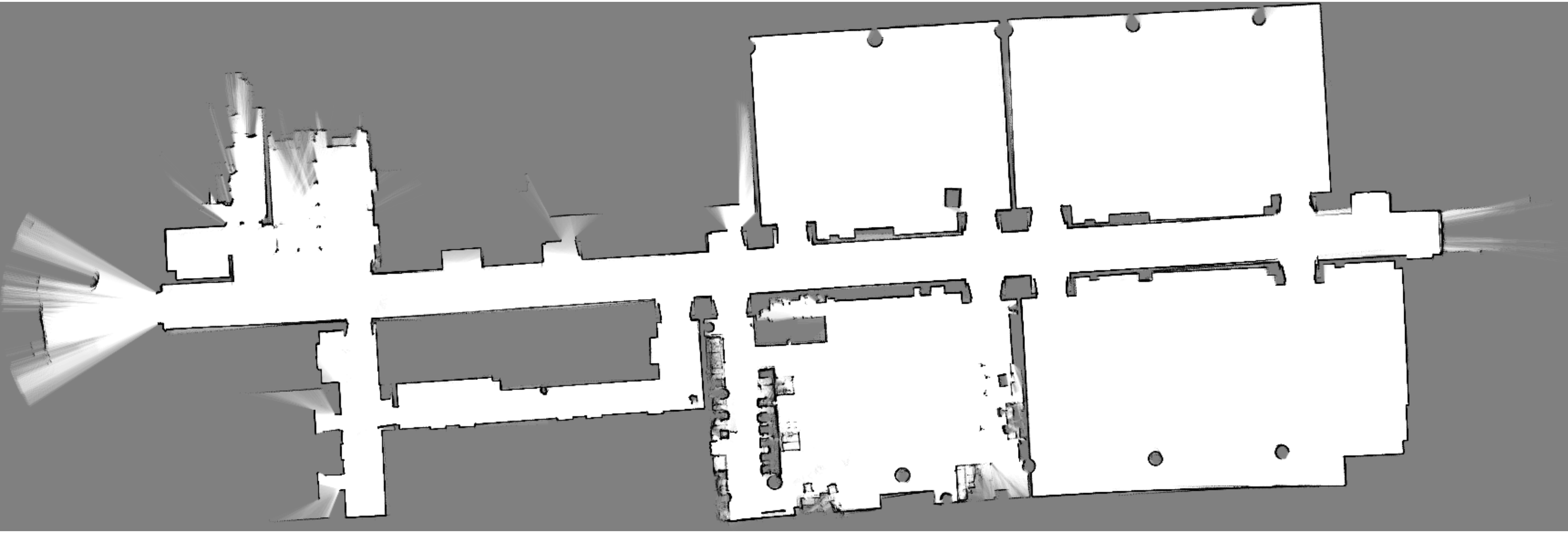
Configuration Space for a Mobile Robot

- Mobile robots operating on a flat ground (2D) have 3 DoF: (x, y, θ)
- Differential Drive: only two motors => only 2 degrees of freedom directly controlled (forward/ backward + turn) => non-holonomic
- Simplification: assume robot is holonomic and it is a point => configuration space is reduced to 2D (x,y)
- => inflate obstacle by size of the robot radius to avoid crashes => obstacle growing



Typical Configuration Space: Occupancy grid

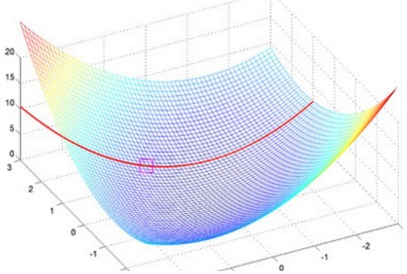
- Fixed cell decomposition: occupancy grid example: STAR Center



Path Planning: Overview of Algorithms

1. Optimal Control

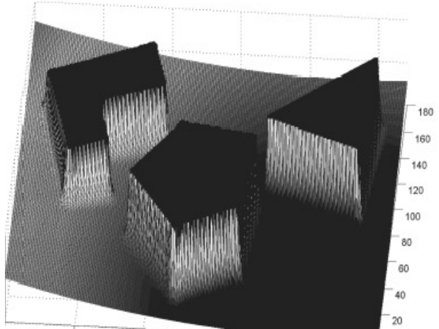
- Solves truly optimal solution
- Becomes intractable for even moderately complex as well as nonconvex problems



Source: http://mitocw.udsm.ac.tz

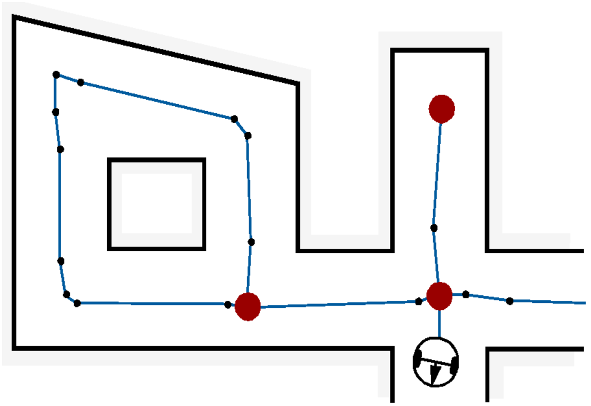
2. Potential Field

- Imposes a mathematical function over the state/configuration space
- Many physical metaphors exist
- Often employed due to its simplicity and similarity to optimal control solutions

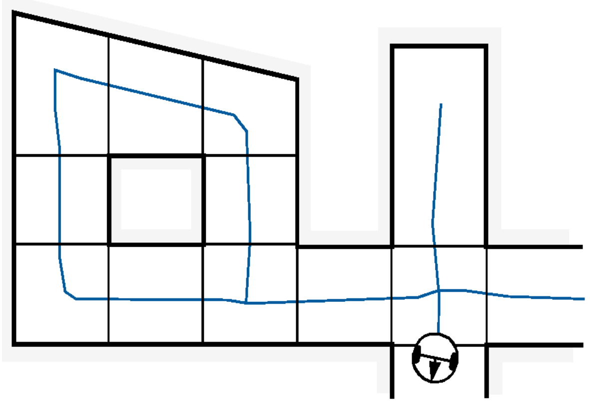


3. Graph Search

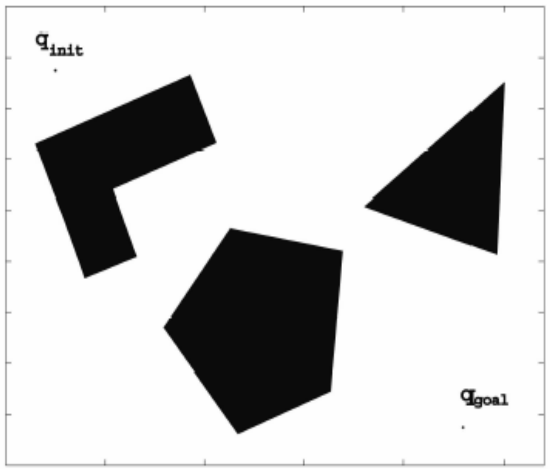
- Identify a set edges between nodes within the free space



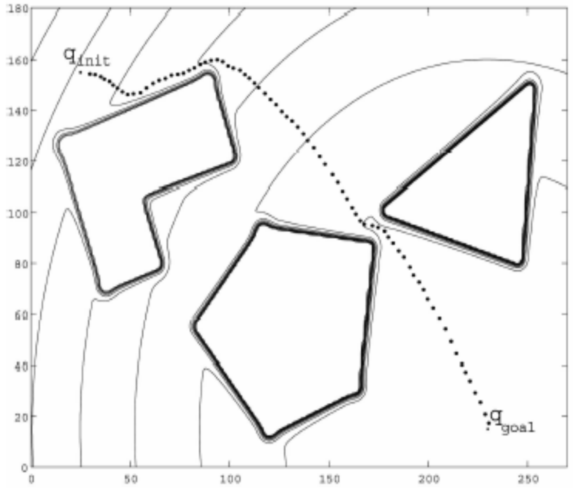
- Where to put the nodes?

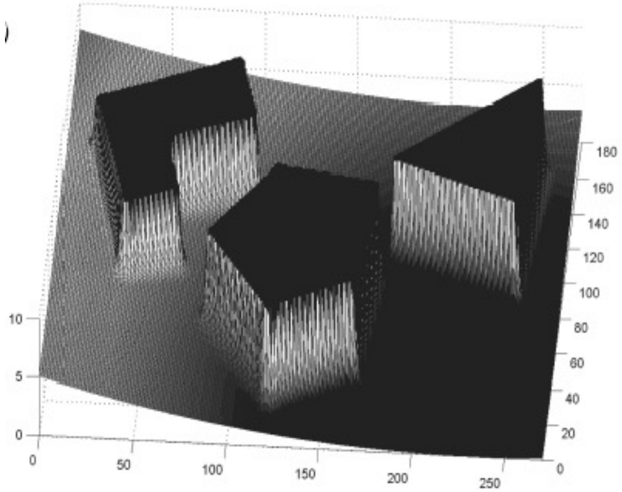


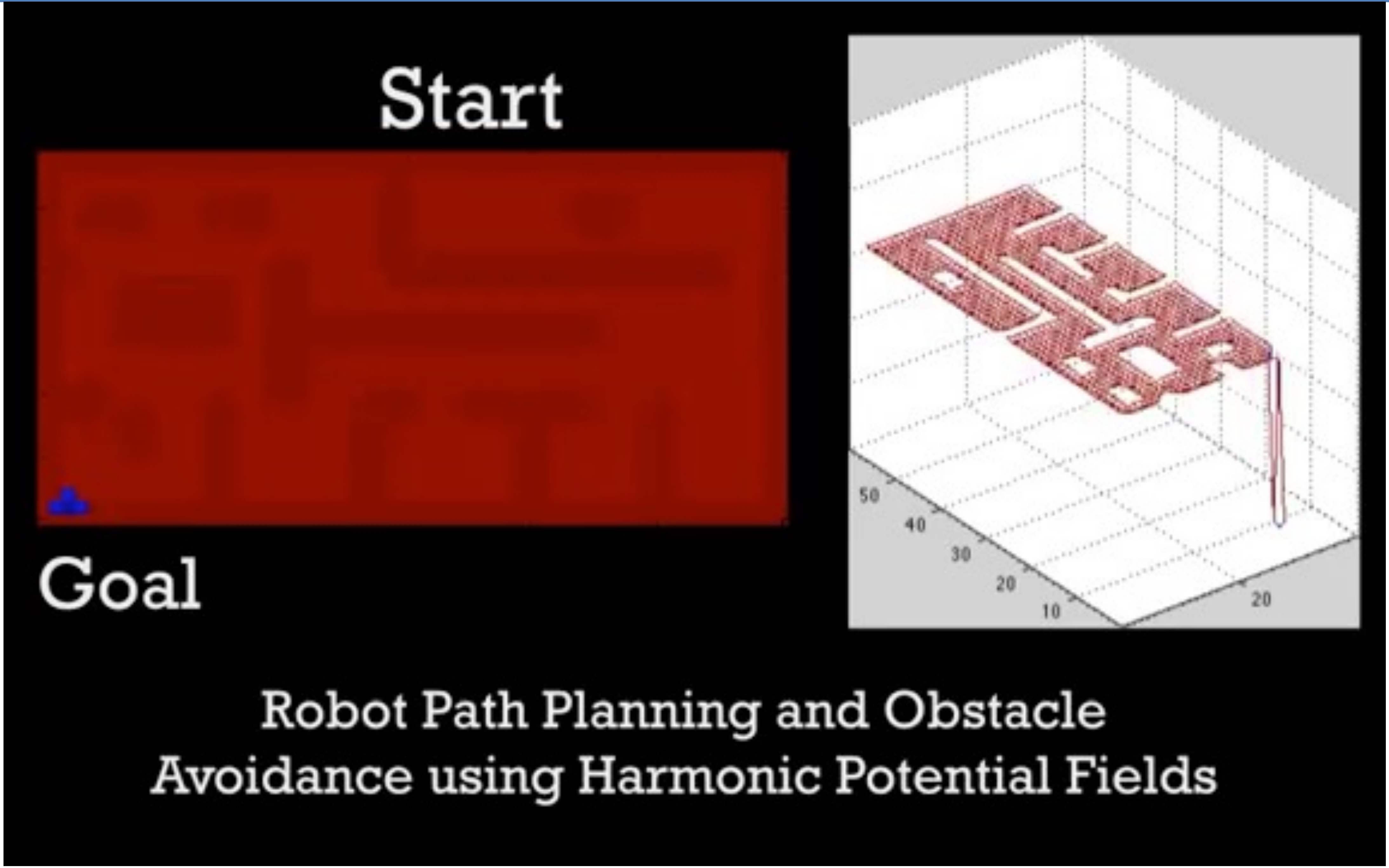
Potential Field Path Planning Strategies



- Robot is treated as a *point under the influence* of an artificial potential field.
- Operates in the continuum
  - Generated robot movement is similar to a ball rolling down the hill
  - Goal generates attractive force
  - Obstacle are repulsive forces






Start
Goal
50
40
30
20
10
20
Robot Path Planning and Obstacle
Avoidance using Harmonic Potential Fields

Potential Field Path Planning: Potential Field Generation

- Generation of potential field function *U(q)*
  - attracting (goal) and repulsing (obstacle) fields
  - summing up the fields
  - functions must be differentiable
- Generate artificial force field *F(q)*

$$F(q) = -\nabla U(q) = -\nabla U_{att}(q) - \nabla U_{rep}(q) = \begin{bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial U}{\partial y} \end{bmatrix}$$

- Set robot speed (v_x, v_y) proportional to the force *F*(*q*) generated by the field
  - the force field drives the robot to the goal
  - if robot is assumed to be a point mass
  - Method produces both a plan *and* the corresponding control

Potential Field Path Planning: Attractive Potential Field

- Parabolic function representing the Euclidean distance $\rho_{goal} = \left\| q - q_{goal} \right\|$ to the goal

$$U_{att}(q) = \frac{1}{2} k_{att} \cdot \rho_{goal}^2(q) = \frac{1}{2} k_{att} \cdot (q - q_{goal})^2$$

- Attracting force converges linearly towards 0 (goal)

$$F_{att}(q) = -\nabla U_{att}(q) = k_{att} \cdot (q - q_{goal})$$

Potential Field Path Planning: Repulsing Potential Field

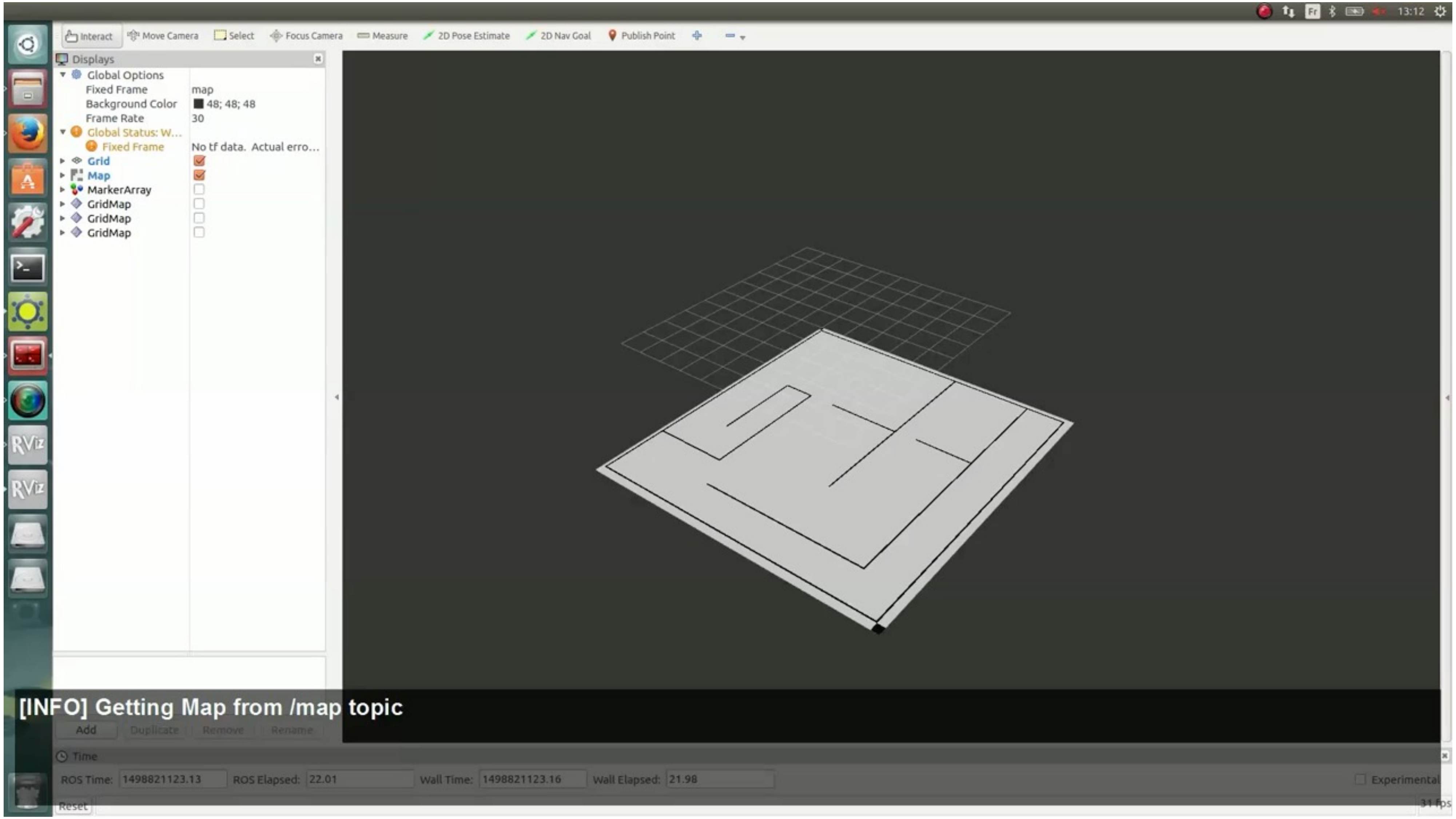
- Should generate a barrier around all the obstacle
  - strong if close to the obstacle
  - not influence if far from the obstacle

$$U_{rep}(q) = \begin{cases} \frac{1}{2}k_{rep}\left(\frac{1}{\rho(q)} - \frac{1}{\rho_0}\right)^2 & \text{if } \rho(q) \leq \rho_0 \\ 0 & \text{if } \rho(q) \geq \rho_0 \end{cases}$$

  - $\rho(q)$: minimum distance to the object
  - Field is positive or zero and *tends to infinity* as q gets closer to the object

ROS Grid Map Package

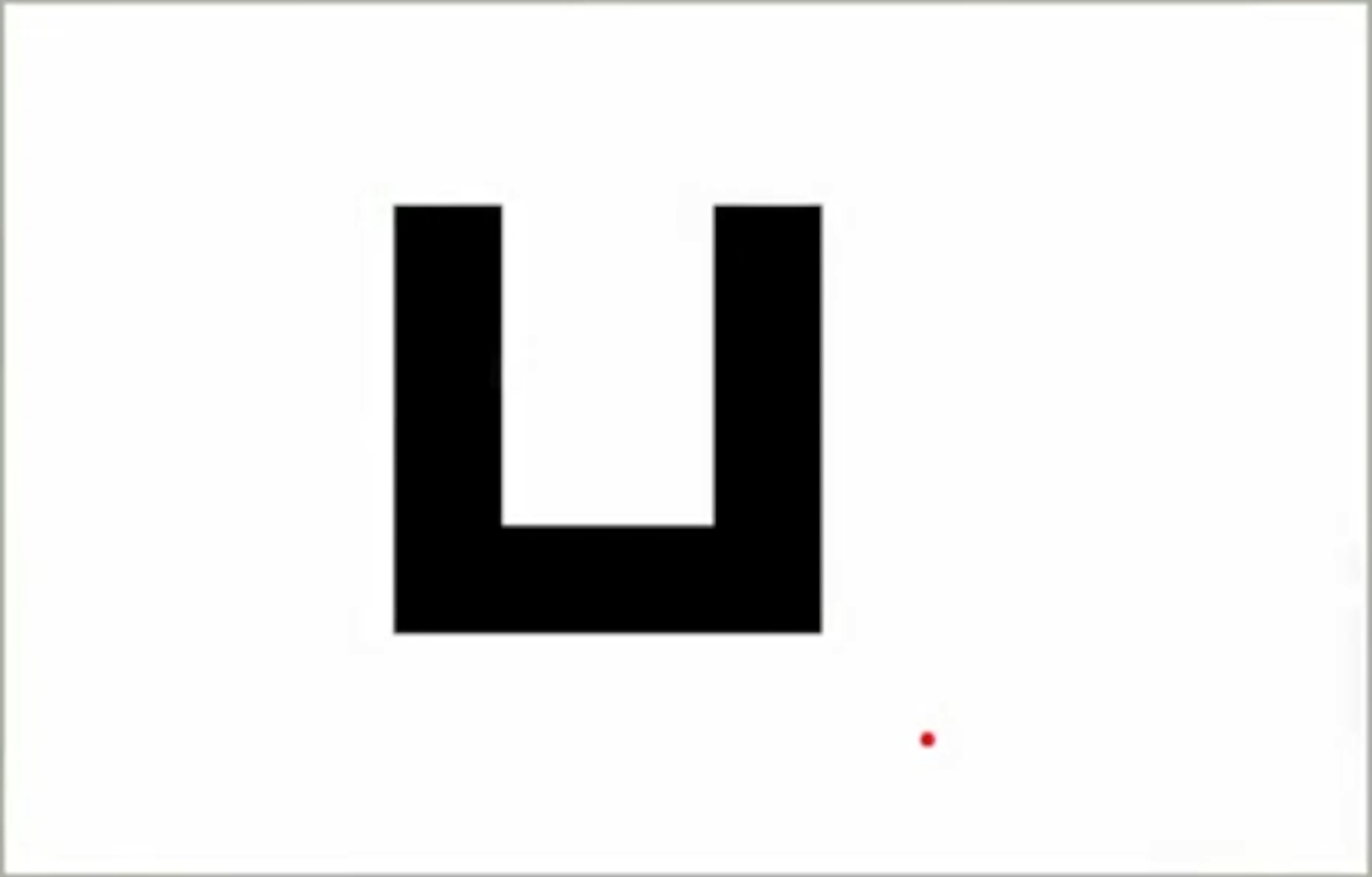
http://wiki.ros.org/grid_map



Potential Field Path Planning:

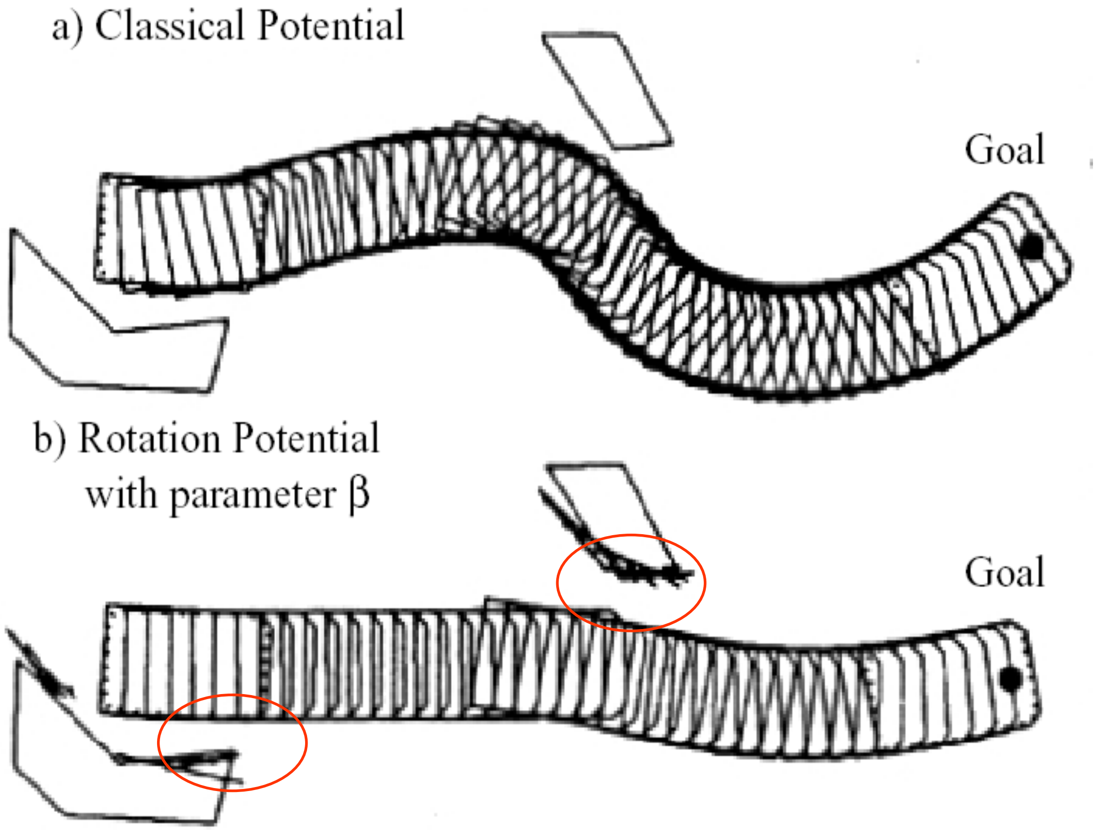
- Notes:
  - Local minima problem exists
  - Problem is getting more complex if the robot is not considered as a point mass
  - If objects are non-convex there exists situations where several minimal distances exist $\rightarrow$ can result in oscillations

Example Configuration Space



Potential Field Path Planning: Extended Potential Field Method

- Additionally a *rotation potential field* and a *task potential field* is introduced
- Rotation potential field
  - force is also a function of robots orientation relative to the obstacles. This is done using a gain factor that reduces the repulsive force when obstacles are parallel to robot's direction of travel
- Task potential field
  - Filters out the obstacles that should not influence the robots movements, i.e. only the obstacles in the sector in front of the robot are considered



Khatib and Chatila

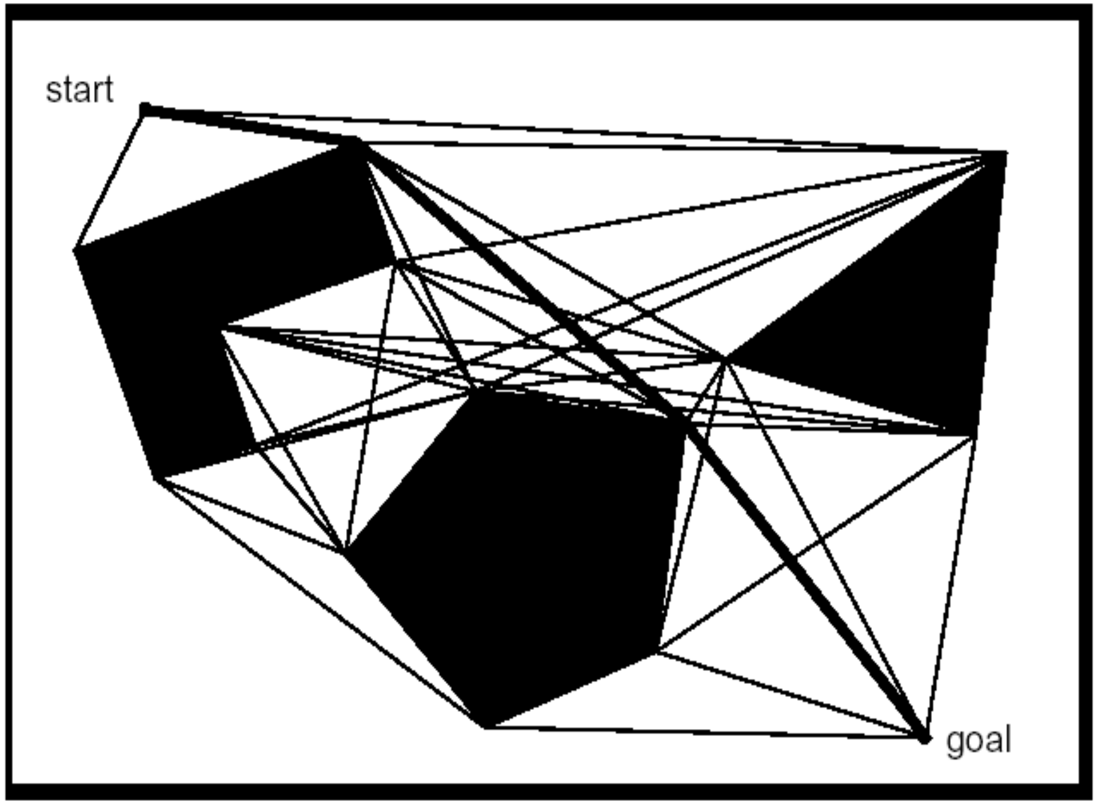
Graph Search

- Overview
  - Solves a least cost problem between two states on a (directed) graph
  - Graph structure is a discrete representation

- Limitations
  - State space is discretized → completeness is at stake
  - Feasibility of paths is often not inherently encoded

- Algorithms
  - (Preprocessing steps)
  - Breath first
  - Depth first
  - Dijkstra
  - A* and variants
  - D* and variants

Graph Construction: Visibility Graph



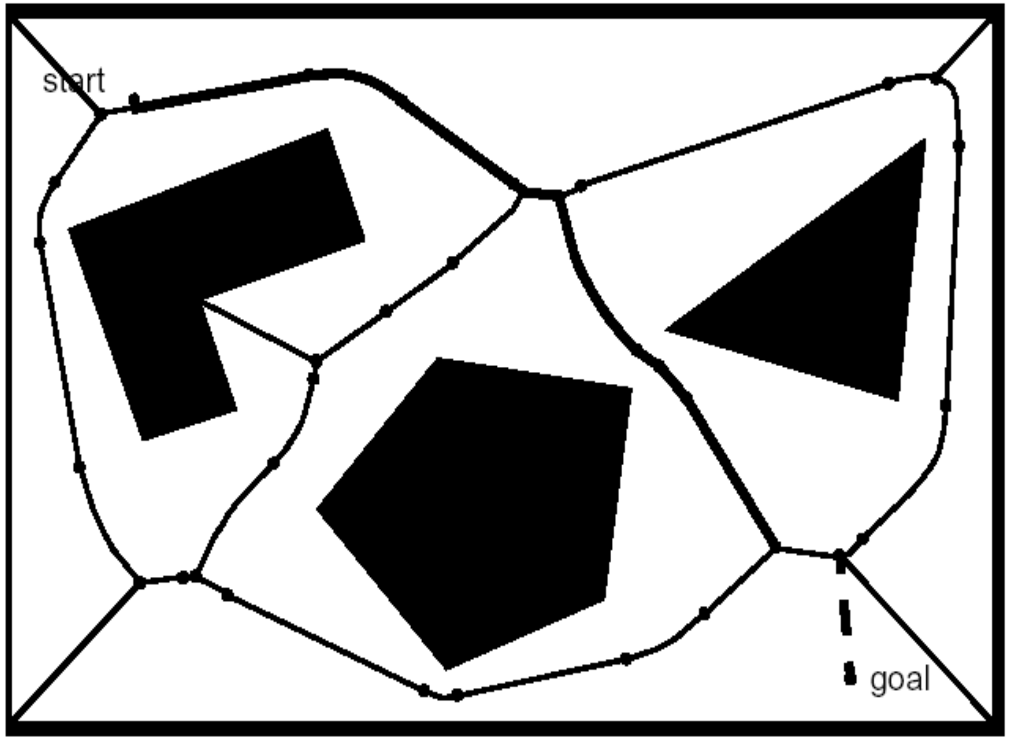
- Particularly suitable for polygon-like obstacles
- Shortest path length
- Grow obstacles to avoid collisions

Graph Construction: Visibility Graph

- Pros
  - The found path is optimal because it is the shortest length path
  - Implementation simple when obstacles are polygons

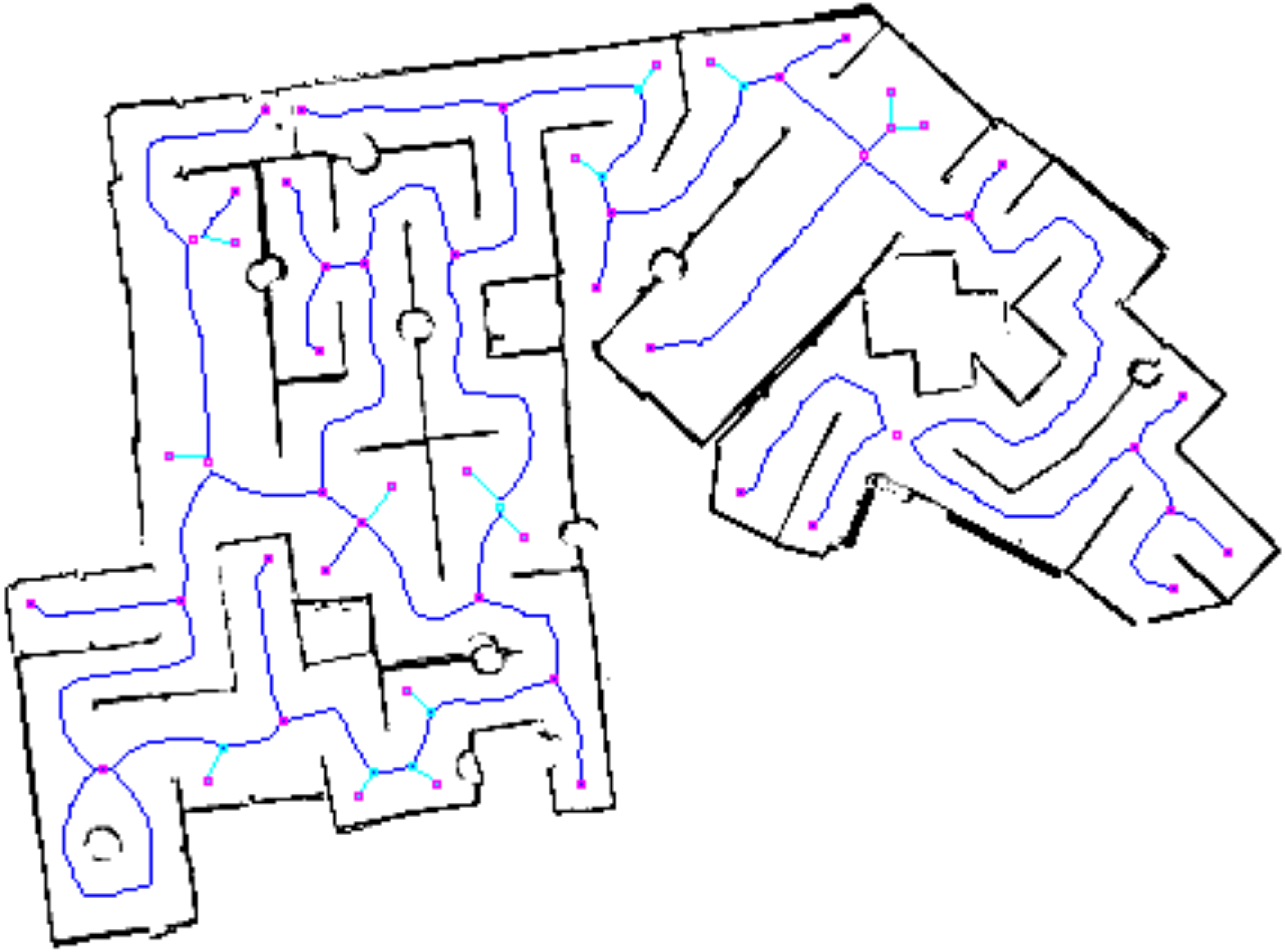
- Cons
  - The solution path found by the visibility graph tend to take the robot as close as possible to the obstacles: the common solution is to grow obstacles by more than robot’s radius
  - Number of edges and nodes increases with the number of polygons
  - Thus it can be inefficient in densely populated environments

Graph Construction: Voronoi Diagram



- Tends to maximize the distance between robot and obstacles

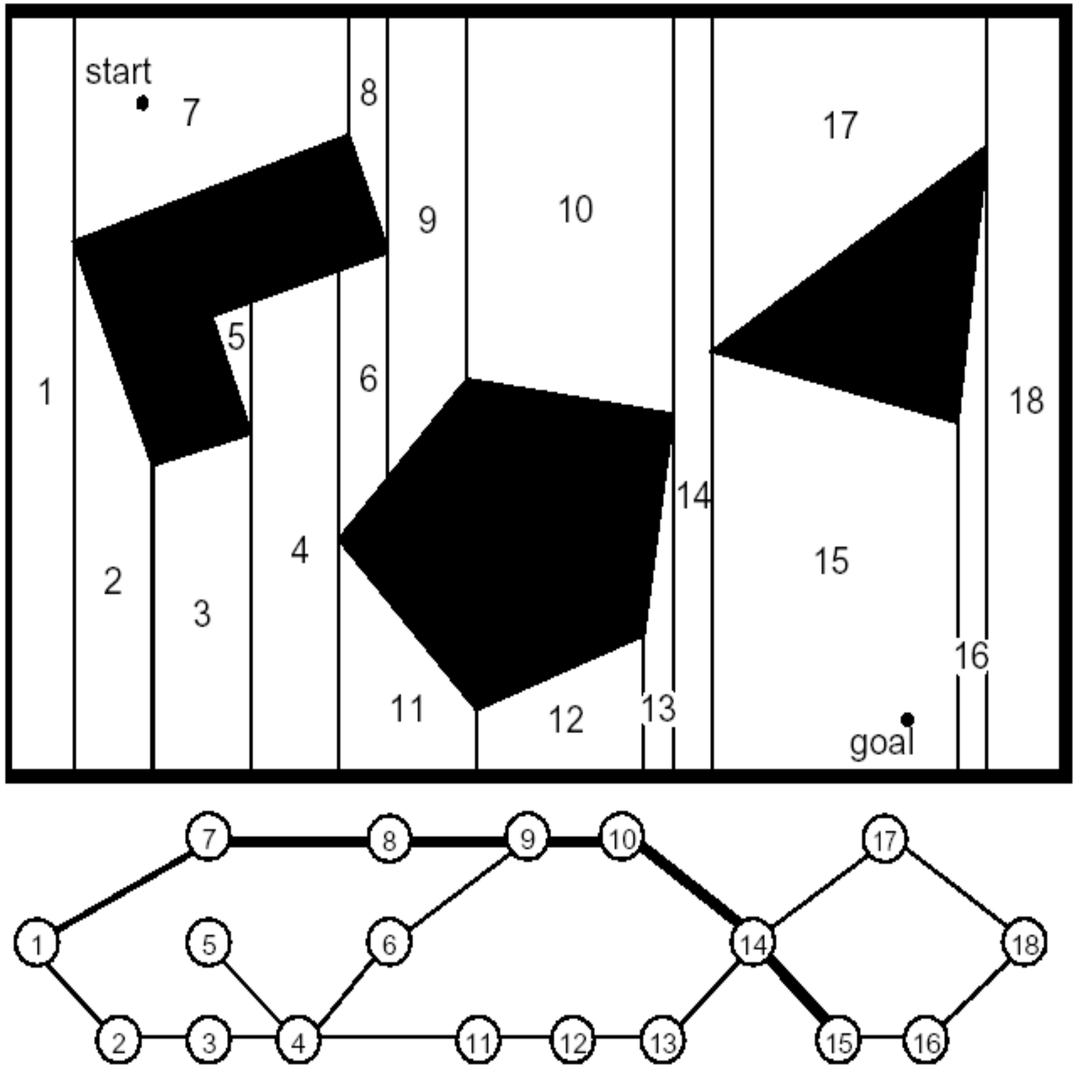
Topology Graph



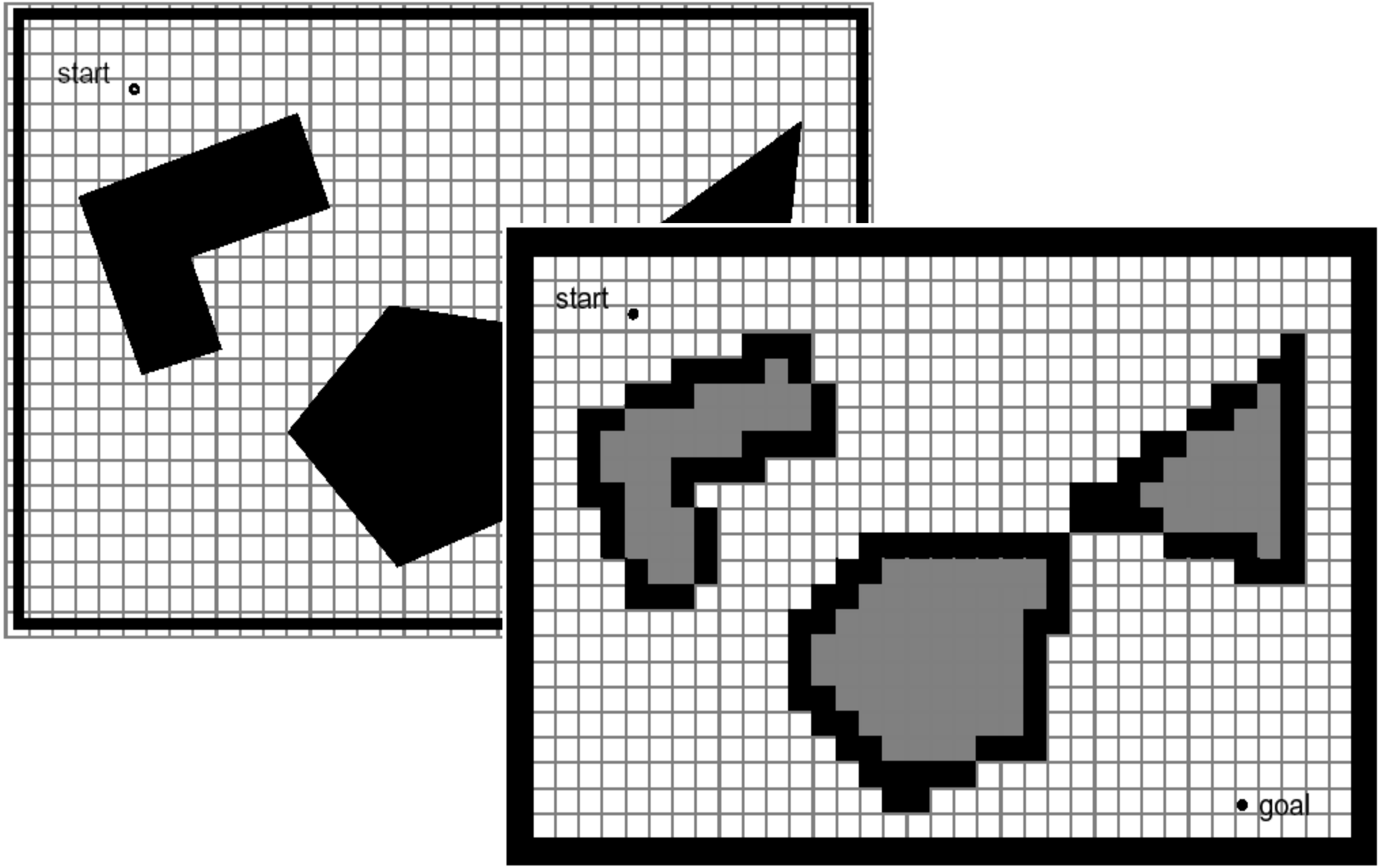
Graph Construction: Voronoi Diagram

- Pros
  - Using range sensors like laser or sonar, a robot can navigate along the Voronoi diagram using simple control rules
- Cons
  - Because the Voronoi diagram tends to keep the robot as far as possible from obstacles, any short range sensor will be in danger of failing
  - Voronoi diagram can change drastically in open areas

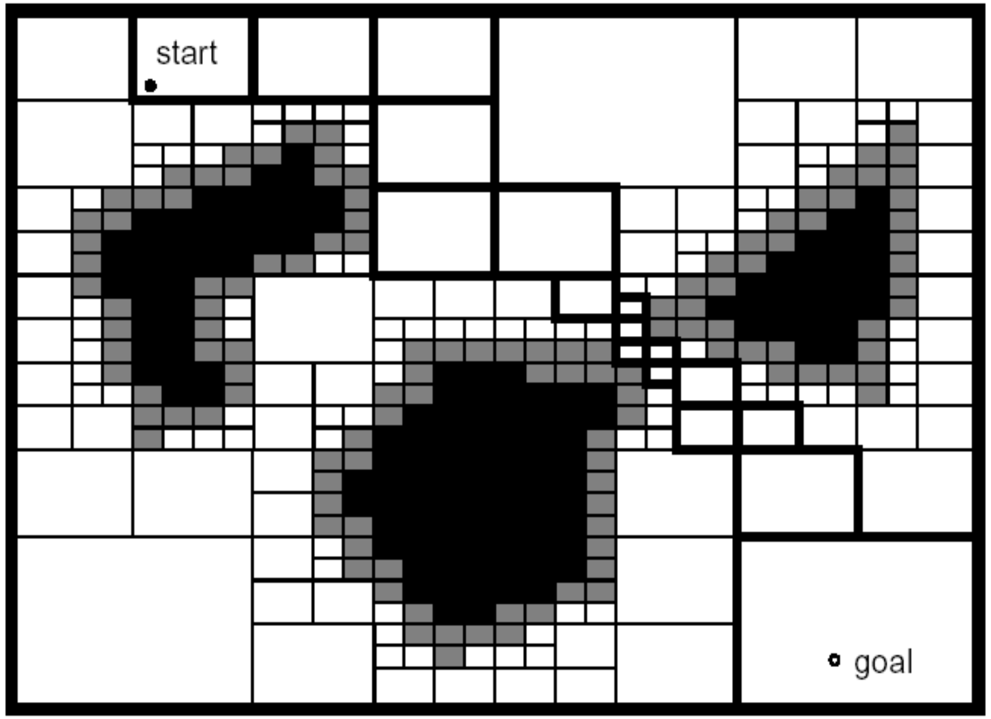
Graph Construction: Exact Cell Decomposition (2/4)



Graph Construction: Approximate Cell Decomposition (3/4)



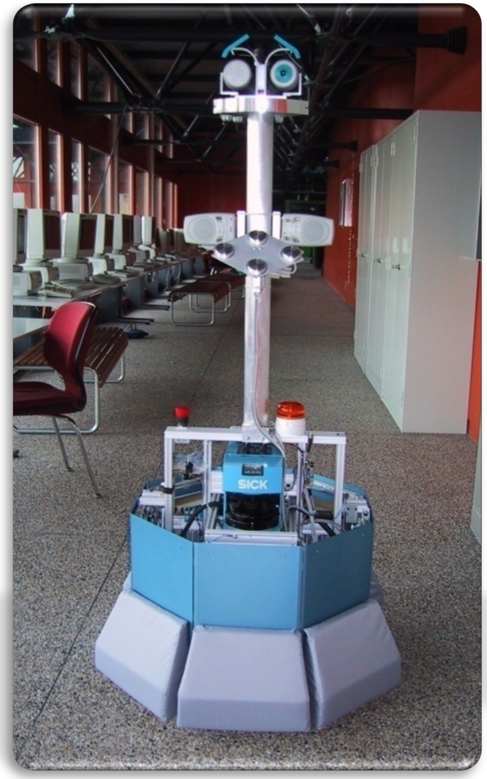
Graph Construction: Adaptive Cell Decomposition (4/4)



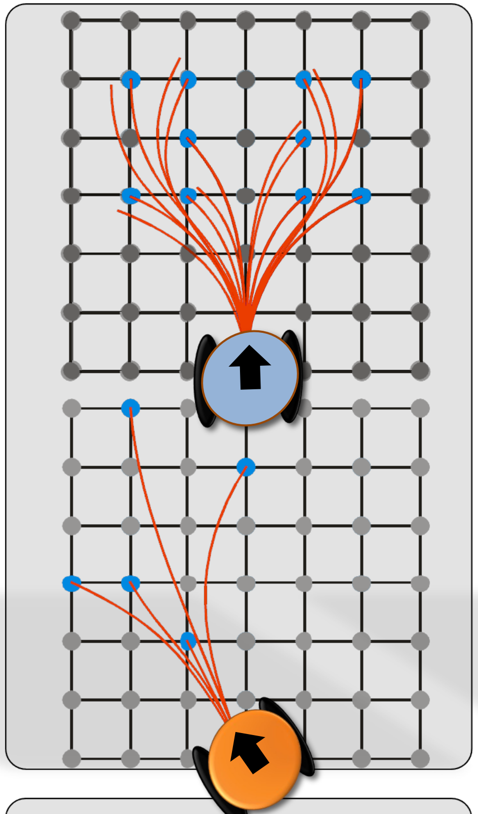
- Close relationship with map representation (Quadtree)!

Graph Construction: State Lattice Design (1/2)

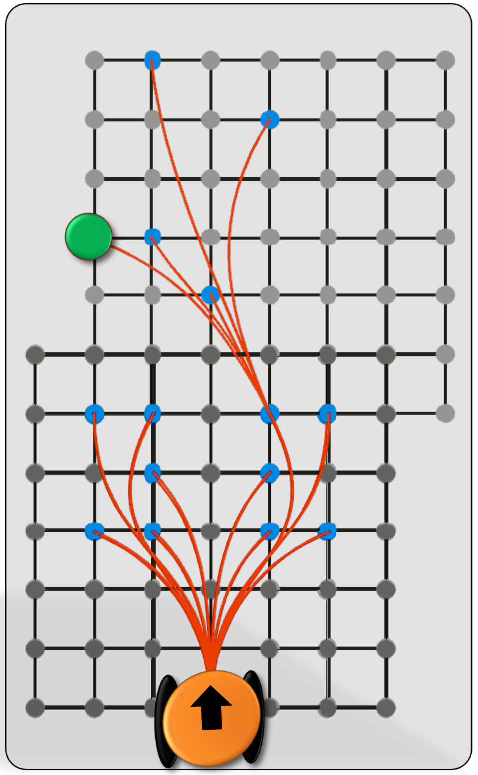
- Enforces edge feasibility



Offline:
Motion Model



Offline:
Lattice Gen.

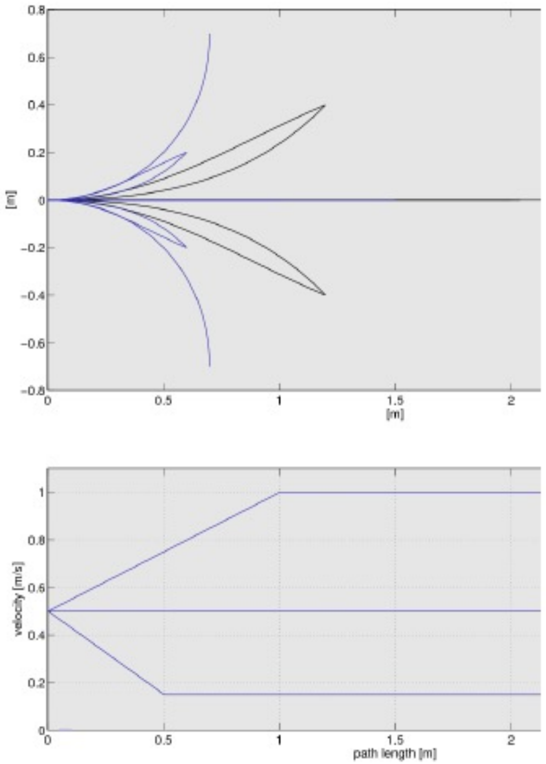


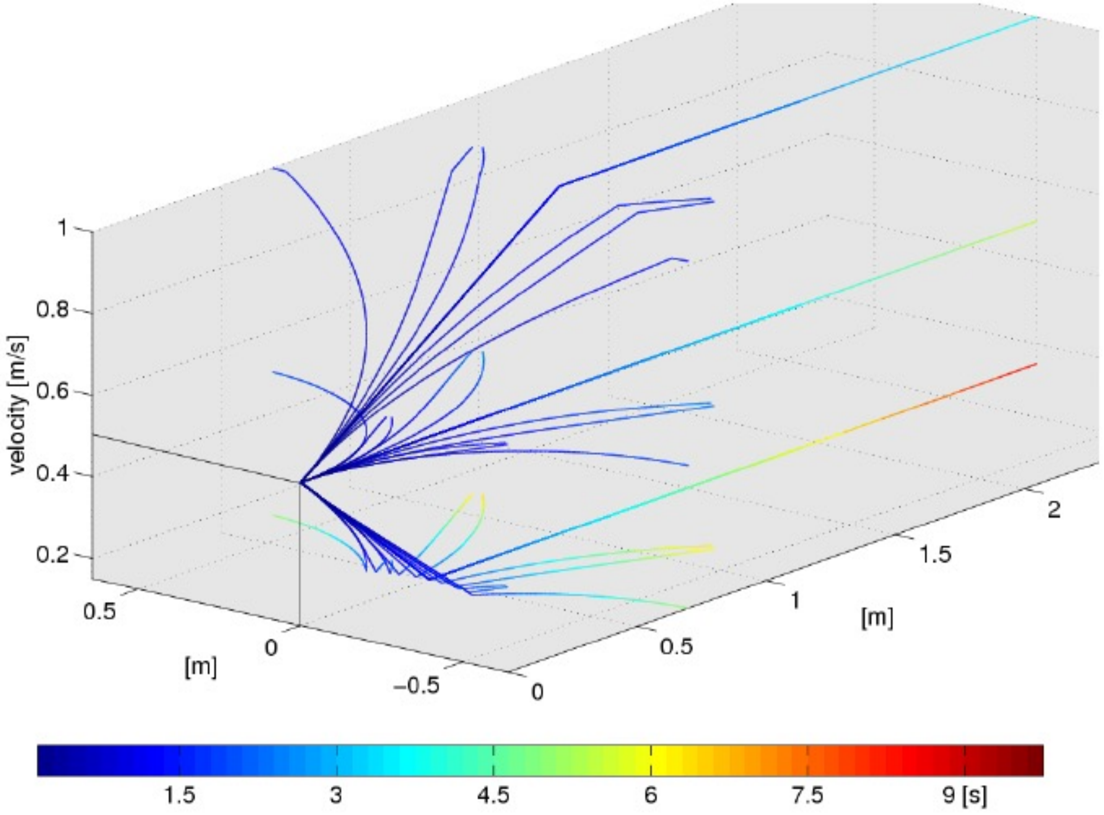
Online:
Incremental Graph
Constr.

Graph Construction: State Lattice Design (2/2)

Martin Rufli

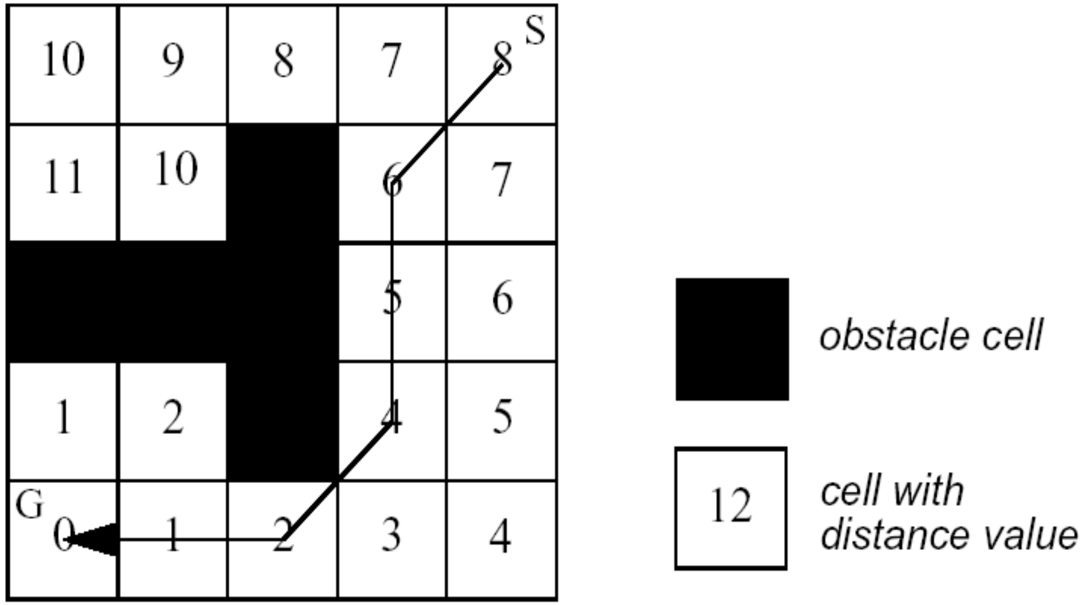
- State lattice encodes only kinematically feasible edges





Deterministic Graph Search

- Methods
  - Breath First
  - Depth First
  - **Dijkstra**
  - A* and variants
  - D* and variants
  - ...



DIJKSTRA‘S ALGORITHM

EDSGER WYBE DIJKSTRA

1930 - 2002

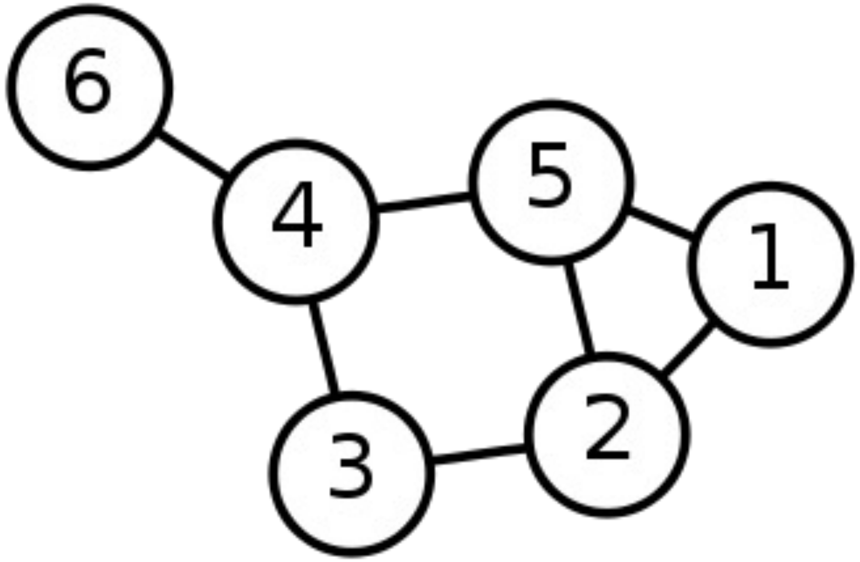
"Computer Science is no more about computers than astronomy is about telescopes."

http://www.cs.utexas.edu/~EWD/

SINGLE-SOURCE SHORTEST PATH PROBLEM

- **Single-Source Shortest Path Problem** - The problem of finding shortest paths from a source vertex *v* to all other vertices in the graph.

- **Graph**
- Set of vertices and edges
- Vertex:
  - Place in the graph; connected by:
- Edge: connecting two vertices
  - Directed or undirected (undirected in Dijkstra's Algorithm)
  - Edges can have weight/ distance assigned



Dijkstra material from http://www.cs.utexas.edu/~tandy/barrera.ppt

Diklstra's Algorithm

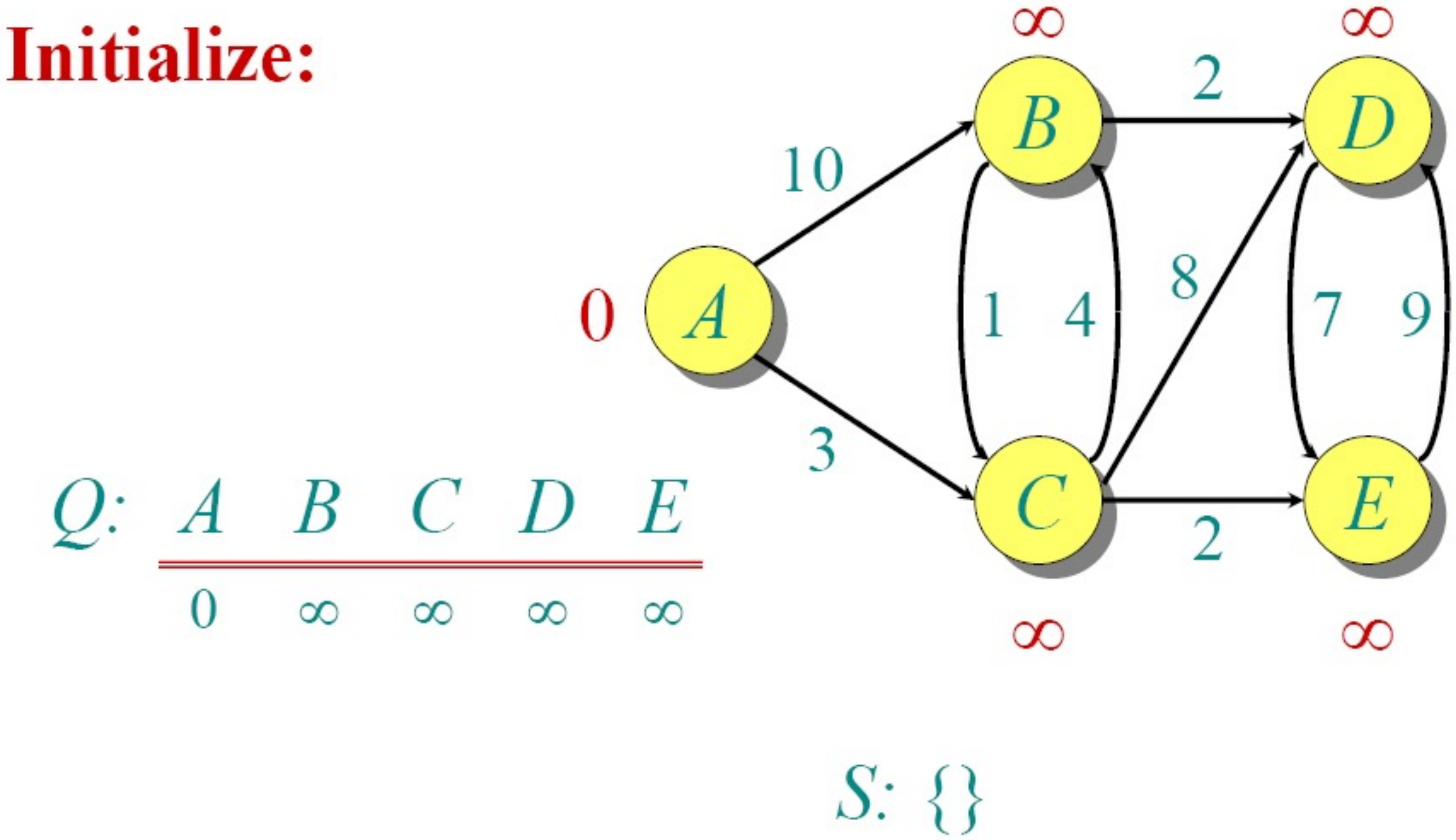
- Assign all vertices infinite distance to goal
- Assign 0 to distance from start
- Add all vertices to the queue

- While the queue is not empty:
  - Select vertex with smallest distance and remove it from the queue
  - Visit all neighbor vertices of that vertex,
  - calculate their distance and
  - update their (the neighbors) distance if the new distance is smaller

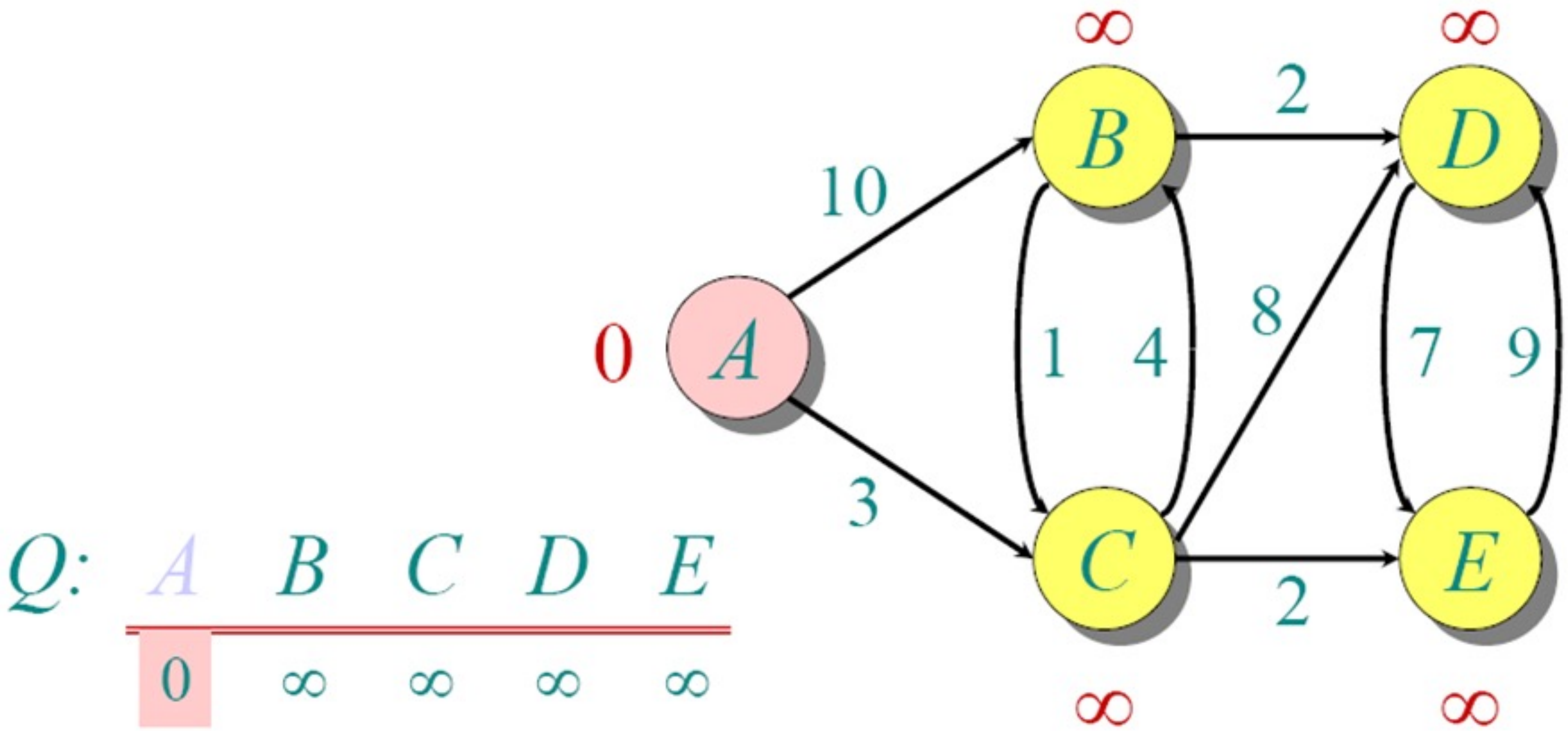
Diklstra's Algorithm - Pseudocode

```
dist[s] ← 0                                  (distance to source vertex is zero)
for  all v ∈ V–{s}
        do  dist[v] ← ∞                      (set all other distances to infinity)
S ← Ø                                        (S, the set of visited vertices is initially empty)
Q← V                                         (Q, the queue initially contains all vertices)
while Q ≠Ø                                   (while the queue is not empty)
do   u ← mindistance(Q, dist)                (select the element of Q with the min. distance)
      S←S∪{u}                                (add u to list of visited vertices)
      for all v ∈ neighbors[u]
            do  if   dist[v] > dist[u] + w(u, v)         (if new shortest path found)
                        then     d[v] ←d[u] + w(u, v)    (set new value of shortest path)
         (if desired, add traceback code)
return dist
```

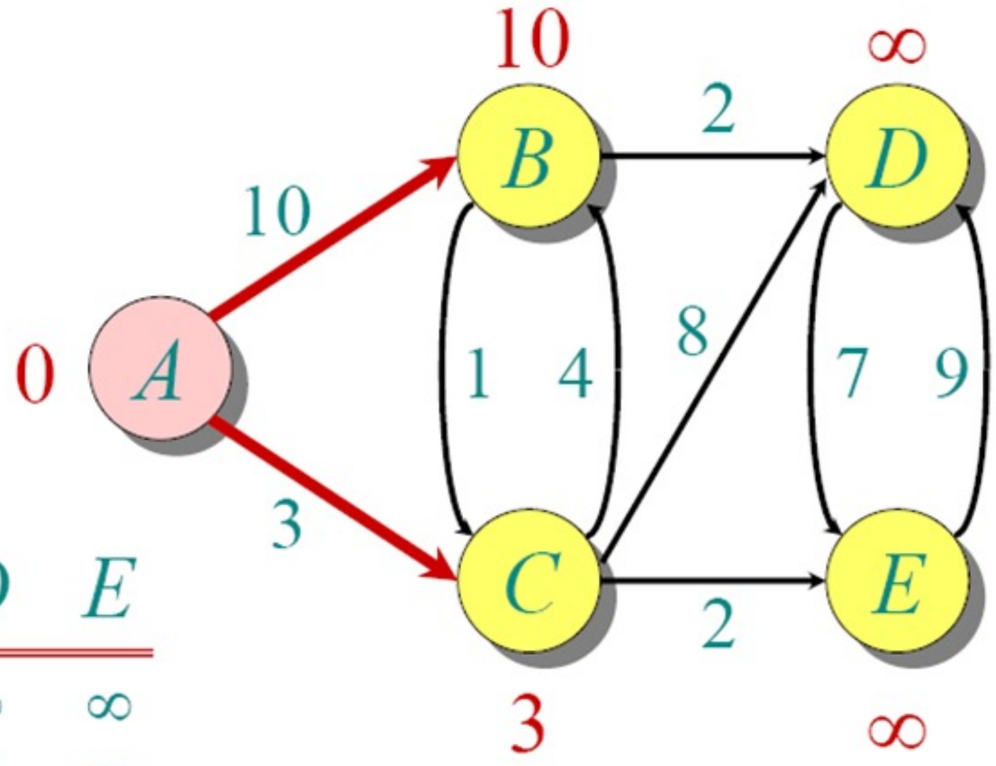
Dijkstra Example



Dijkstra Example



Dijkstra Example

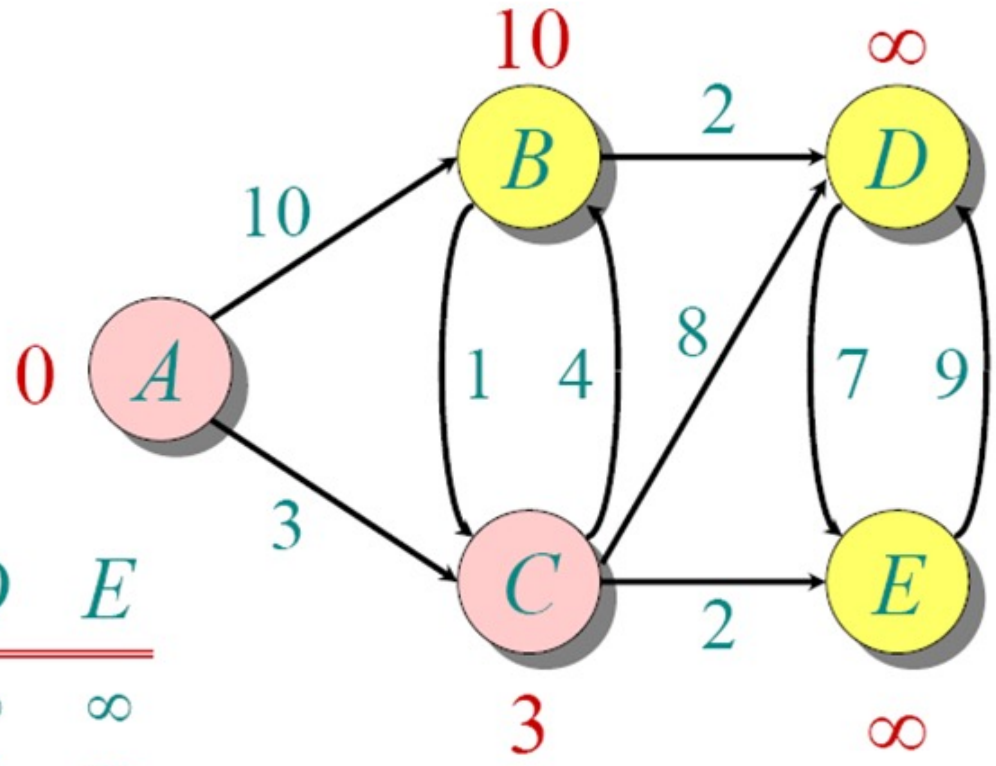


Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞

S: { A }

Dijkstra Example

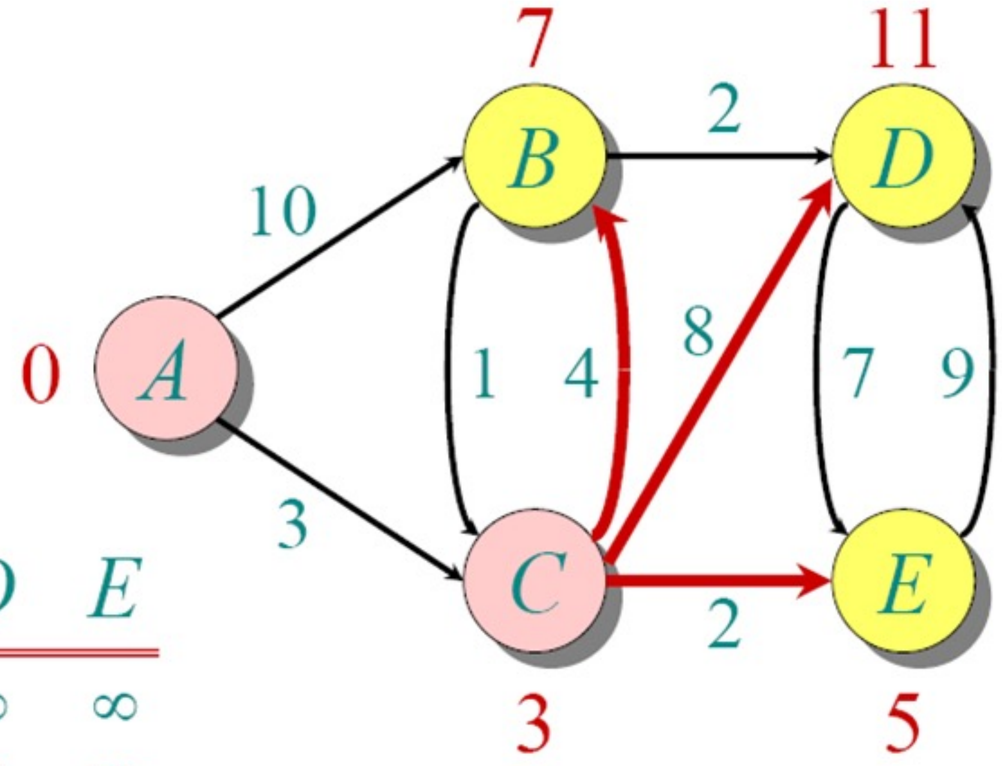


Q:

A	B	C	D	E
0	∞	∞	∞	∞
	10	3	∞	∞

S: { A, C }

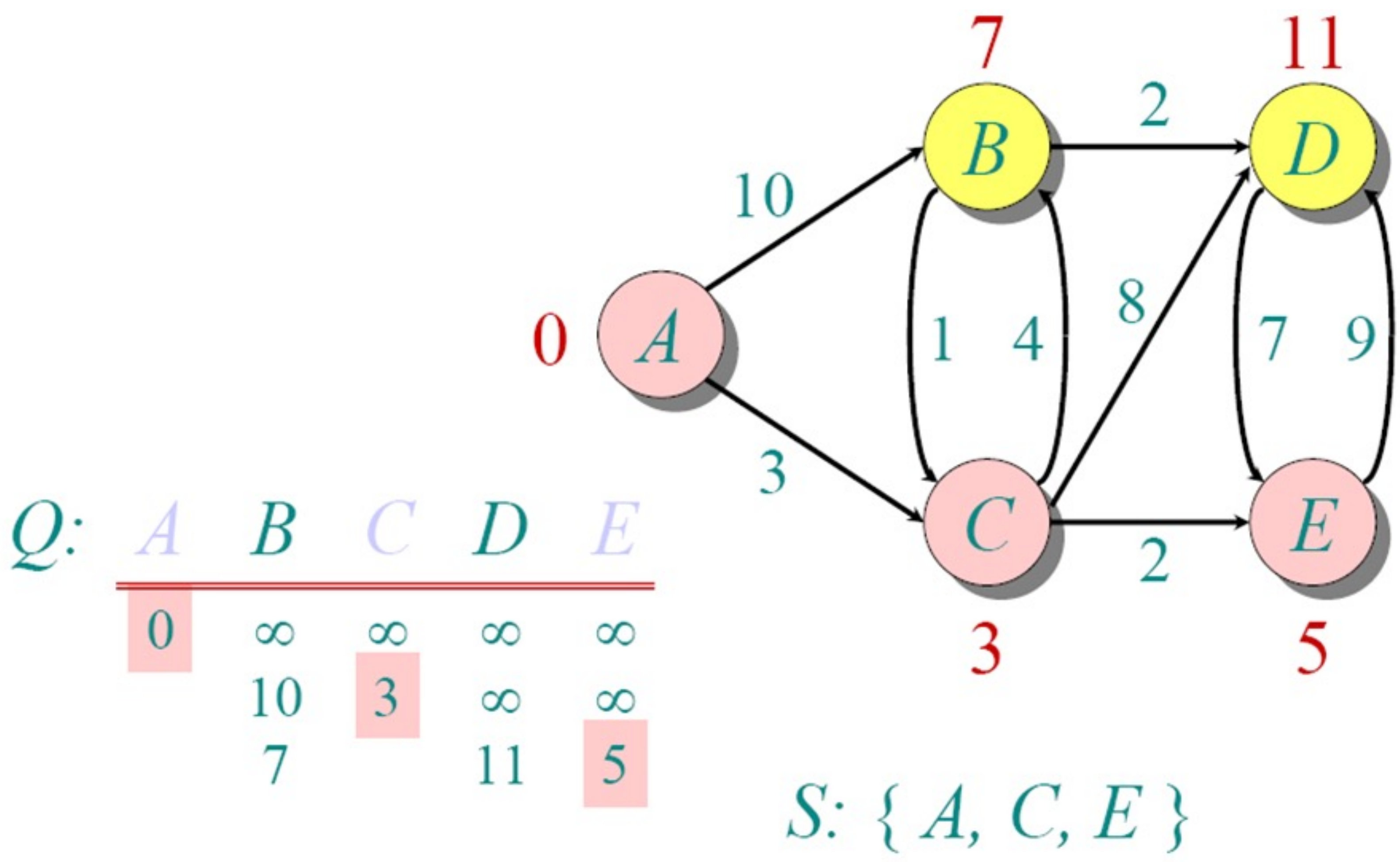
Dijkstra Example



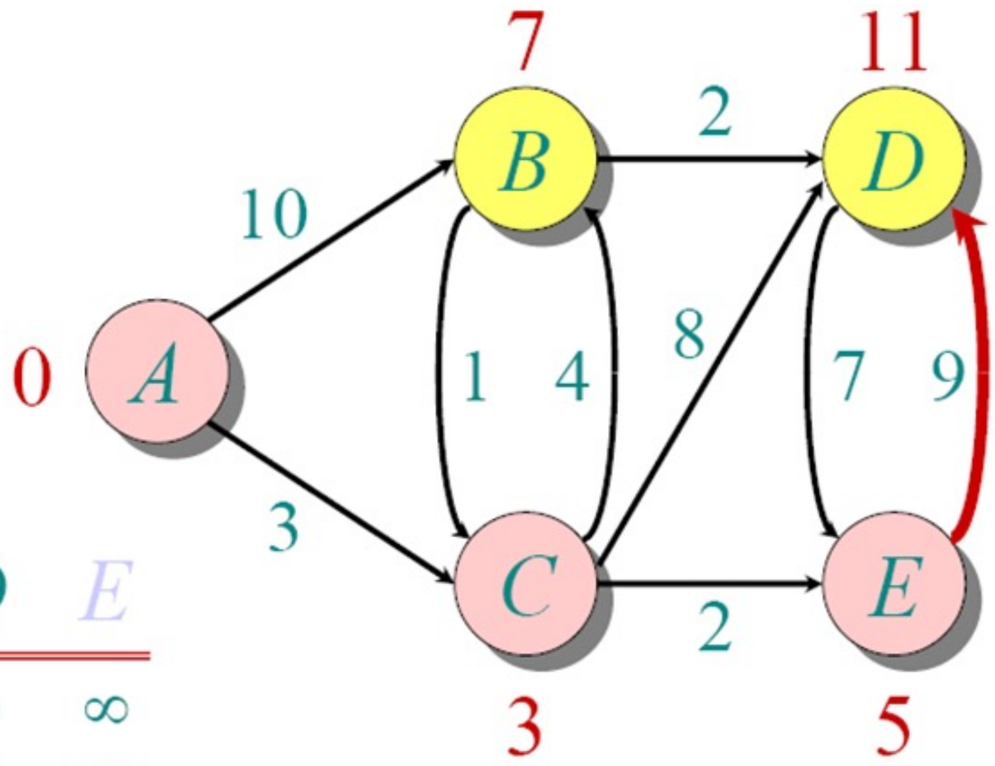
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C }

Dijkstra Example



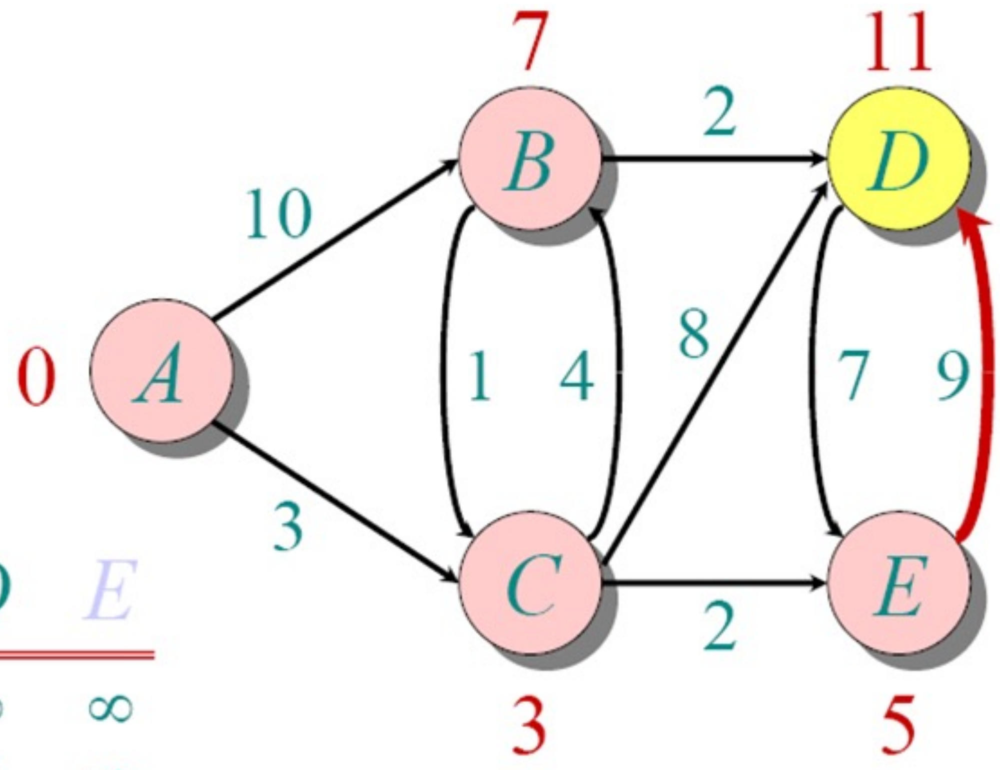
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E }

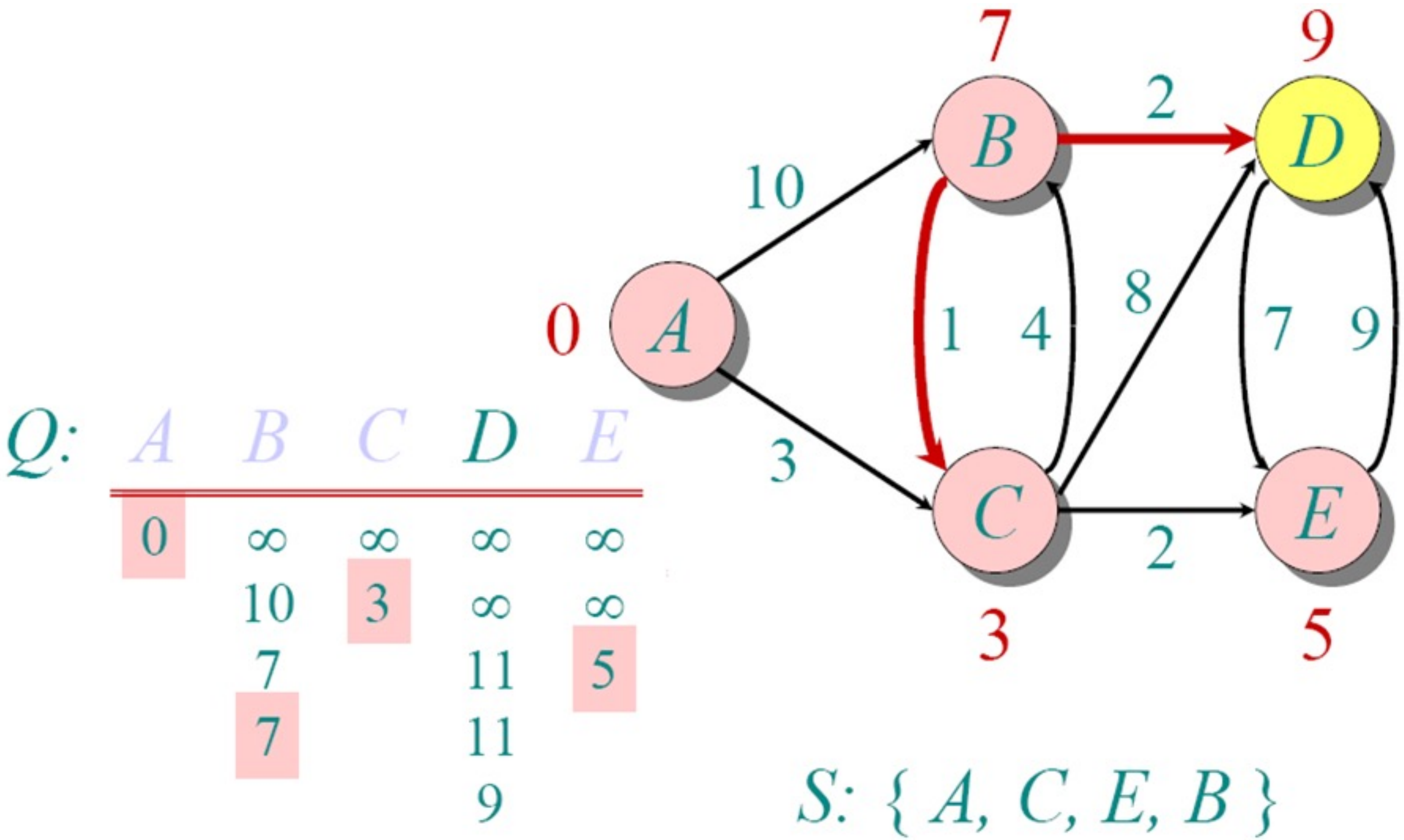
Dijkstra Example



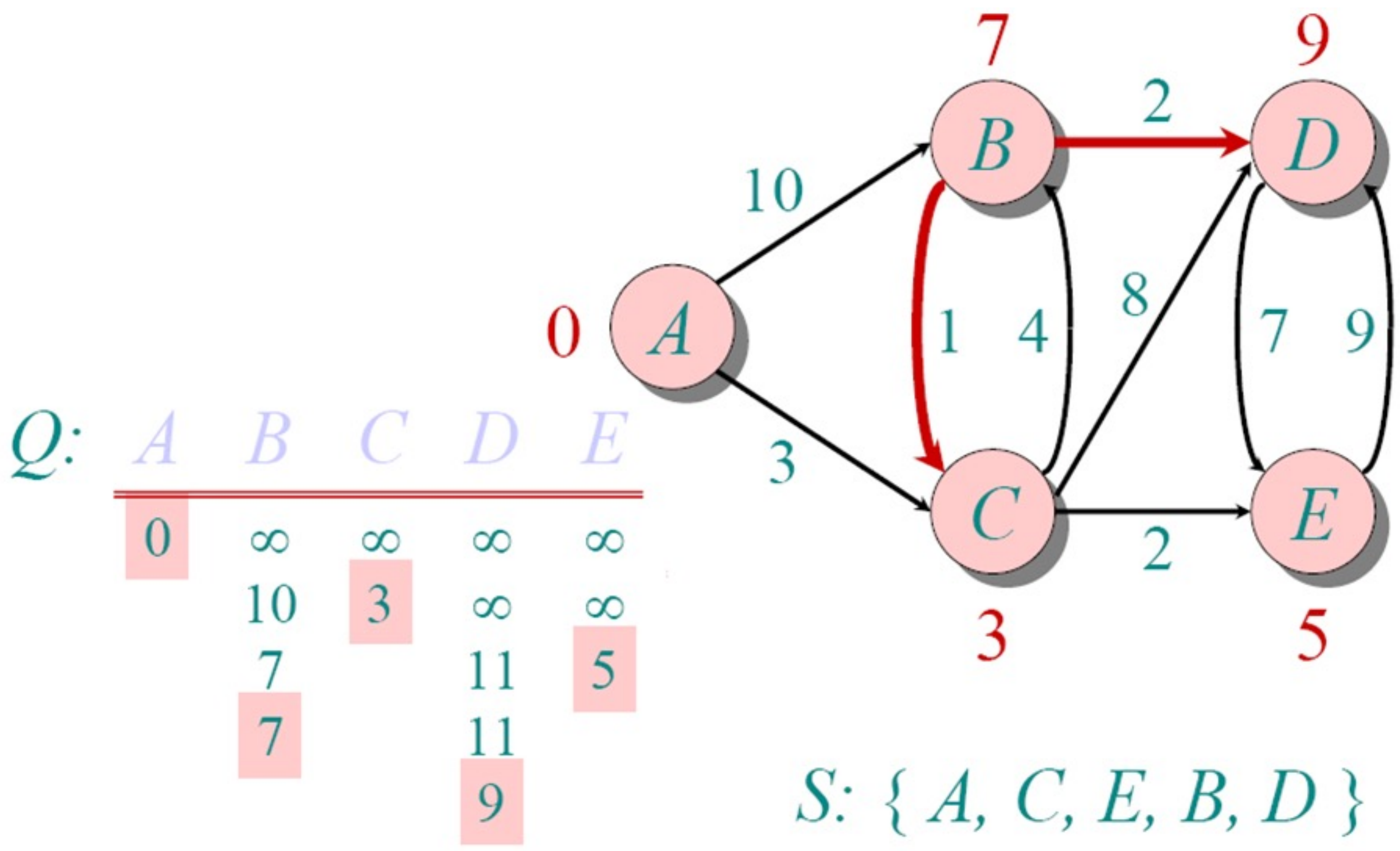
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E, B }

Dijkstra Example

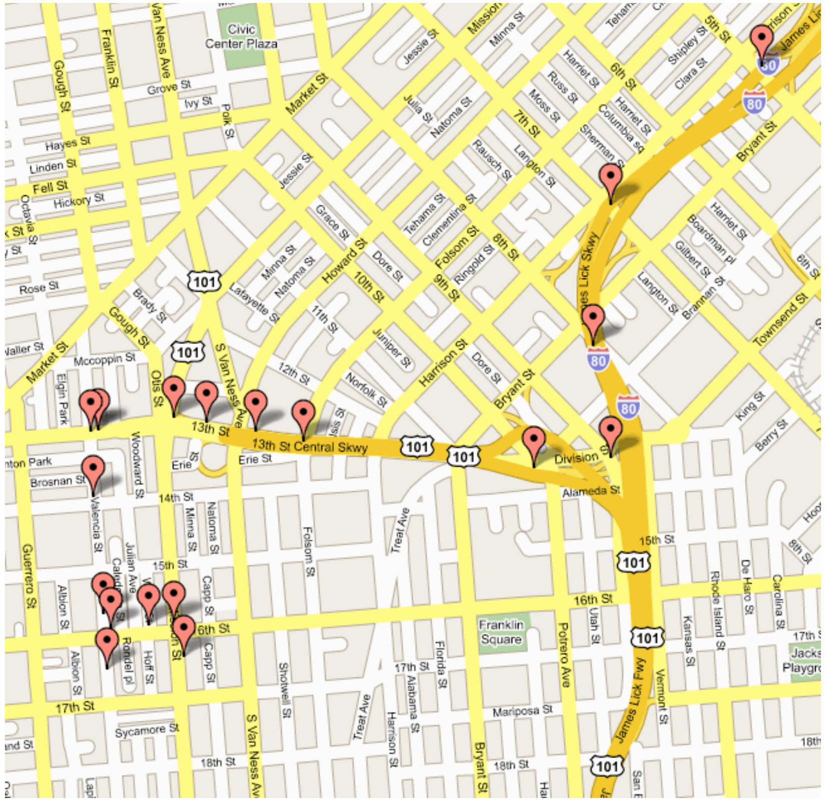


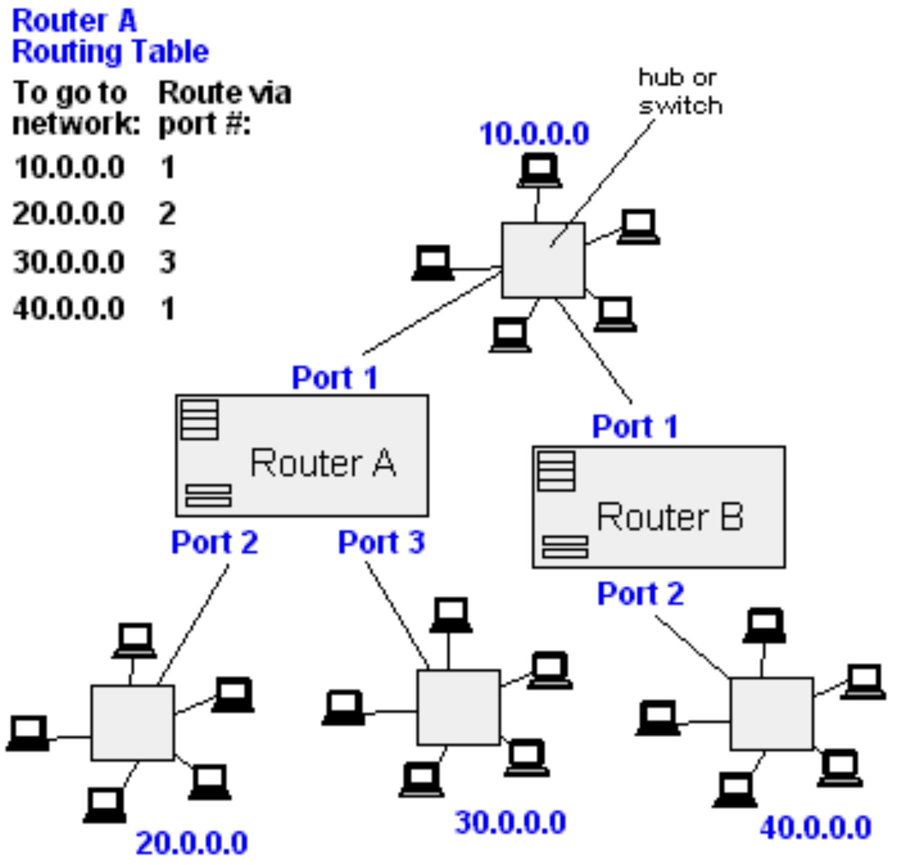
Dijkstra Example



APPLICATIONS OF DIJKSTRA'S ALGORITHM

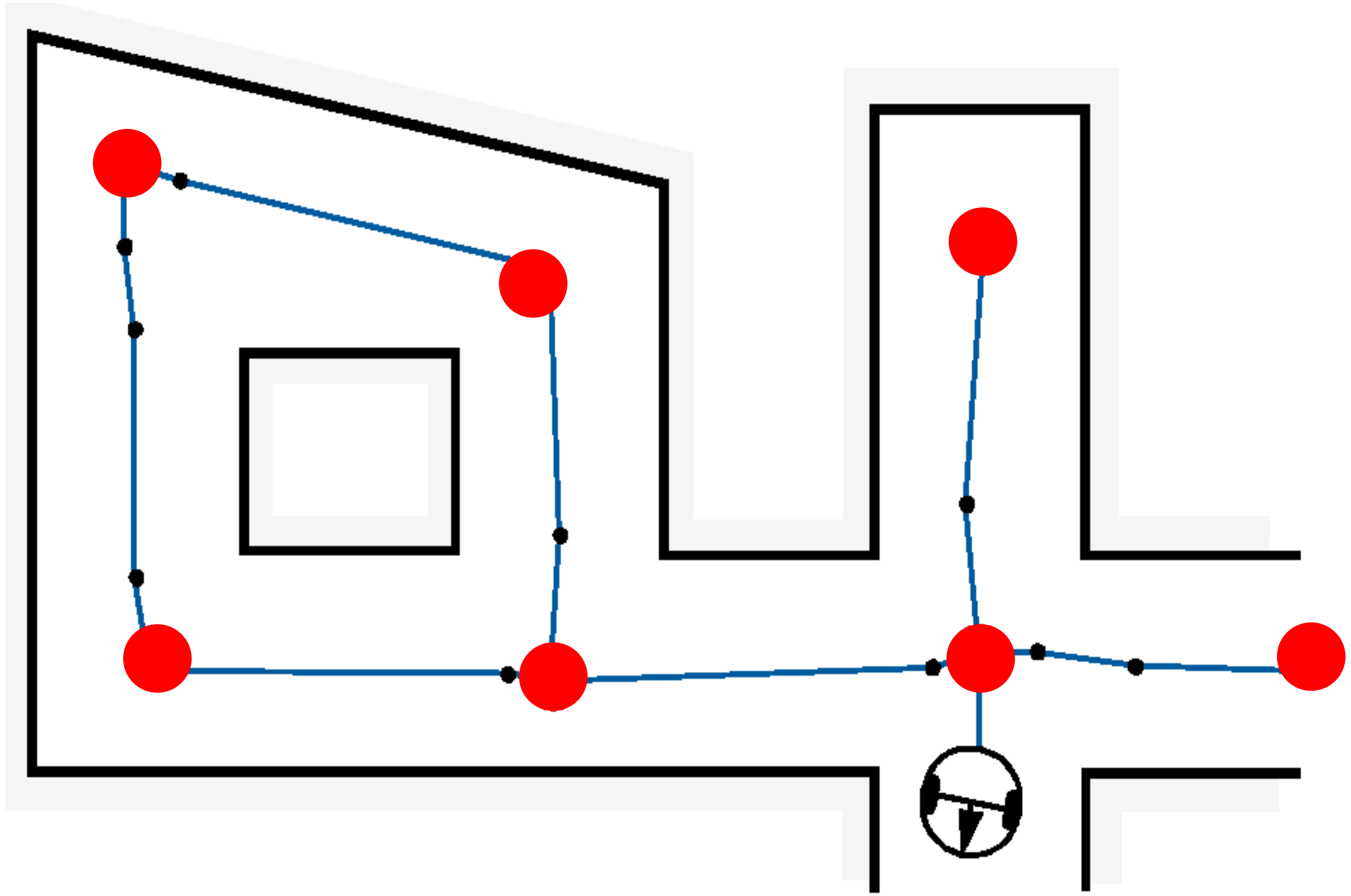
- Navigation Systems
- Internet Routing





Dijkstra's Algorithm for Path Planning: Topological Maps

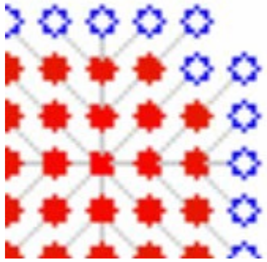
- Topological Map:
  - Places (vertices) in the environment (red dots)
  - Paths (edges) between them (blue lines)
  - Length of path = weight of edge
- => Apply Dijkstra's Algorithm to find path from start vertex to goal vertex



Dijkstra's Algorithm for Path Planning: Grid Maps

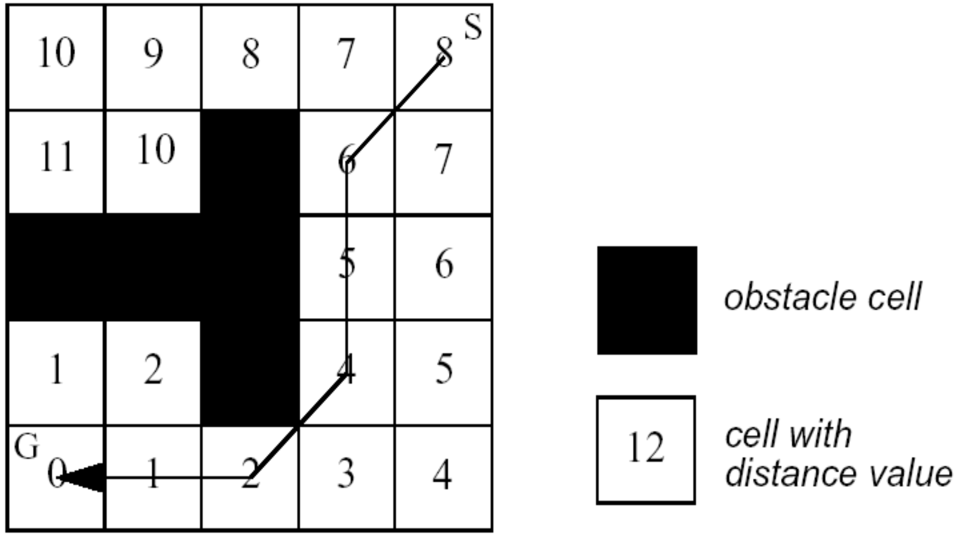
- Graph:
  - Neighboring free cells are connected:
    - 4-neighborhood: up/ down/ left right
    - **8-neighborhood**: also diagonals
  - All edges have weight 1

- Stop once goal vertex is reached
- Per vertex: save edge over which the shortest distance from start was reached => Path

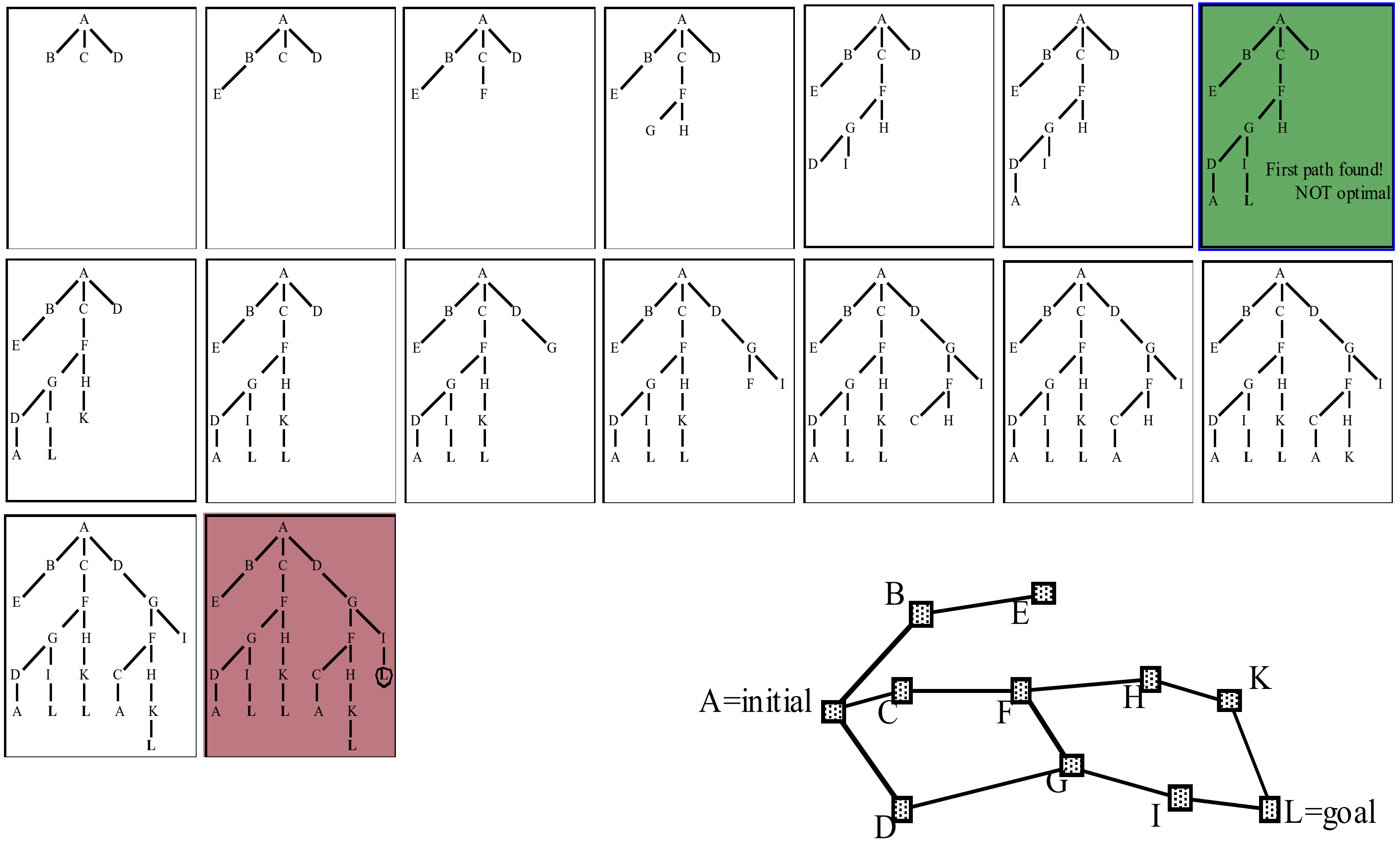


Graph Search Strategies: Breath-First Search

- Corresponds to a wavefront expansion on a 2D grid
- Breath-First: Dijkstra‘s search where all edges have weight 1

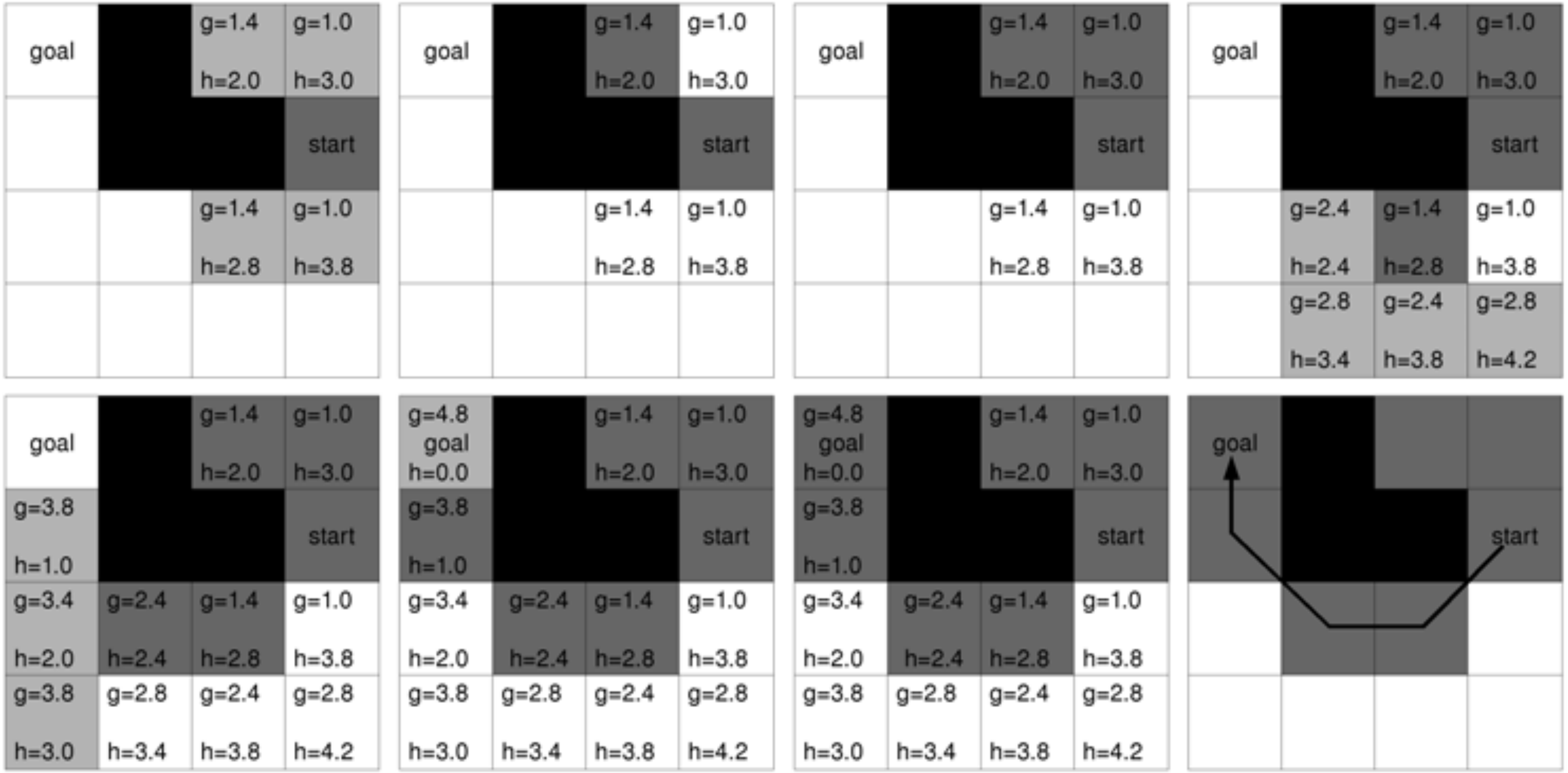


Graph Search Strategies: Depth-First Search



Graph Search Strategies: A* Search

- Similar to Dijkstra's algorithm, except that it uses a heuristic function h(n)
- f(n) = g(n) + h(n)



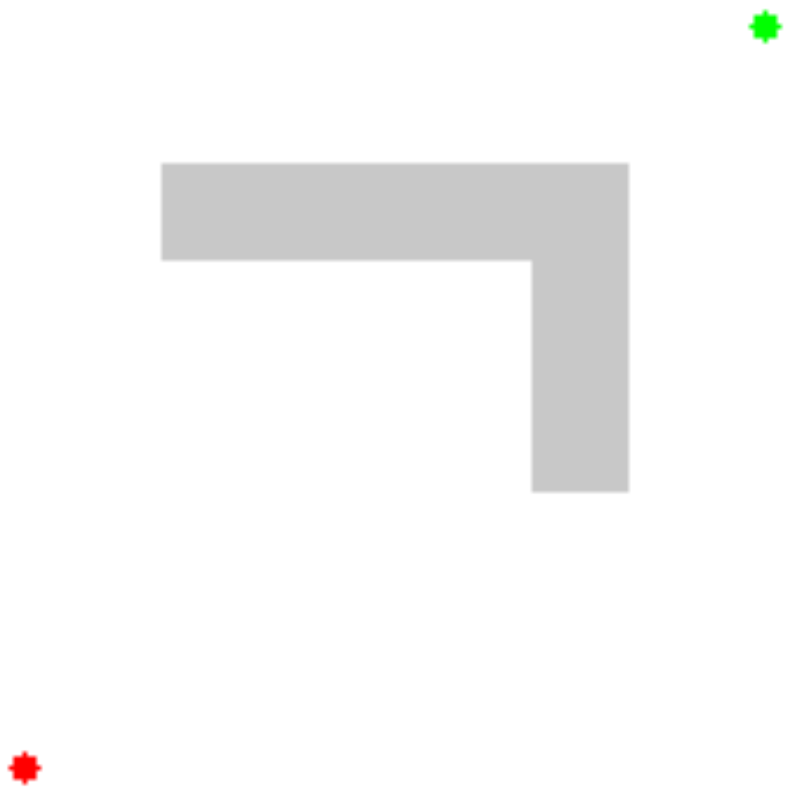
A*

- Developed 1986 as part of the Shakey project!
- Complexity:

Worst-case performance	$O(\lvert E \rvert) = O(b^d)$
Worst-case space complexity	$O(\lvert V \rvert) = O(b^d)$

  b: branching factor
  d: depth

- Good heuristic => small branching factor

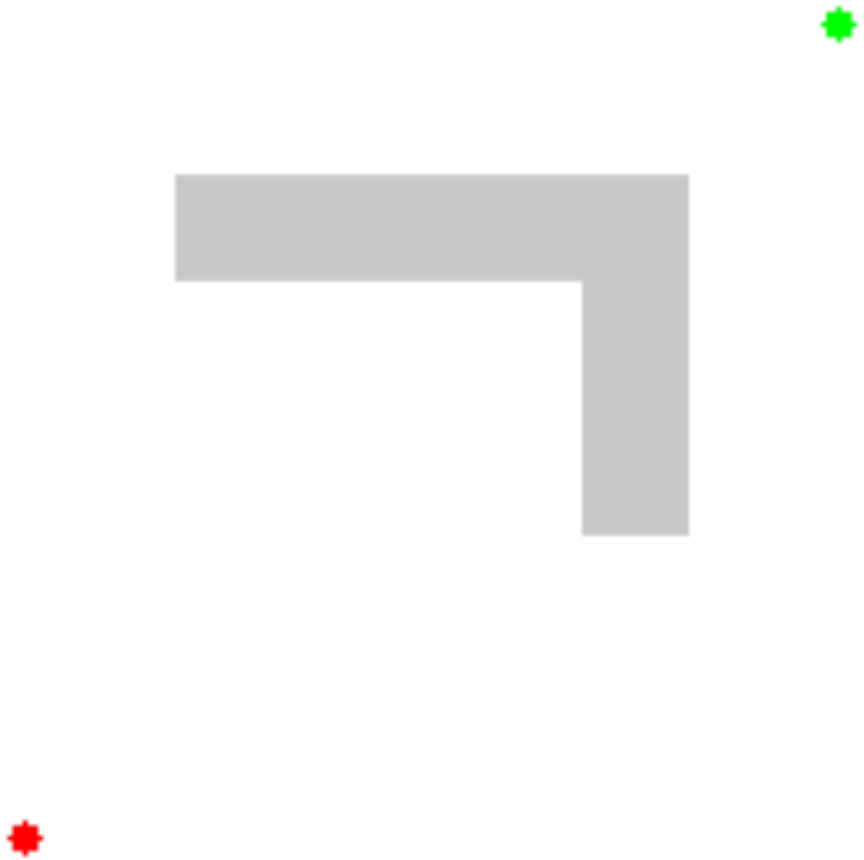


Optimal Planning

- Dijkstra finds the optimal path
- What about A*?
  - Find admissible heuristic h(n)
  - Admissible: do not overestimate the true cost-to-go
  - A* is optimal (finds optimal/ shortest path) if h(n) is admissible for all n
  - Admissible example: use geometric distance for h(n):

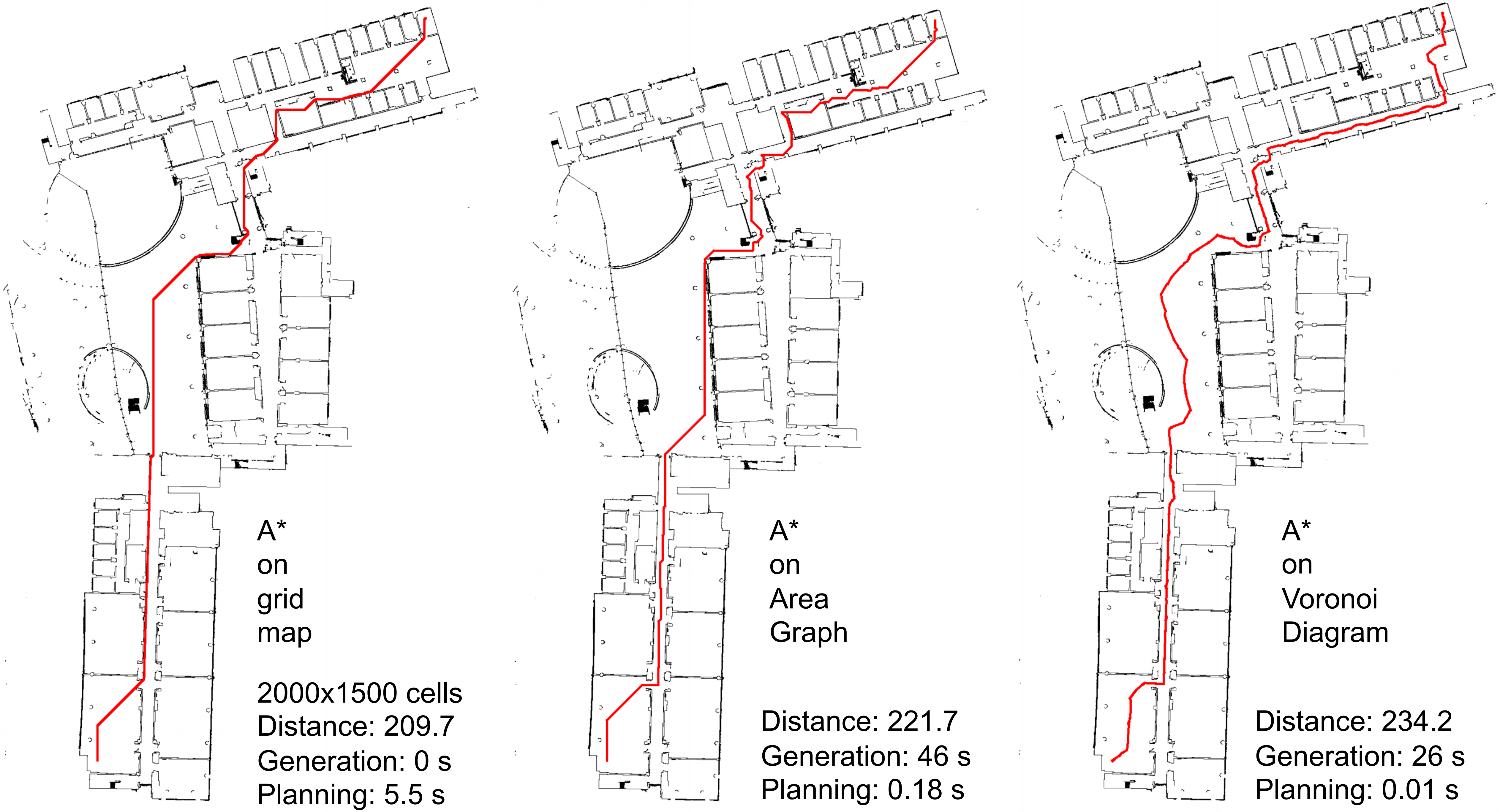
$$h(n) = \sqrt{(x_{goal} - x)^2 + (y_{goal} - y)^2}\ ).$$

  - Example: heuristic 5x geometric distance
  - h(n) = 0 => Dijkstra's Algorithm



A*

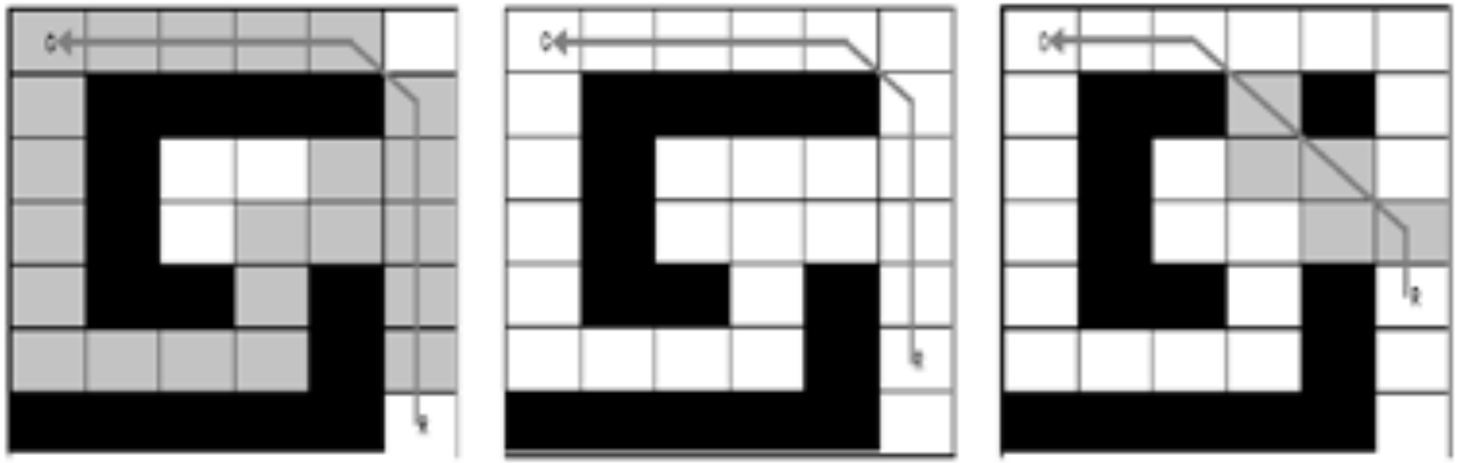
- Hierarchical planning possible e.g.: Go to the library:
  - First plan how to get from SIST building to library
  - Then plan how to get from entrance of library to goal room (campus level vs. library level)
- Many variants of A* algorithms exist – with different properties
- A* as graph search: applications outside of robotics/ path planning
  - Video games
  - Parsing with stochastic grammars in natural language processing
- Graph on which planning is done matters!
  - E.g.: Grid map; Pose Graph; Topological Graph; Open Street Map; Lattice Graphs; …



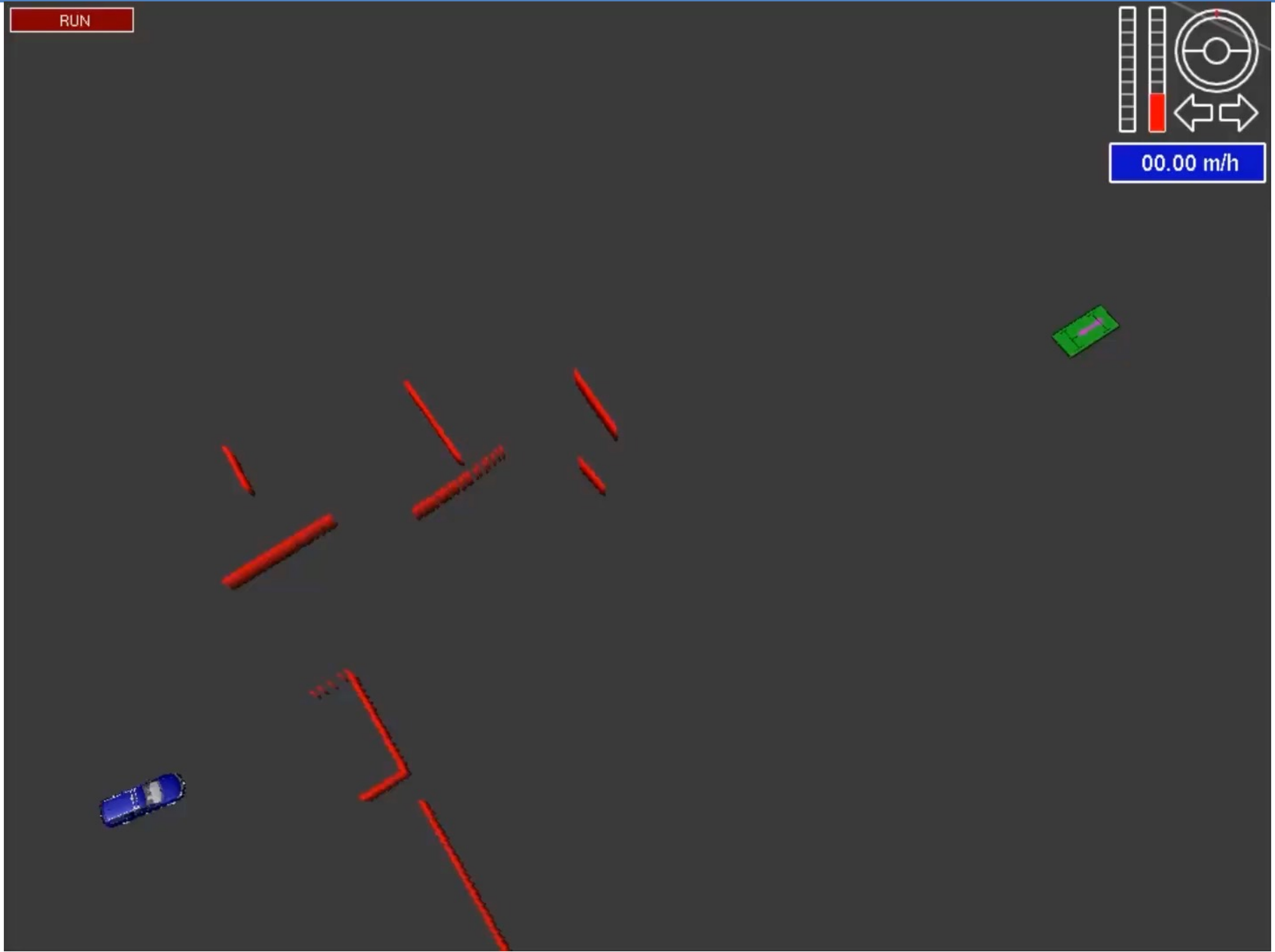
A*
on
grid
map
2000x1500 cells
Distance: 209.7
Generation: 0 s
Planning: 5.5 s
A*
on
Area
Graph
Distance: 221.7
Generation: 46 s
Planning: 0.18 s
A*
on
Voronoi
Diagram
Distance: 234.2
Generation: 26 s
Planning: 0.01 s

Graph Search Strategies: D* Search

- Similar to A* search, except that the search starts from the goal outward
- $f(n) = g(n) + \varepsilon\, h(n)$
- First pass is identical to A*
- Subsequent passes reuse information from previous searches



$\epsilon = 1.0$ $\epsilon = 1.0$ $\epsilon = 1.0$


RUN
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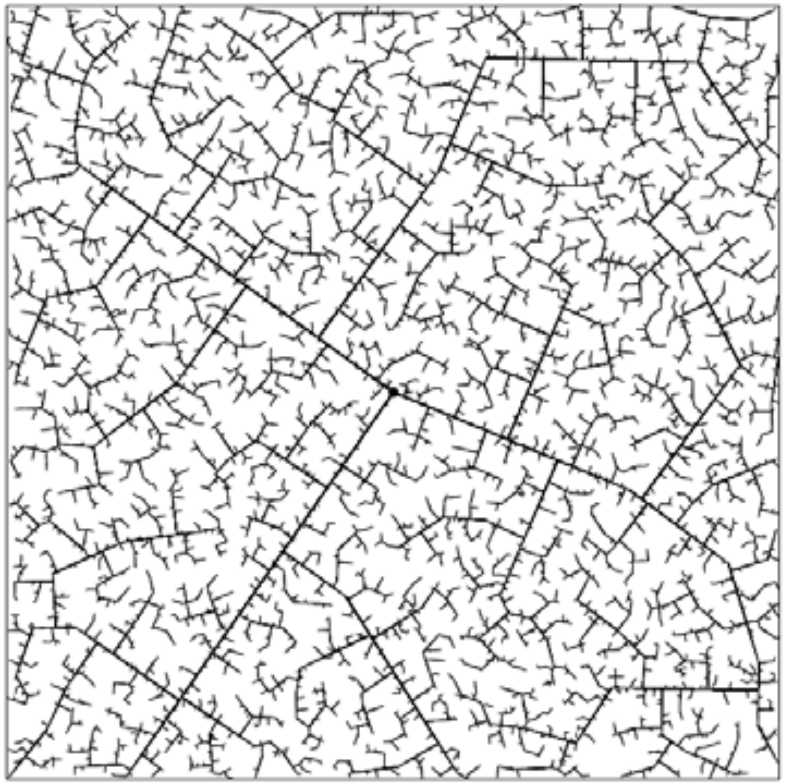
Graph Search Strategies: Randomized Search

- Most popular version is the rapidly exploring random tree (RRT)
  - Well suited for high-dimensional search spaces
  - Often produces highly suboptimal solutions

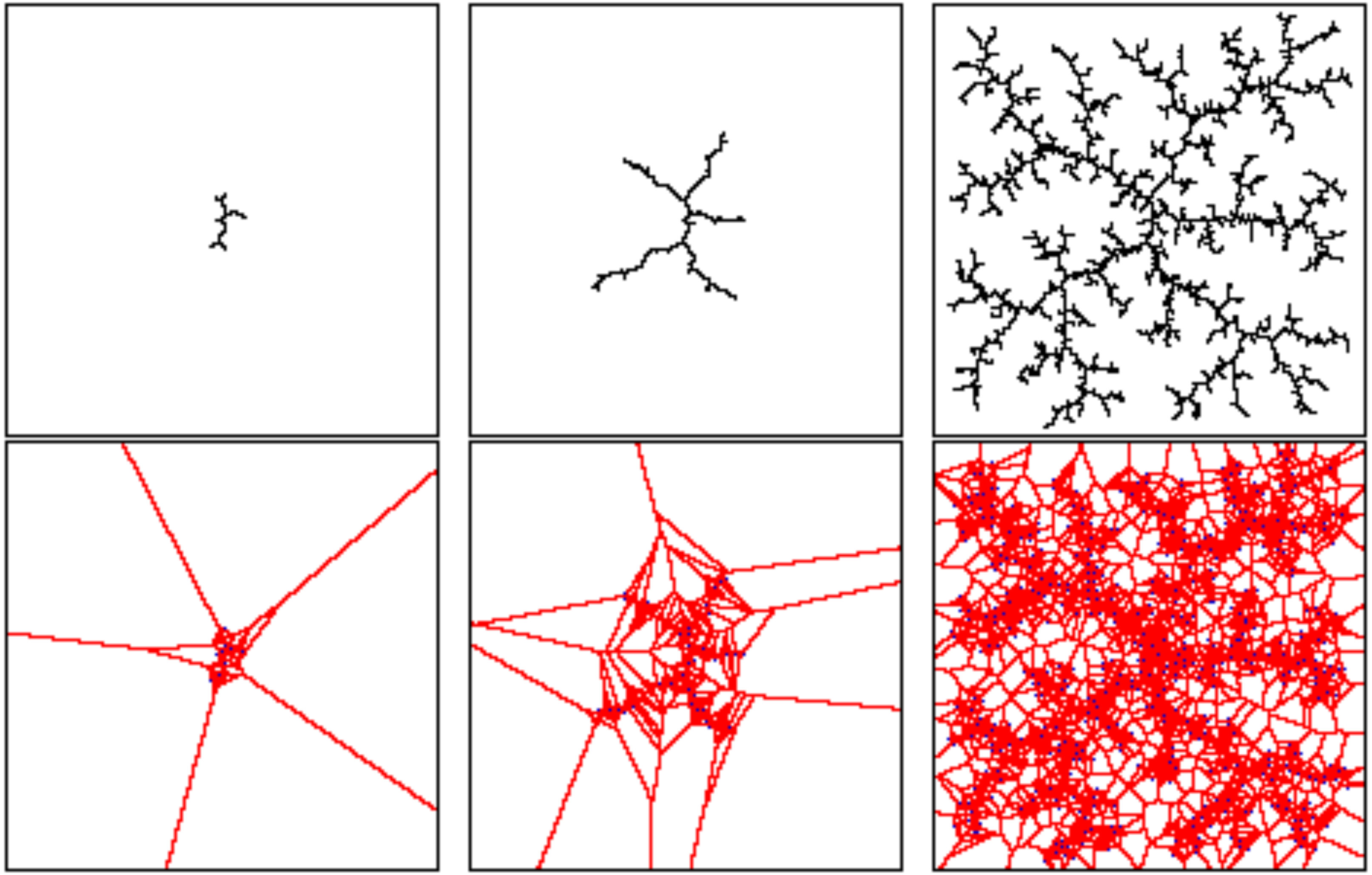
45 iterations

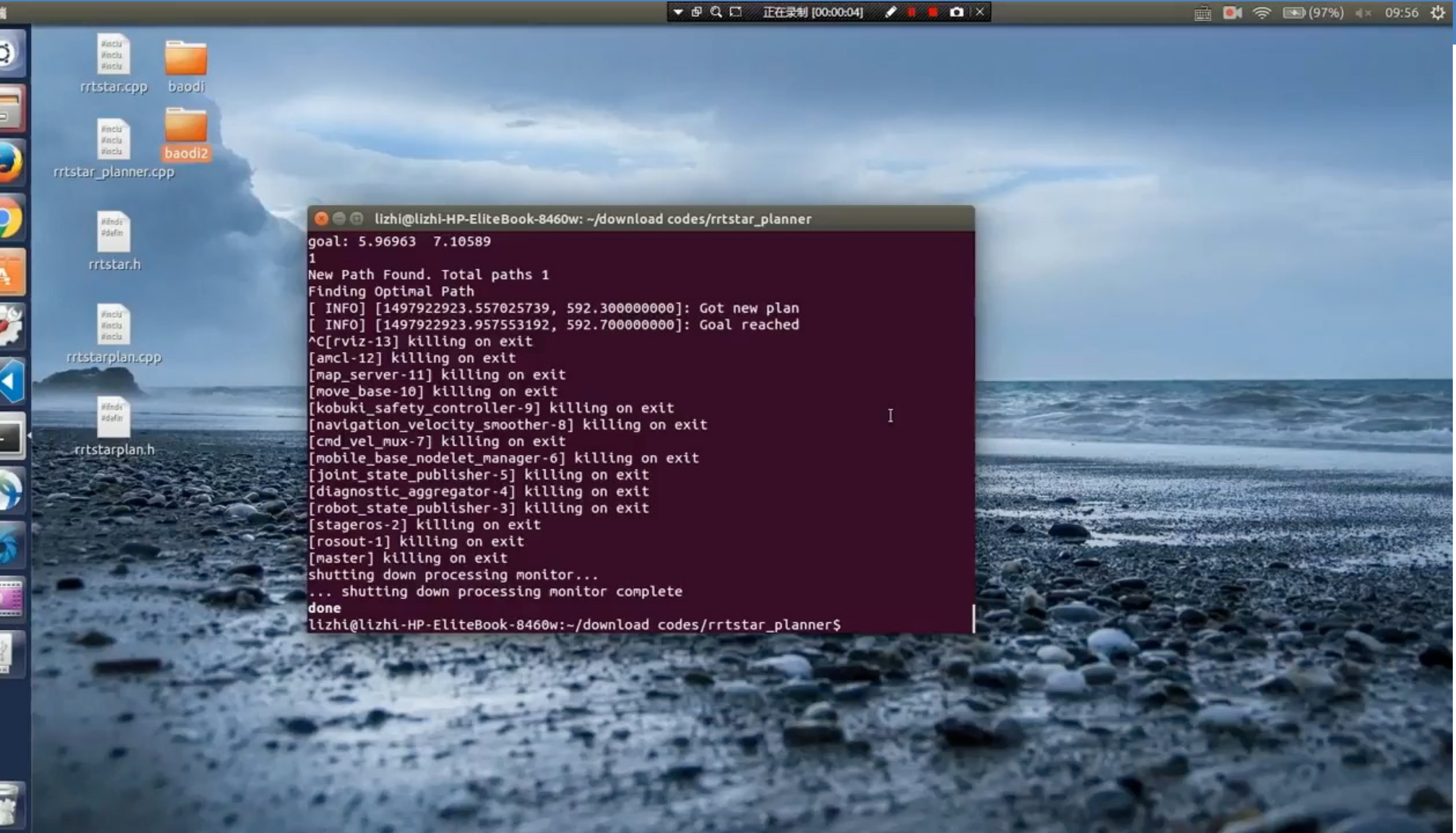


2345 iterations

Why are RRT's rapidly exploring?

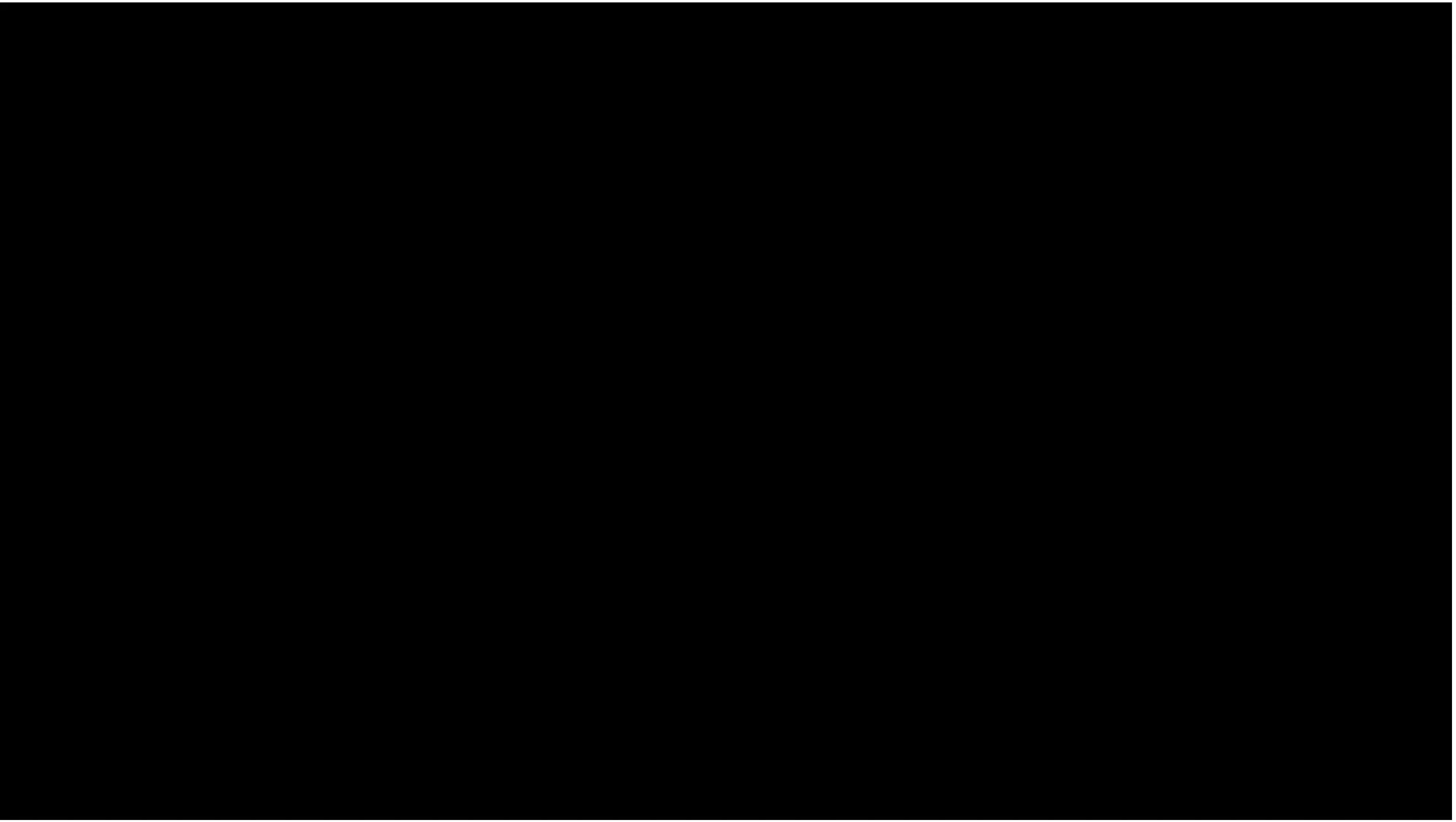
The probability of a node to be selected for expansion is proportional to the area of its Voronoi region

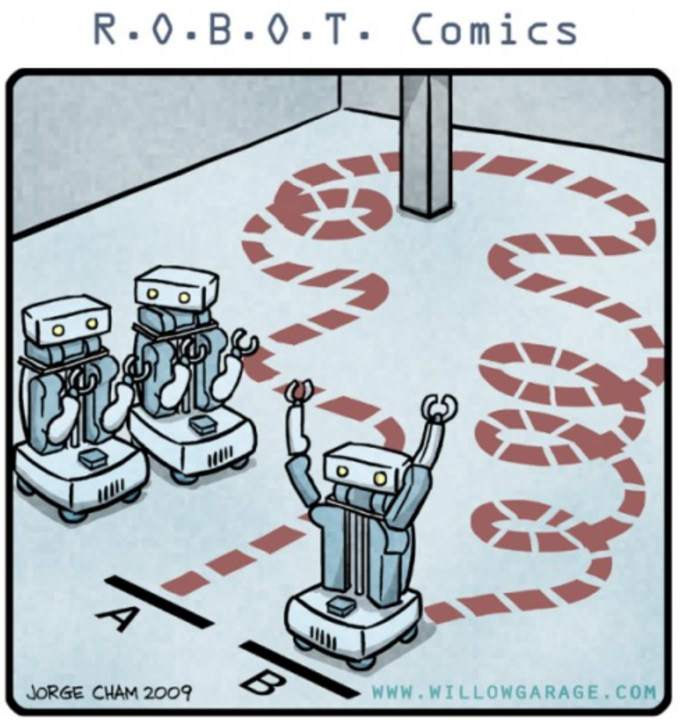



正在录制 [00:00:04]
(97%)
09:56
rrtstar.cpp
baodi
baodi2
rrtstar_planner.cpp
rrtstar.h
rrtstarplan.cpp
rrtstarplan.h
lizhi@lizhi-HP-EliteBook-8460w: ~/download codes/rrtstar_planner
goal: 5.96963 7.10589
1
New Path Found. Total paths 1
Finding Optimal Path
[INFO] [1497922923.557025739, 592.300000000]: Got new plan
[INFO] [1497922923.957553192, 592.700000000]: Goal reached
^C[rviz-13] killing on exit
[amcl-12] killing on exit
[map_server-11] killing on exit
[move_base-10] killing on exit
[kobuki_safety_controller-9] killing on exit
[navigation_velocity_smoother-8] killing on exit
[cmd_vel_mux-7] killing on exit
[mobile_base_nodelet_manager-6] killing on exit
[joint_state_publisher-5] killing on exit
[diagnostic_aggregator-4] killing on exit
[robot_state_publisher-3] killing on exit
[stageros-2] killing on exit
[rosout-1] killing on exit
[master] killing on exit
shutting down processing monitor...
... shutting down processing monitor complete
done
lizhi@lizhi-HP-EliteBook-8460w:~/download codes/rrtstar_planner$

ROS Navigation

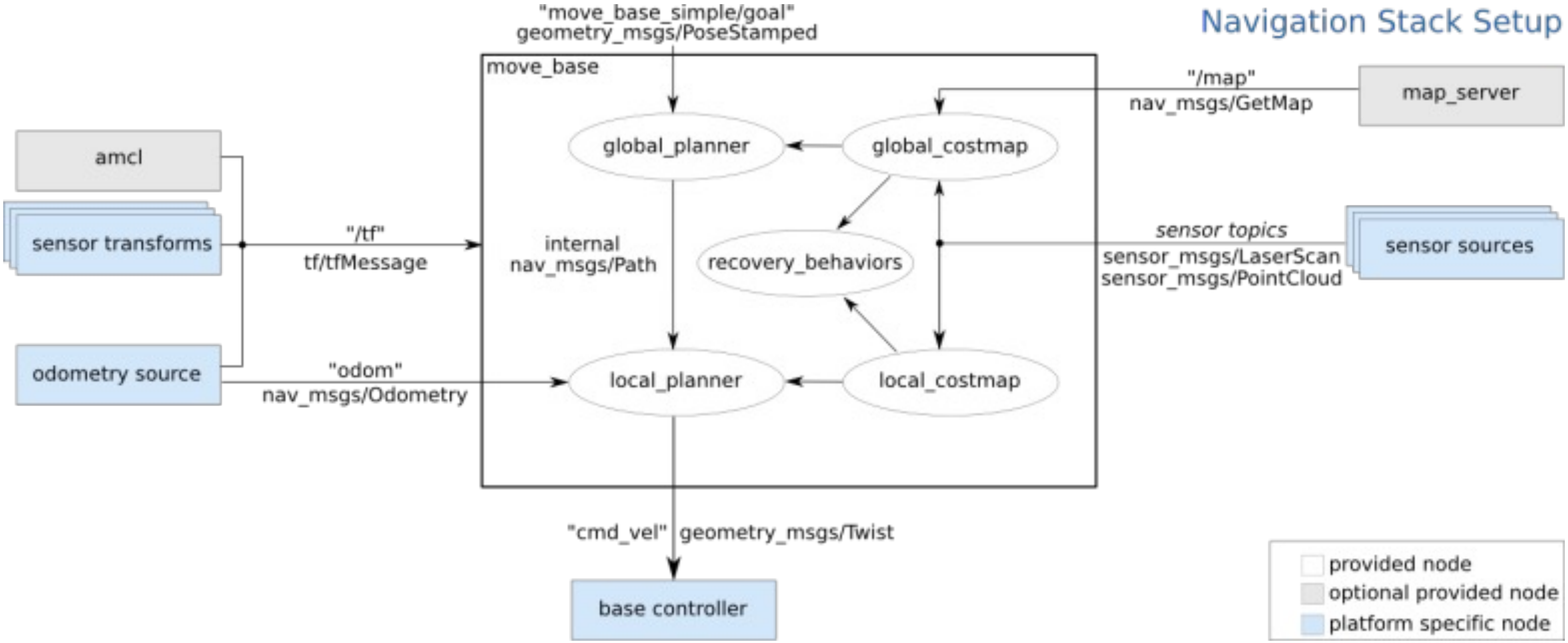
- http://wiki.ros.org/navigation





"HIS PATH-PLANNING MAY BE SUB-OPTIMAL, BUT IT'S GOT FLAIR."

Path Planning in ROS: move_base



teb_local_planner

An optimal trajectory planner for mobile robots based on Timed-Elastic-Bands

http://wiki.ros.org/teb_local_planner