上海科技大学
ShanghaiTech University

CS283: **Robotics Spring 2024: SLAM II**

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OVERVIEW:
THREE SLAM PARADIGMS

The Three SLAM Paradigms

- Most of the SLAM algorithms are based on the following three different approaches:
  - Extended Kalman Filter SLAM: (called EKF SLAM)
  - Particle Filter SLAM: (called FAST SLAM)
  - Graph-Based SLAM

EKF SLAM: overview

- **Extended state vector** $\boldsymbol{y_t}$: robot pose $\boldsymbol{x_t}$ + position of all the features $\boldsymbol{m_i}$ in the map:

$$y_t = [x_t, m_0, \dots, m_{n-1}]^T$$

- Example: 2D line-landmarks, size of $\boldsymbol{y_t}$ = $3+2n$: three variables to represent the robot pose + $2n$ variables for the n line-landmarks having vector components

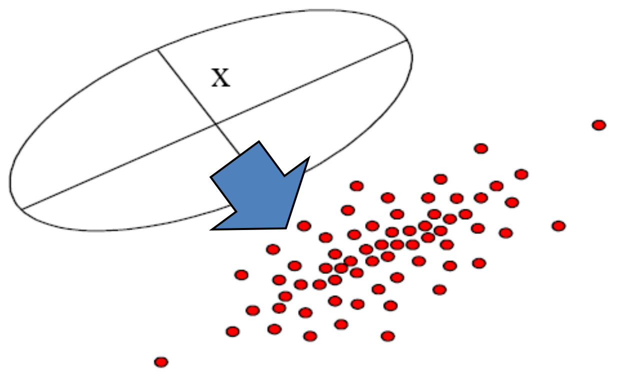
$$(\alpha_i, r_i)$$

$$y_t = [x_t, y_t, \theta_t, \alpha_0, r_0, \dots, \alpha_{n-1}, r_{n-1}]^T$$

- As the robot moves and takes measurements, the state vector and covariance matrix are updated using the standard equations of the extended Kalman filter
- Drawback: EKF SLAM is computationally very expensive.

Particle Filter SLAM: FastSLAM

- **FastSLAM approach**
  - Using particle filters.
  - Particle filters: mathematical models that represent probability distribution as a set of discrete particles that occupy the state space.



probability distribution (ellipse) as particle set (red dots)

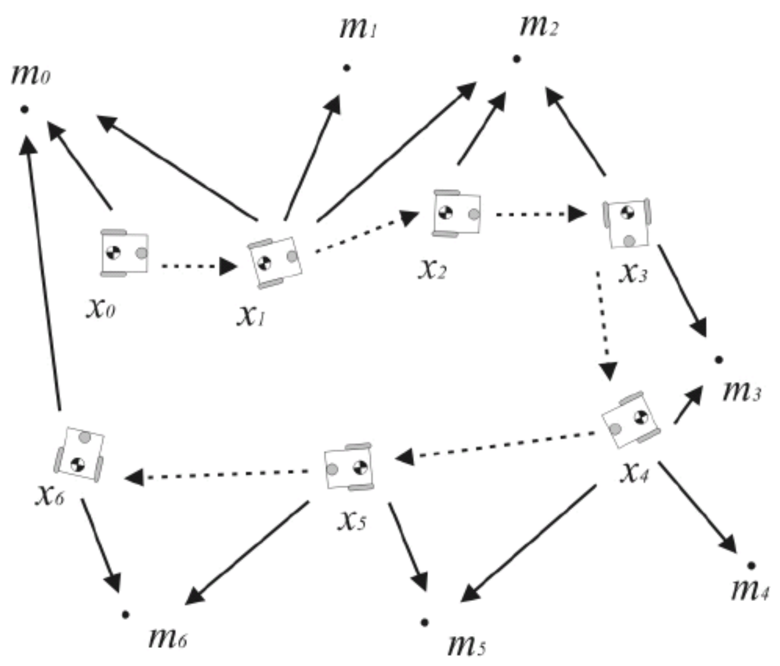
- Particle filter update
  - Generate new particle distribution using motion model and controls

  a) For each particle:
    1. Compare particle's prediction of measurements with actual measurements
    2. Particles whose predictions match the measurements are given a high weight

  b) Filter resample:
    - Resample particles based on weight
    - Filter resample
      - Assign each particle a weight depending on how well its estimate of the state agrees with the measurements and randomly draw particles from previous distribution based on weights creating a new distribution.

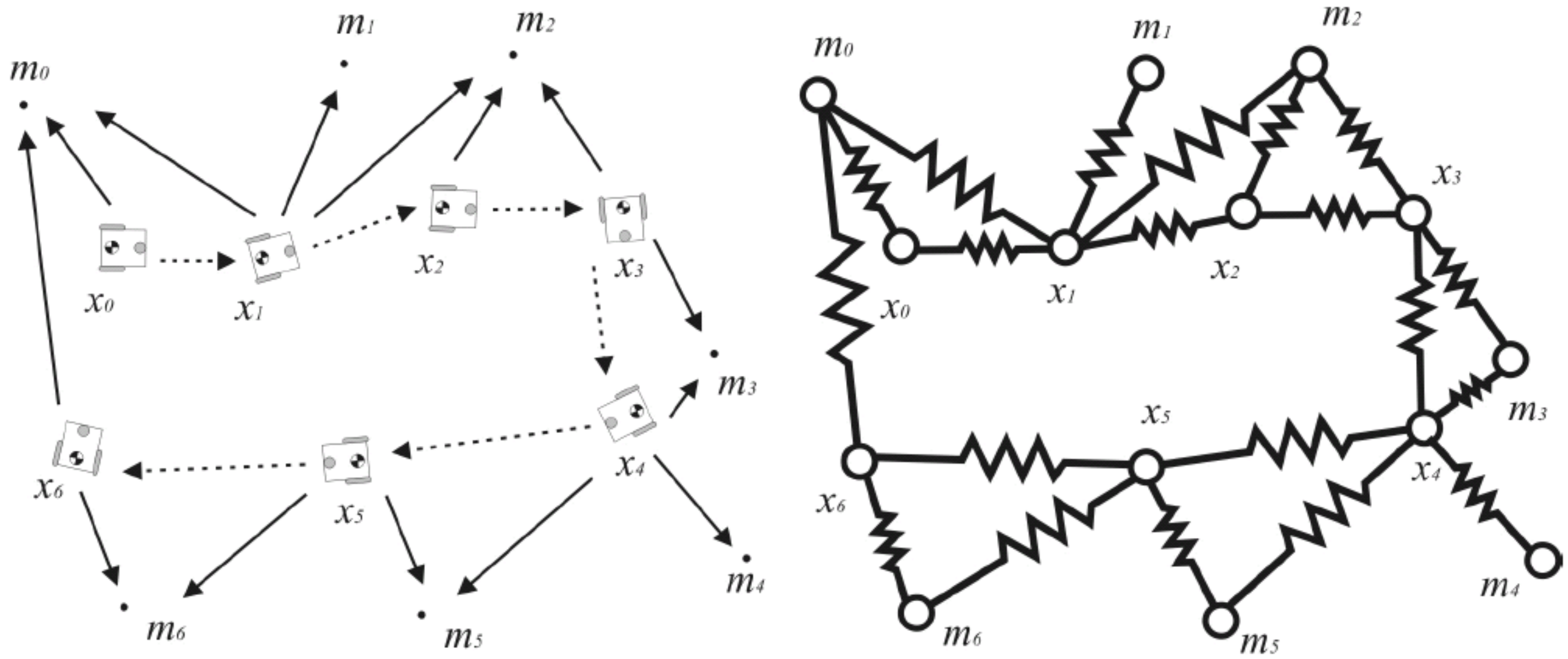
Graph-Based SLAM (1/3)

- SLAM problem can be interpreted as a sparse graph of nodes and constraints between nodes.
- The nodes of the graph are the robot locations and the features in the map.
- Constraints: relative position between consecutive robot poses , (given by the odometry input ***u***) and the relative position between the robot locations and the features observed from those locations.



Graph-Based SLAM (2/3)

- Constraints are not rigid but soft constraints!
- Relaxation: compute the solution to the full SLAM problem =>
  - Compute best estimate of the robot path and the environment map.
  - Graph-based SLAM represents robot locations and features as the nodes of an elastic net. The SLAM solution can then be found by computing the state of minimal energy of this net

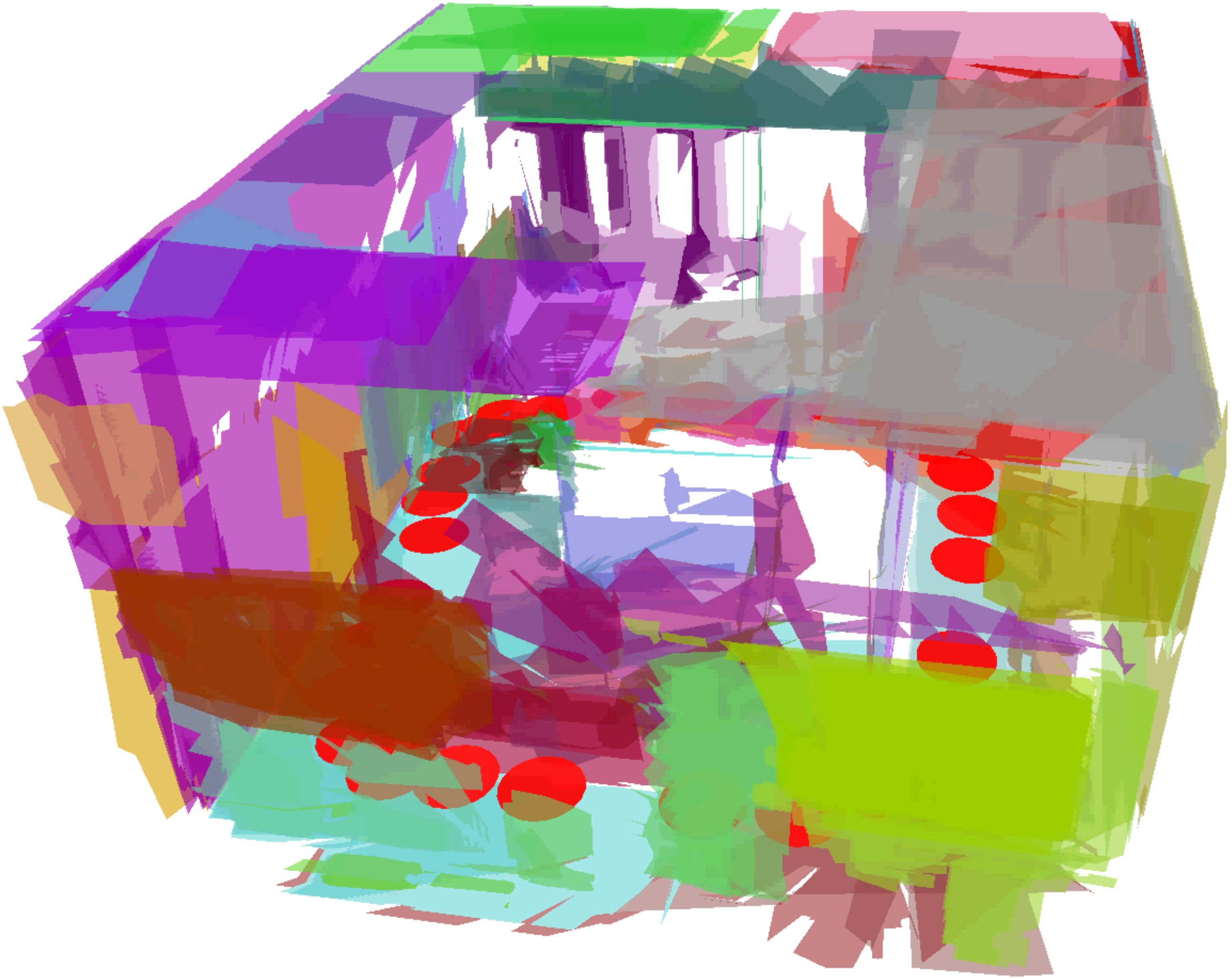


Graph-Based SLAM (3/3)

- Significant advantage of graph-based SLAM techniques over EKF SLAM:
  - EKF SLAM: computation and memory for to update and store the covariance matrix is quadratic with the number of features.
  - Graph-based SLAM: update time of the graph is constant and the required memory is linear in the number of features.
- However, the final graph optimization can become computationally costly if the robot path is long.
- Libraries for graph-based slam: g2o, ceres

SLAM EXAMPLES

Experiment Lab Run: 29 3D point-clouds; size of each: 541 x 361 = 195,301



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ROBOTICS UNIVERSIDAD DE ZARAGOZA

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ORB-SLAM2: an Open-Source SLAM System for Monocular, Stereo and RGB-D Cameras

Raúl Mur-Artal and Juan D. Tardós

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LOAM: Lidar Odometry and Mapping in Real-time

(Open Source Code and Open Datasets)

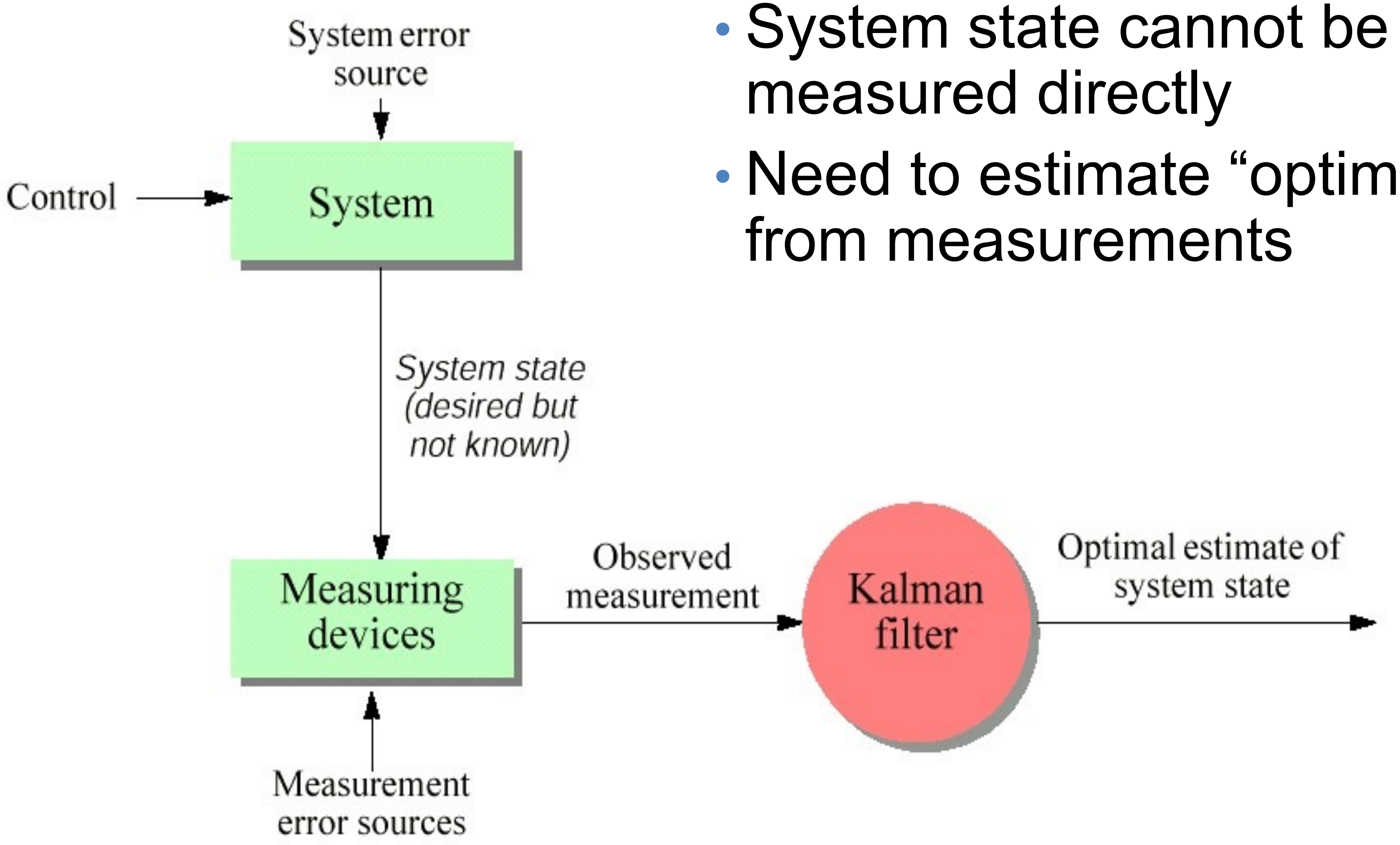
The Field Robotics Center
At the Robotics Institute of Carnegie Mellon University

KALMAN FILTER OVERVIEW

Following Material:

- Michael Williams, Australian National University
- Cornelia Fermüller, University of Maryland

The Problem

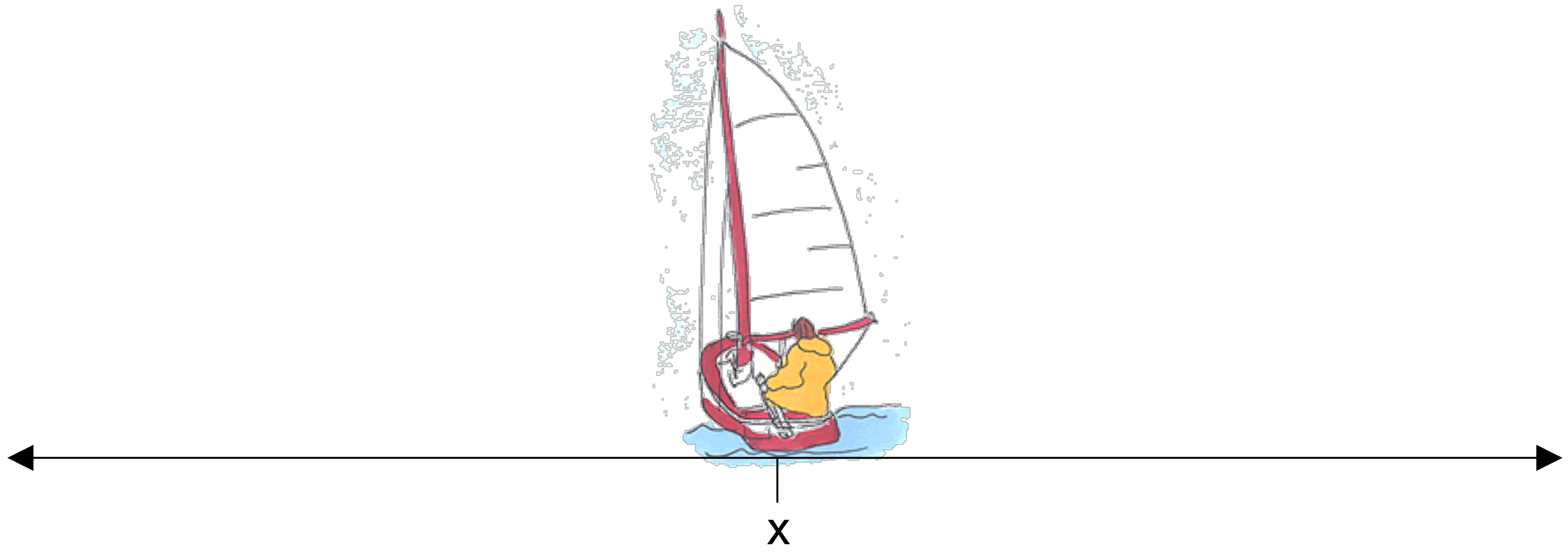


- System state cannot be measured directly
- Need to estimate “optimally” from measurements

What is a Kalman Filter?

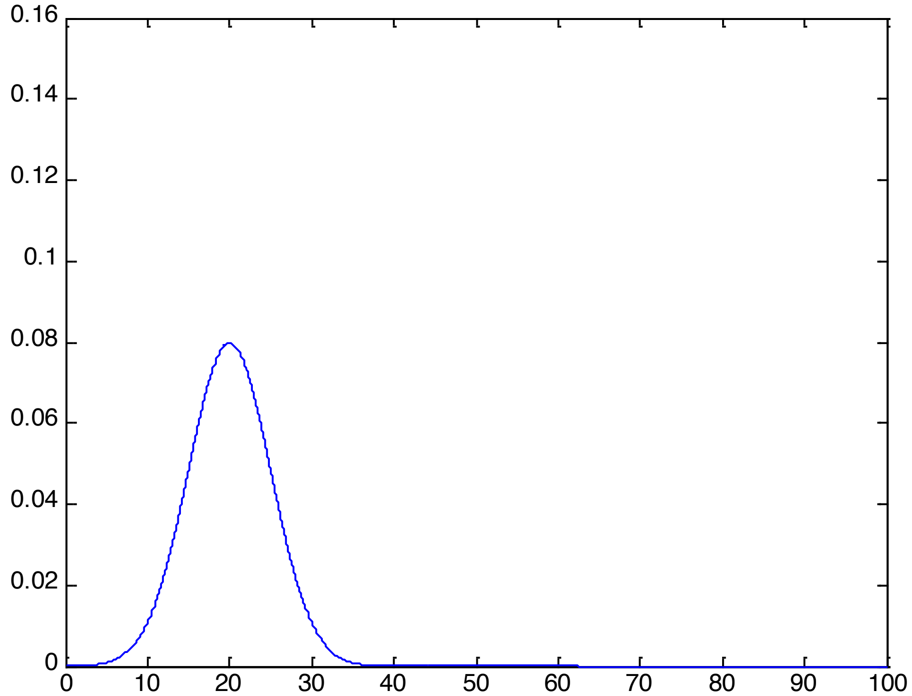
- Recursive data processing algorithm
- Generates optimal estimate of desired quantities given the set of measurements
- Optimal?
  - For linear system and white Gaussian errors, Kalman filter is “best” estimate based on all previous measurements
  - For non-linear system optimality is ‘qualified’
- Recursive?
  - Doesn’t need to store all previous measurements and reprocess all data each time step

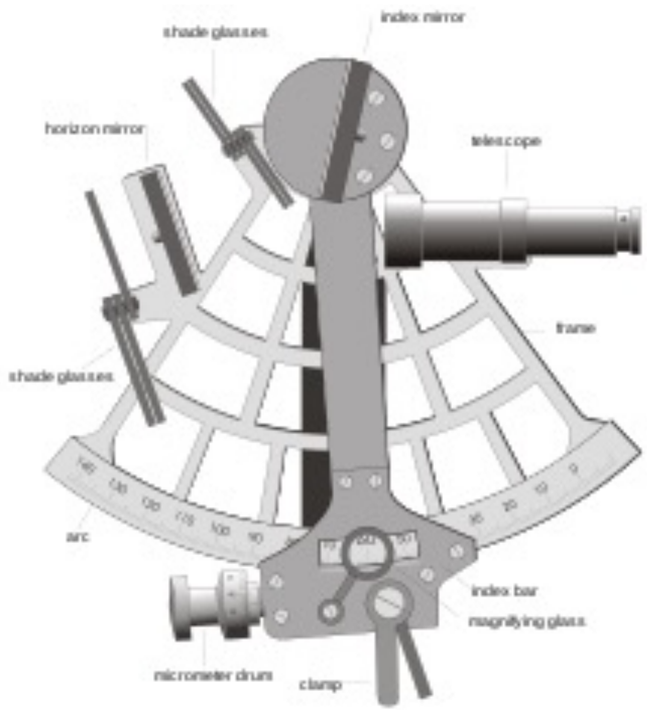
Conceptual Overview



- Lost on the 1-dimensional line
- Position – x(t)
- Assume Gaussian distributed measurements

Conceptual Overview

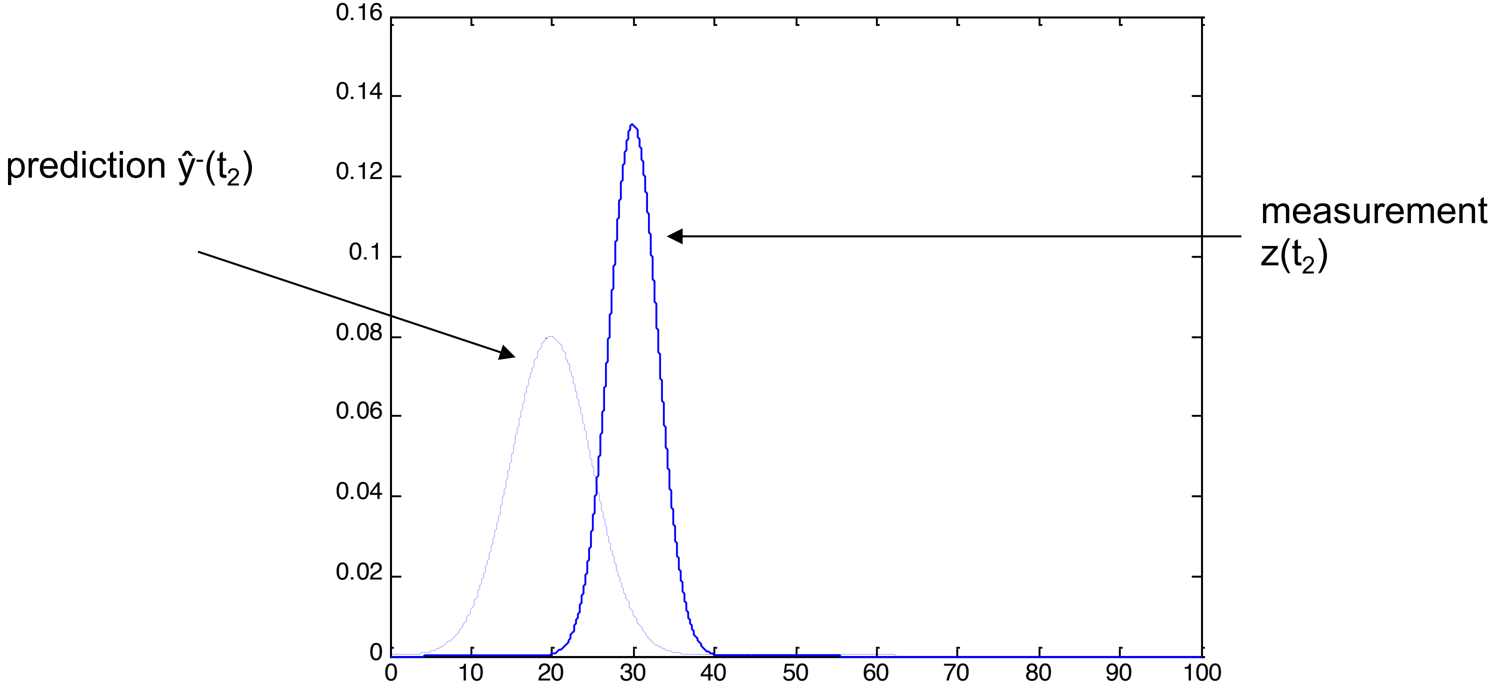




Sextant

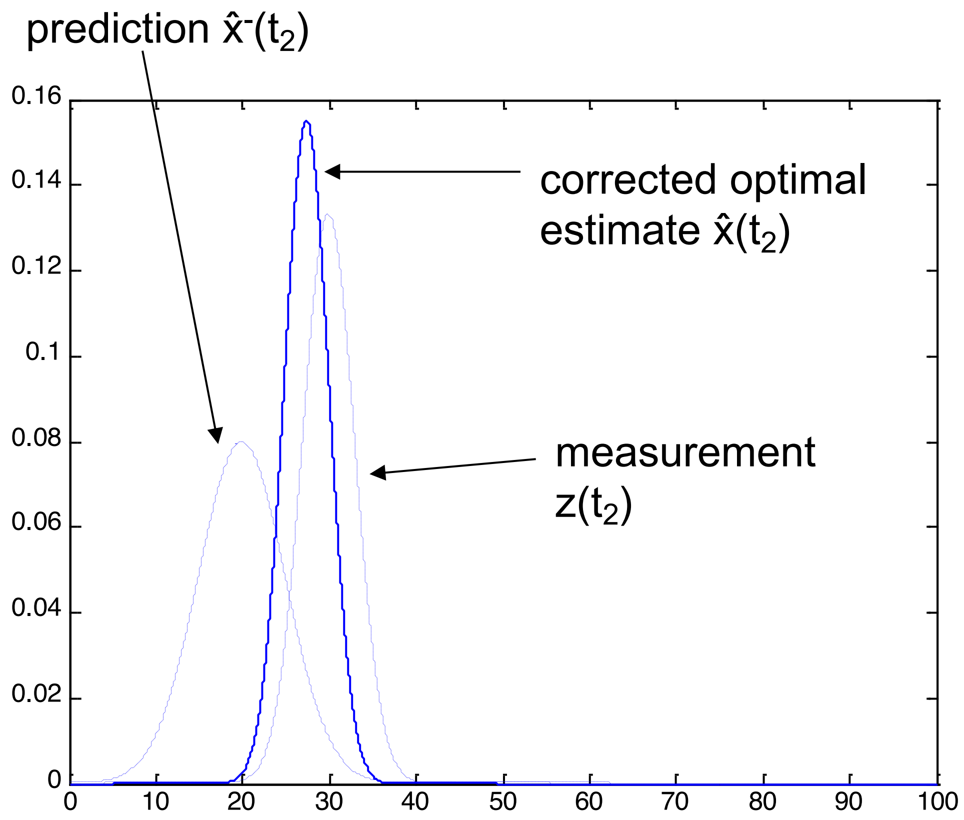
- Sextant Measurement at t_1: Mean = z_1 and Variance = σ_{z1}
- Optimal estimate of position is: $\hat{x}(t_1) = z_1$
- Variance of error in estimate: $\sigma^2_x (t_1) = \sigma^2_{z1}$
- Boat in same position at time t_2 - <u>Predicted</u> position is z_1

Conceptual Overview



- So we have the prediction $\hat{x}^{-}(t_2)$
- GPS Measurement at t_2: Mean = z_2 and Variance = σ_{z2}
- Need to <u>correct</u> the prediction due to measurement to get $\hat{x}(t_2)$
- Closer to more trusted measurement – linear interpolation?

Conceptual Overview



- Corrected mean is the new optimal estimate of position
- New variance is smaller than either of the previous two variances

Conceptual Overview

- Lessons so far:

Make prediction based on previous data: $\hat{x}^-$, σ^-

↓

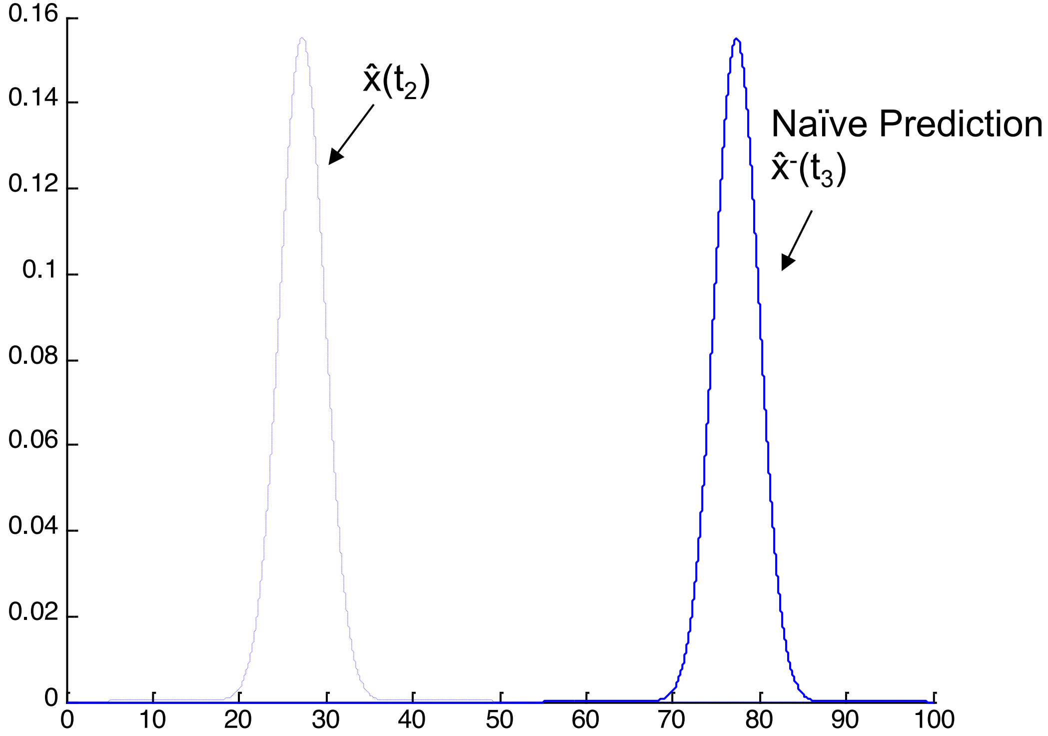
Take measurement: z_k, σ_z

↓

Optimal estimate ($\hat{x}$) = Prediction + (Kalman Gain) * (Measurement - Prediction)

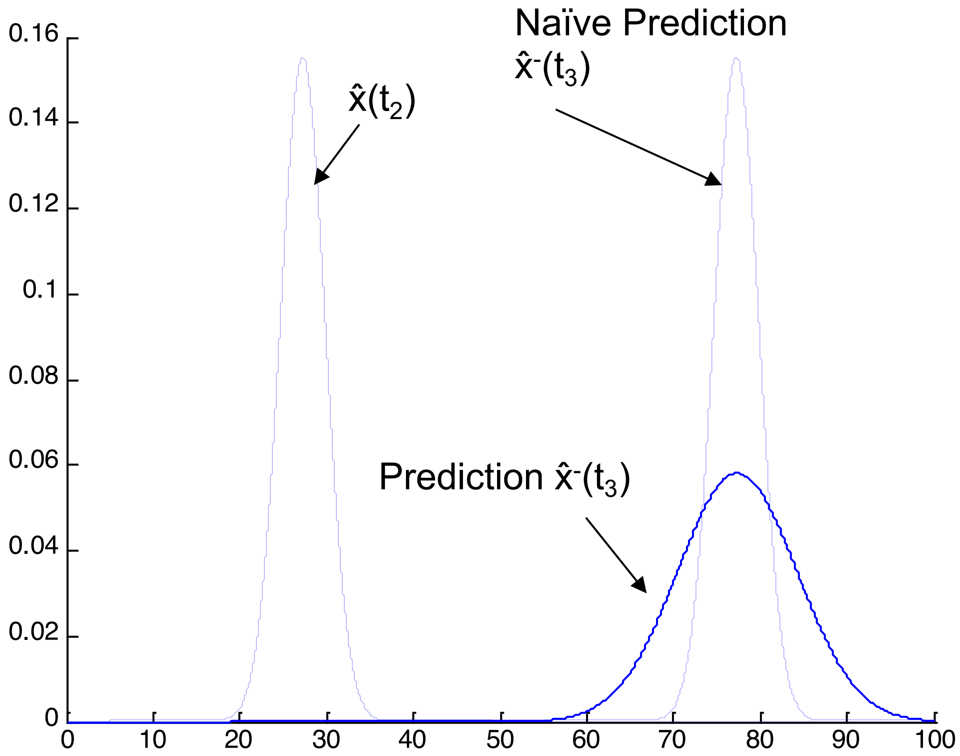
Variance of estimate = Variance of prediction * (1 – Kalman Gain)

Conceptual Overview



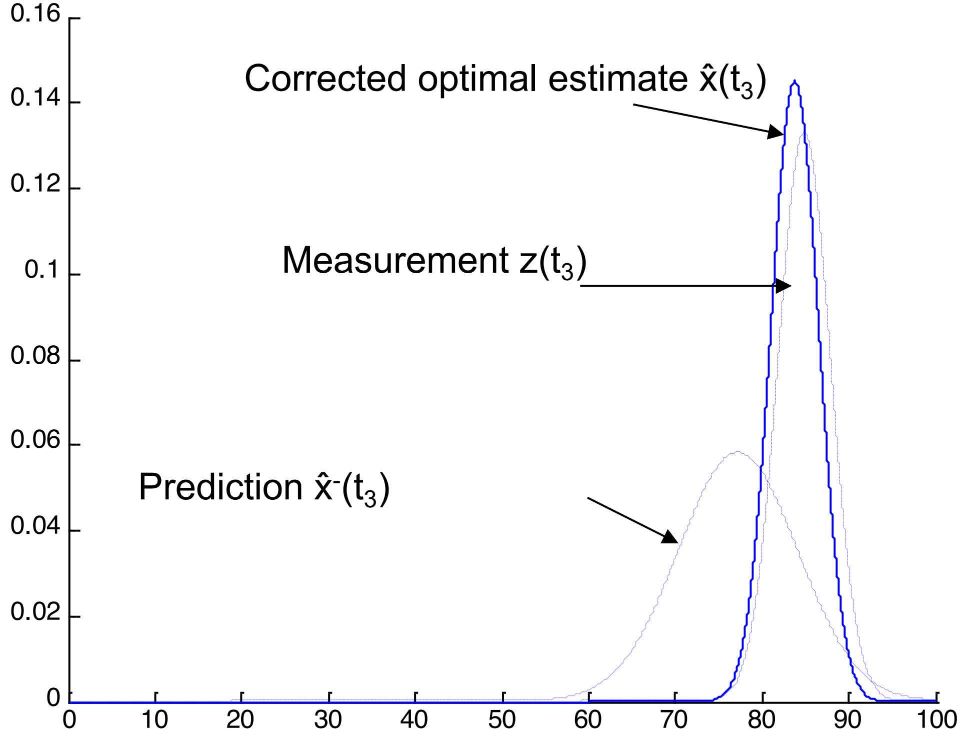
- At time t_3, boat moves with velocity dx/dt=u
- Naïve approach: Shift probability to the right to predict
- This would work if we knew the velocity exactly (perfect model)

Conceptual Overview



- Better to assume imperfect model by adding Gaussian noise
- dx/dt = u + w
- Distribution for prediction moves and spreads out

Conceptual Overview



- Now we take a measurement at t_3
- Need to once again correct the prediction
- Same as before

Conceptual Overview

- Lessons learnt from conceptual overview:
  - Initial conditions ($\hat{x}_{k-1}$ and σ_{k-1})
  - Prediction ($\hat{x}^{-}_{k}$, σ^{-}_{k})
    - Use initial conditions and model (eg. constant velocity) to make prediction
  - Measurement (z_k)
    - Take measurement
  - Correction ($\hat{x}_k$, σ_k)
    - Use measurement to correct prediction by ‘blending’ prediction and residual – always a case of merging only two Gaussians
    - Optimal estimate with smaller variance

Theoretical Basis

- Process to be estimated:

$\hat{x}_k = Ax_{k-1} + Bu_k + w_{k-1}$ Process Noise (w) with covariance Q

$z_k = Hx_k + v_k$ Measurement Noise (v) with covariance R

- Kalman Filter

Predicted: $\hat{y}^-_k$ is estimate based on measurements at previous time-steps

$$\hat{x}^-_k = Ax_{k-1} + Bu_k$$

$$P^-_k = AP_{k-1}A^T + Q$$

Corrected: $\hat{y}_k$ has additional information – the measurement at time k

$$\hat{x}_k = \hat{x}^-_k + K(z_k - H\,\hat{x}^-_k)$$

$$K = P^-_k H^T (HP^-_k H^T + R)^{-1}$$

$$P_k = (I - KH)P^-_k$$

Blending Factor

- If we are sure about measurements:
  - Measurement error covariance (R) decreases to zero
  - K decreases and weights residual more heavily than prediction
- If we are sure about prediction
  - Prediction error covariance P^-_k decreases to zero
  - K increases and weights prediction more heavily than residual

Theoretical Basis

Prediction (Time Update)

(1) Project the state ahead

$$\hat{x}^-_k = Ay_{k-1} + Bu_k$$

(2) Project the error covariance ahead

$$P^-_k = AP_{k-1}A^T + Q$$

Correction (Measurement Update)

(1) Compute the Kalman Gain

$$K = P^-_k H^T (HP^-_k H^T + R)^{-1}$$

(2) Update estimate with measurement z_k

$$\hat{x}_k = \hat{x}^-_k + K(z_k - H\hat{x}^-_k)$$

(3) Update Error Covariance

$$P_k = (I - KH)P^-_k$$

RECURSIVE STATE ESTIMATION

Following Material:

- Cyrill Stachniss, University of Bonn

State Estimation

- Estimate the state x of a system given observations z and controls u
- **Goal:**

$$p(x \mid z, u)$$

State Estimation

- Estimate the state x of a system given observations z and controls u
- **Goal:**

$$p(x_t | z_{1:t}, u_{1:t})$$

(reminder)

Tiny Reminder (Probability Theory)

(reminder)

Bayes' Rule

$$p(x, y) = p(x \mid y)\, p(y)$$

$$p(x, y) = p(y \mid x)\, p(x)$$

⇓

$$p(x \mid y) = \frac{p(y \mid x)\, p(x)}{p(y)} = \frac{\text{likelihood prior}}{\text{evidence}}$$

Bayes' Rule with Background Knowledge z

$$p(x \mid y) = \frac{p(y \mid x)\, p(x)}{p(y)}$$

$$p(x \mid y, z) = \frac{p(y \mid x, z)\, p(x \mid z)}{p(y \mid z)}$$

(reminder)

Law of Total Probability and Marginalization

Law of Total Probability

$$p(x) = \sum_y p(x \mid y)\, p(y) \qquad p(x) = \int p(x \mid y)\, p(y)\, dy$$

Marginalization

$$p(x) = \sum_y p(x, y) \qquad p(x) = \int p(x, y)\, dy$$

Markov Property/ Assumption

- “The future is independent from the past - given the **current** state.”
- Markov property = the conditional probability distribution of future states depends only upon the present state, not on the sequence of events that preceded it.
- Such a process has no memory

State Estimation

- Estimate the state x of a system given observations z and controls u

- **Goal:**

$$p(x_t | z_{1:t}, u_{1:t})$$

Recursive Bayes Filter 1

$$bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$$

definition of the belief

Recursive Bayes Filter 2

$$bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$$
$$= \underline{\eta\, p(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, p(x_t \mid z_{1:t-1}, u_{1:t})}$$

Bayes' rule

Recursive Bayes Filter 3

$$\begin{aligned} bel(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\ &= \eta\, p(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, p(x_t \mid z_{1:t-1}, u_{1:t}) \\ &= \eta\, \underline{p(z_t \mid x_t)}\, p(x_t \mid z_{1:t-1}, u_{1:t}) \end{aligned}$$

Markov assumption

Recursive Bayes Filter 4

$$
\begin{aligned}
bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t}) \\
&= \eta \, p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) \, p(x_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta \, p(z_t \mid x_t) \, p(x_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta \, p(z_t \mid x_t) \int \underline{p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t})} \\
&\qquad \underline{p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) \, dx_{t-1}}
\end{aligned}
$$

Law of total probability

Recursive Bayes Filter 5

$$bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$$
$$= \eta \, p(z_t \mid x_t, z_{1:t-1}, u_{1:t}) \, p(x_t \mid z_{1:t-1}, u_{1:t})$$
$$= \eta \, p(z_t \mid x_t) \, p(x_t \mid z_{1:t-1}, u_{1:t})$$
$$= \eta \, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \, p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) \, dx_{t-1}$$
$$= \eta \, p(z_t \mid x_t) \int \underline{p(x_t \mid x_{t-1}, u_t)} \, p(x_{t-1} \mid z_{1:t-1}, u_{1:t}) \, dx_{t-1}$$

Markov assumption

Recursive Bayes Filter 6

$$
\begin{aligned}
bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, p(x_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t)\, p(x_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \\
&\qquad p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, p(x_{t-1} \mid z_{1:t-1}, u_{1:\underline{t-1}})\, dx_{t-1}
\end{aligned}
$$

independence assumption

Recursive Bayes Filter 7

$$
\begin{aligned}
bel(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, p(x_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t)\, p(x_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \\
&\qquad\qquad p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t-1})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, \underline{bel(x_{t-1})}\, dx_{t-1}
\end{aligned}
$$

recursive term

Complete Derivation of the Recursive Bayes Filter

$$
\begin{aligned}
bel(x_t) &= p(x_t \mid z_{1:t}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, p(x_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t)\, p(x_t \mid z_{1:t-1}, u_{1:t}) \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t}) \\
&\qquad\qquad p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, p(x_{t-1} \mid z_{1:t-1}, u_{1:t-1})\, dx_{t-1} \\
&= \eta\, p(z_t \mid x_t) \int p(x_t \mid x_{t-1}, u_t)\, bel(x_{t-1})\, dx_{t-1}
\end{aligned}
$$

Prediction and Correction Step

- Bayes filter can be written as a two step process

$$bel(x_t) = \underline{\eta \, p(z_t \mid x_t)} \; \underline{\int p(x_t \mid u_t, x_{t-1}) \, bel(x_{t-1}) \, dx_{t-1}}$$

- **Prediction step**

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}) \, bel(x_{t-1}) \, dx_{t-1}$$

- **Correction step**

$$bel(x_t) = \eta \, p(z_t \mid x_t) \, \overline{bel}(x_t)$$

Motion and Observation Model

- Prediction step

$$\overline{bel}(x_t) = \int \underline{p(x_t \mid u_t, x_{t-1})}\; bel(x_{t-1})\, dx_{t-1}$$

motion model

- Correction step

$$bel(x_t) = \eta\, \underline{p(z_t \mid x_t)}\; \overline{bel}(x_t)$$

observation model
(also: measurement or sensor model)

Different Realizations

- The Bayes filter is a **framework** for recursive state estimation
- There are **different realizations**
- **Different properties**
  - Linear vs. non-linear models for motion and observation models
  - Gaussian distributions only?
  - Parametric vs. non-parametric filters
  - …

Popular Filters

- **Kalman filter & EKF**
  - Gaussians
  - Linear or linearized models

- **Particle filter**
  - Non-parametric
  - Arbitrary models (sampling required)

KALMAN FILTER DETAILS

Following Material:

- Michael Williams, Australian National University
- Cornelia Fermüller, University of Maryland

Recursive Bayes Filter

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

1. Algorithm **Bayes_filter**(*Bel(x),d*):
2. η=0
3. If *d* is a perceptual data item *z* then
4. For all *x* do
5. $Bel'(x) = P(z \mid x) Bel(x)$
6. $\eta = \eta + Bel'(x)$
7. For all *x* do
8. $Bel'(x) = \eta^{-1} Bel'(x)$
9. Else if *d* is an action data item *u* then
10. For all *x* do
11. $Bel'(x) = \int P(x \mid u, x') \; Bel(x') \; dx'$
12. Return *Bel′(x)*

Kalman Filter

- Bayes filter with **Gaussians**
- Developed in the late 1950's
- Most relevant Bayes filter variant in practice
- Applications range from economics, wheather forecasting, satellite navigation to robotics and many more.

- The Kalman filter "algorithm" is a couple of **matrix multiplications**!

Gaussians

$$p(x) \sim N(\mu, \sigma^2):$$

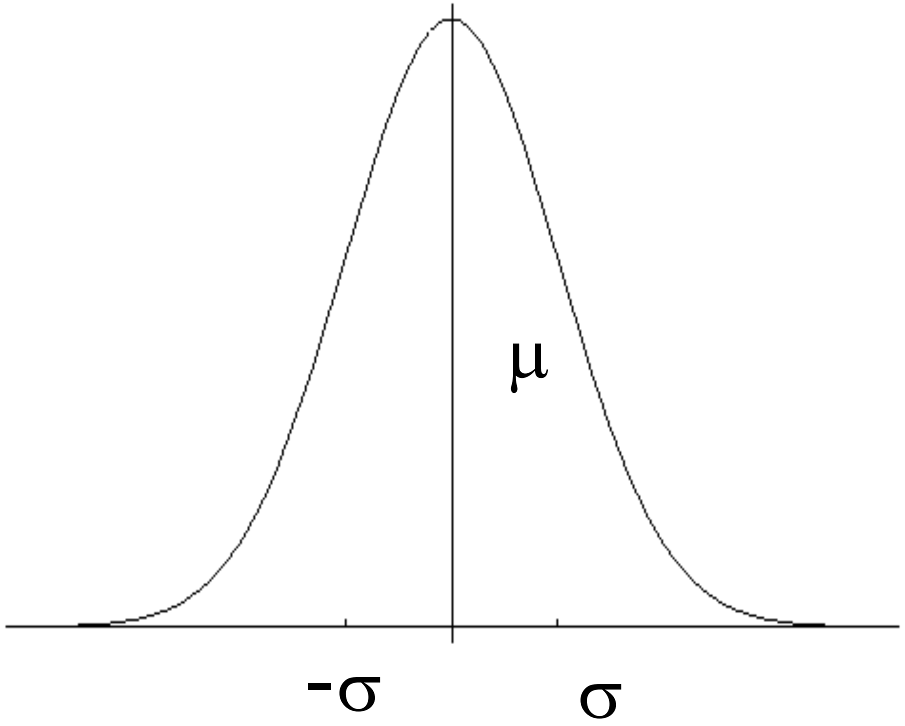
$$p(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

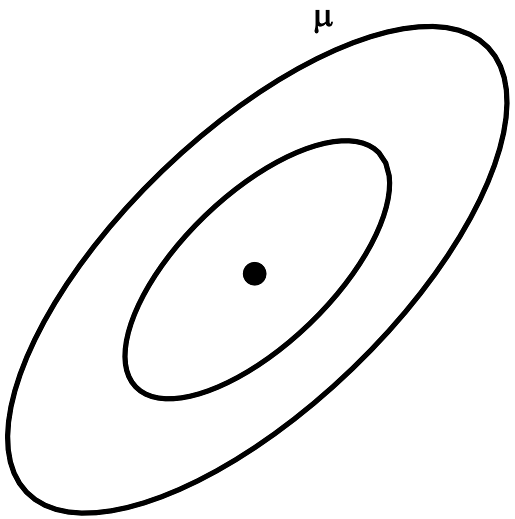
Univariate

$$p(\mathbf{x}) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}):$$

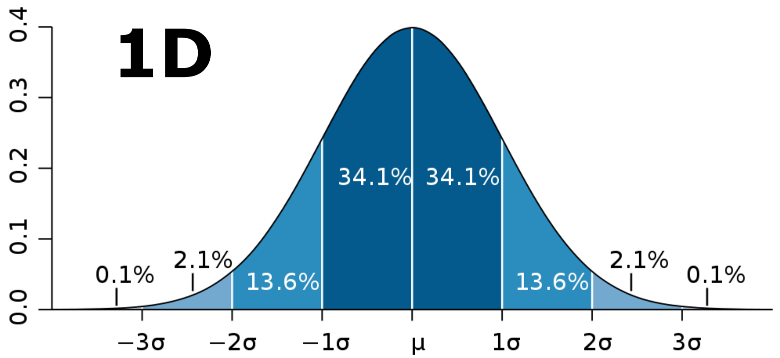
$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^t \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$

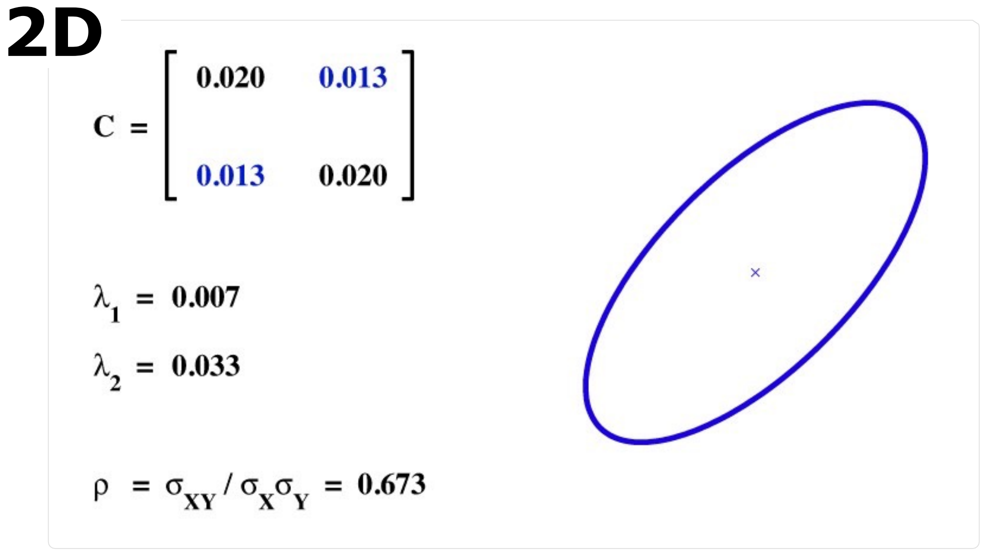
Multivariate

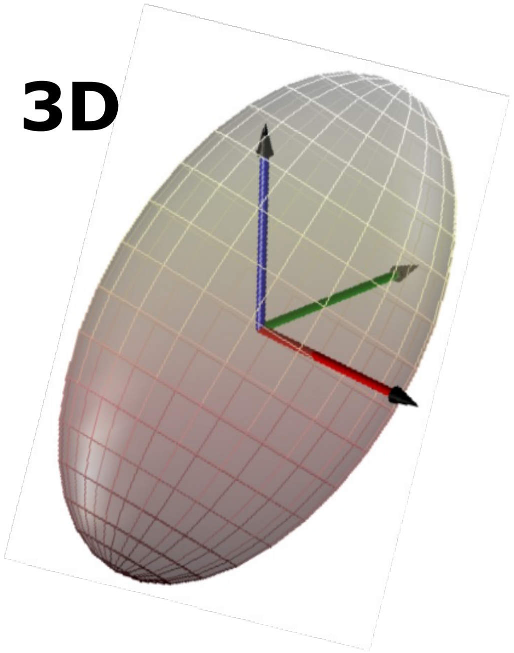




Gaussians


1D
0.0
0.1
0.2
0.3
0.4
0.1%
2.1%
13.6%
34.1%
34.1%
13.6%
2.1%
0.1%
−3σ
−2σ
−1σ
μ
1σ
2σ
3σ


2D
C =
0.020 0.013
0.013 0.020
λ1 = 0.007
λ2 = 0.033
ρ = σXY / σXσY = 0.673


3D

Properties of Gaussians

- Univariate

$$\left.\begin{array}{l} X \sim N(\mu,\sigma^2) \\ Y = aX + b \end{array}\right\} \Rightarrow Y \sim N(a\mu + b, a^2\sigma^2)$$

$$\left.\begin{array}{l} X_1 \sim N(\mu_1,\sigma_1^2) \\ X_2 \sim N(\mu_2,\sigma_2^2) \end{array}\right\} \Rightarrow p(X_1)\cdot p(X_2) \sim N\left(\frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\mu_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\mu_2, \quad \frac{1}{\sigma_1^{-2} + \sigma_2^{-2}}\right)$$

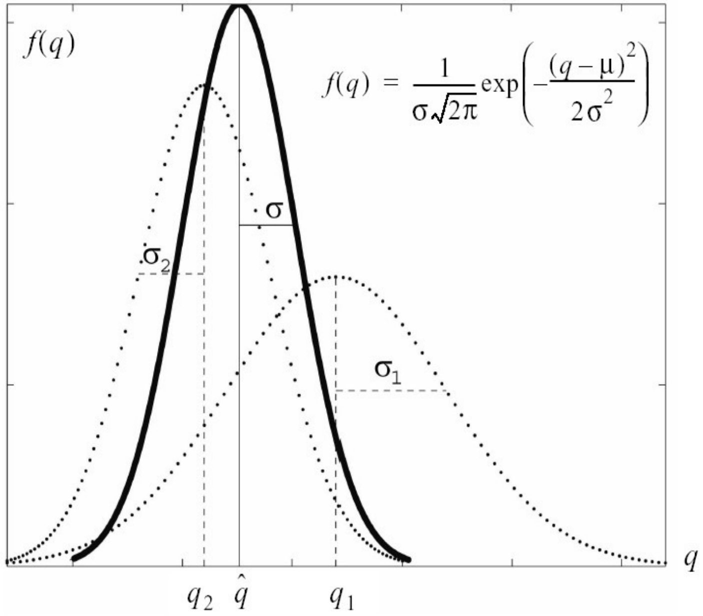
- Multivariate

$$\left.\begin{array}{l} X \sim N(\mu,\Sigma) \\ Y = AX + B \end{array}\right\} \Rightarrow Y \sim N(A\mu + B, A\Sigma A^T)$$

$$\left.\begin{array}{l} X_1 \sim N(\mu_1,\Sigma_1) \\ X_2 \sim N(\mu_2,\Sigma_2) \end{array}\right\} \Rightarrow p(X_1)\cdot p(X_2) \sim N\left(\frac{\Sigma_2}{\Sigma_1 + \Sigma_2}\mu_1 + \frac{\Sigma_1}{\Sigma_1 + \Sigma_2}\mu_2, \quad \frac{1}{\Sigma_1^{-1} + \Sigma_2^{-1}}\right)$$

- We stay in the "Gaussian world" as long as we start with Gaussians and perform only linear transformations

Introduction to Kalman Filter (1)



- Two measurements no dynamics

$$\hat{q}_1 = q_1 \text{ with variance } \sigma_1^2$$

$$\hat{q}_2 = q_2 \text{ with variance } \sigma_2^2$$

- Weighted least-square

$$S = \sum_{i=1}^{n} w_i (\hat{q} - q_i)^2$$

- Finding minimum error

$$\frac{\partial S}{\partial \hat{q}} = \frac{\partial}{\partial \hat{q}} \sum_{i=1}^{n} w_i (\hat{q} - q_i)^2 = 2 \sum_{i=1}^{n} w_i (\hat{q} - q_i) = 0$$

- After some calculation and rearrangements

$$\hat{q} = q_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}(q_2 - q_1)$$

- Another way to look at it – weighted mean

Discrete Kalman Filter

- Estimates the state x of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t$$ ← Process dynamics

- with a measurement

$$z_t = C_t x_t + \delta_t$$ ← Observation model

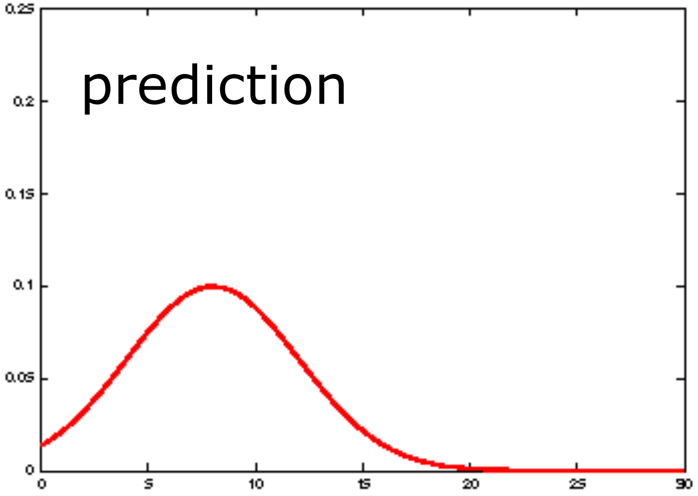
A_t Matrix (nxn) that describes how the state evolves from t - 1 to t without controls or noise.

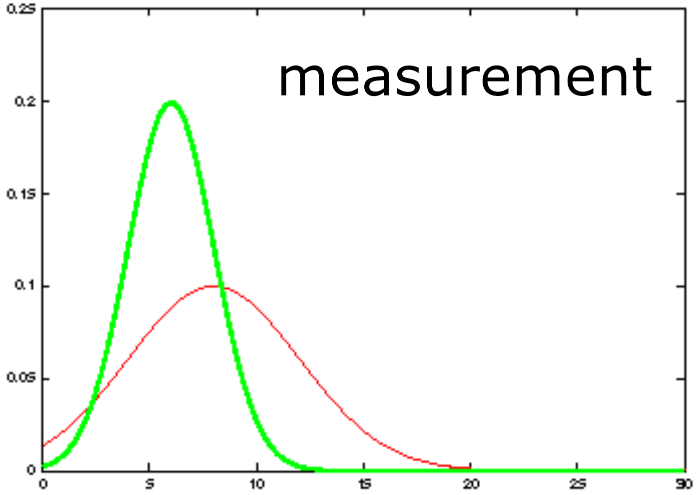
B_t Matrix (nxl) that describes how the control u_t changes the state from t-1 to t.

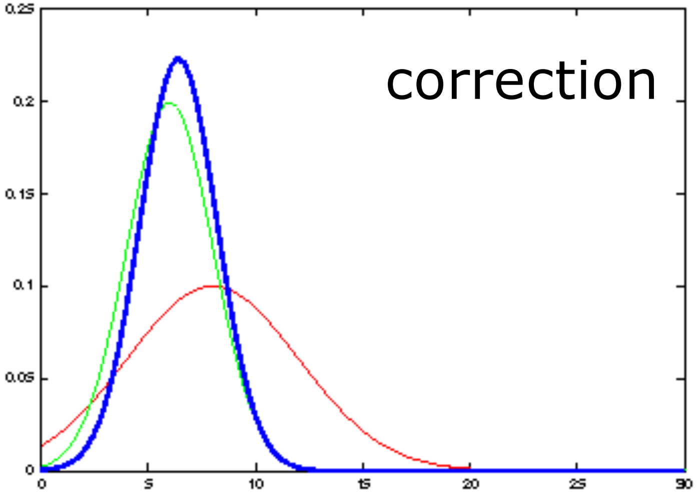
C_t Matrix (kxn) that describes how to map the state x_t to an observation z_t.

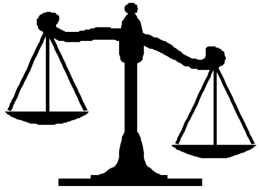
ε_t, δ_t Random variables representing the process and measurement noise that are assumed to be independent and normally distributed with covariance R_t and Q_t respectively.

Kalman Filter Updates in 1D









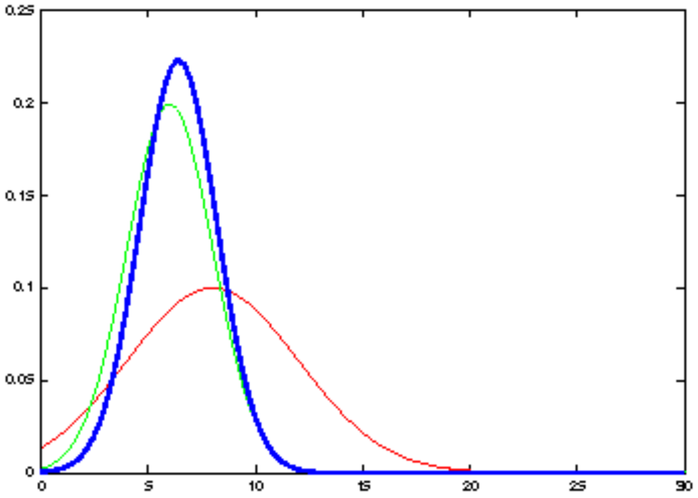
It's a weighted mean!

Kalman Filter Updates in 1D

gain innovation

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - \bar{\mu}_t) \\ \sigma_t^2 = (1 - K_t)\bar{\sigma}_t^2 \end{cases} \quad \text{with} \quad K_t = \frac{\bar{\sigma}_t^2}{\bar{\sigma}_t^2 + \bar{\sigma}_{obs,t}^2}$$

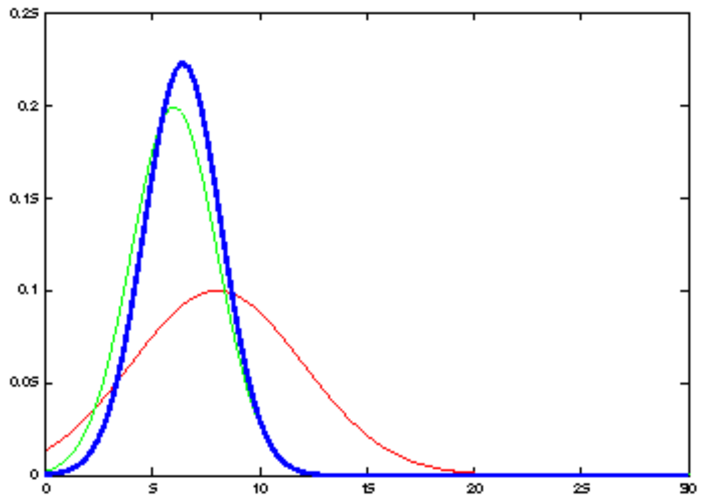
$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases} \quad \text{with} \quad K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

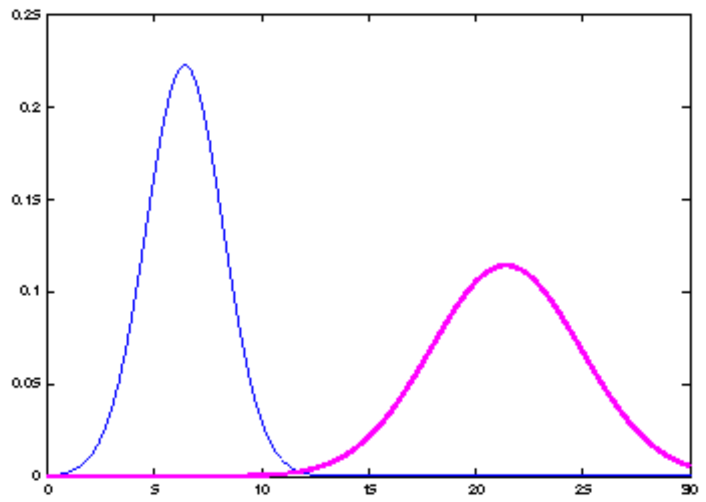


Kalman Filter Updates in 1D

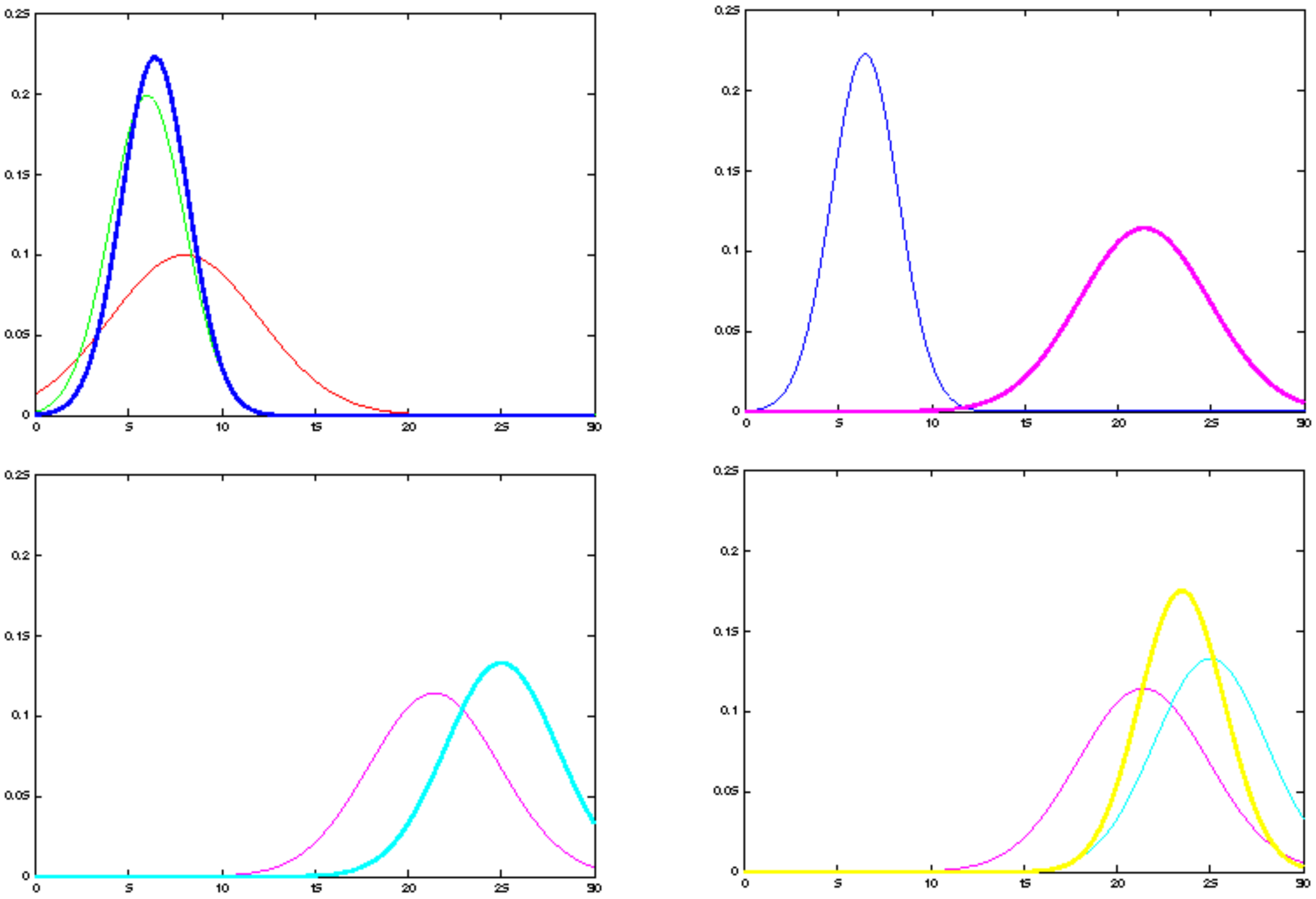
$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = a_t \mu_{t-1} + b_t u_t \\ \bar{\sigma}_t^2 = a_t^2 \sigma_t^2 + \sigma_{act,t}^2 \end{cases}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$





Kalman Filter Updates



Kalman Filter Algorithm

1. Algorithm **Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):

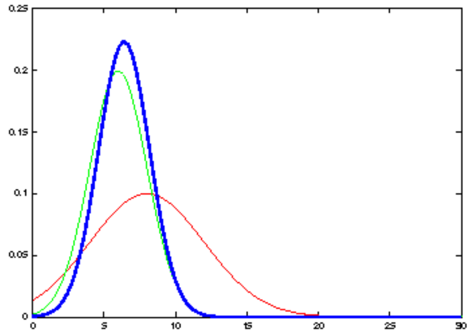
   Prediction:

2. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
3. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

   Correction:

4. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
5. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
6. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$
7. Return μ_t, Σ_t

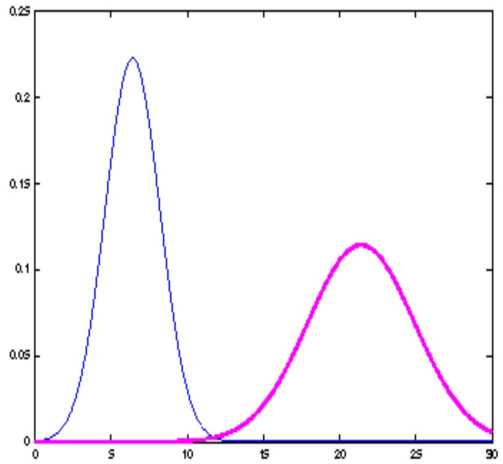
The Prediction-Correction-Cycle



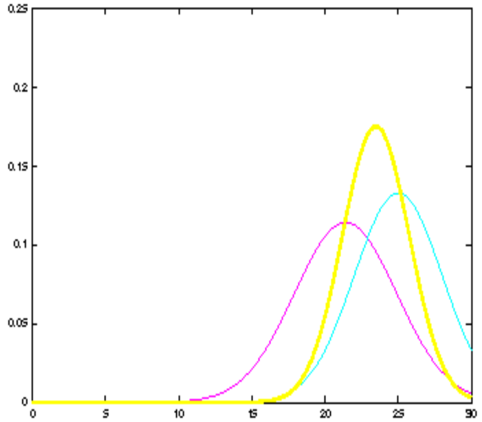
Prediction

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = a_t \mu_{t-1} + b_t u_t \\ \bar{\sigma}_t^2 = a_t^2 \sigma_t^2 + \sigma_{act,t}^2 \end{cases}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$

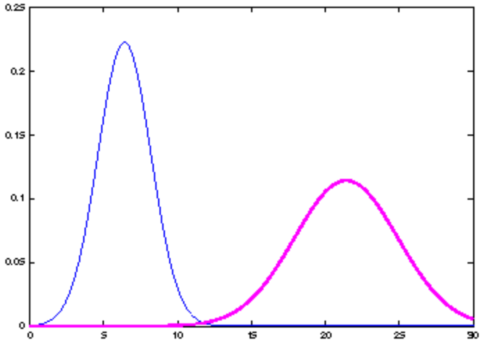


The Prediction-Correction-Cycle



$$bel(x_t) = \begin{cases} \mu_t = \mu_t \quad K_t(z_t \quad \mu_t) \\ \sigma_t^2 = (1 \quad K_t)\sigma_t^2 \end{cases}, K_t = \frac{\bar{\sigma}_t^2}{\bar{\sigma}_t^2 + \bar{\sigma}_{obs,t}^2}$$

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases}, K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{\ 1}$$



The Prediction-Correction-Cycle

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - \bar{\mu}_t) \\ \sigma_t^2 = (1 - K_t)\bar{\sigma}_t^2 \end{cases}, K_t = \frac{\bar{\sigma}_t^2}{\bar{\sigma}_t^2 + \bar{\sigma}_{obs,t}^2}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = a_t \mu_{t-1} + b_t u_t \\ \bar{\sigma}_t^2 = a_t^2 \sigma_t^2 + \sigma_{act,t}^2 \end{cases}$$

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - C_t \bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases}, K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$

Kalman Filter Summary

- Highly efficient: Polynomial in measurement dimensionality k and state dimensionality n:

$$O(k^{2.376} + n^2)$$

- Optimal for linear Gaussian systems!
- Most robotics systems are nonlinear!