



上海科技大学
ShanghaiTech University

CS283: Robotics Fall 2020: Sensors & Perception

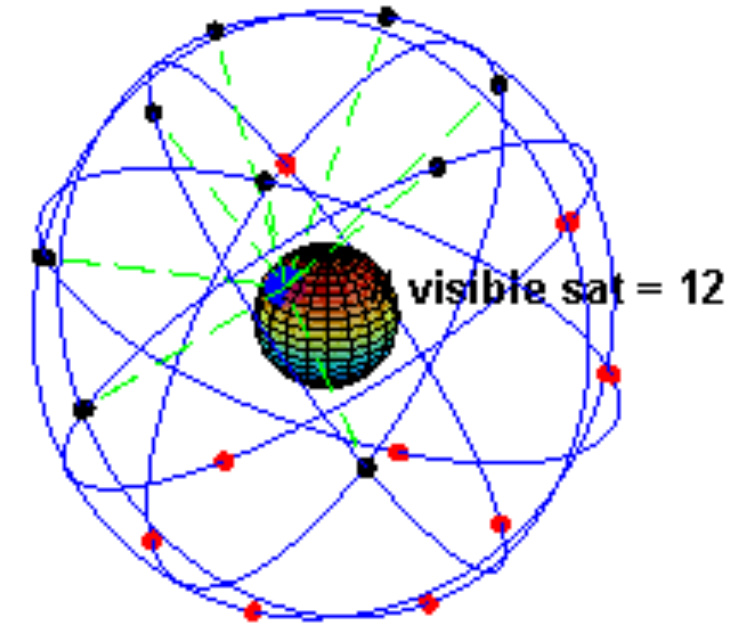
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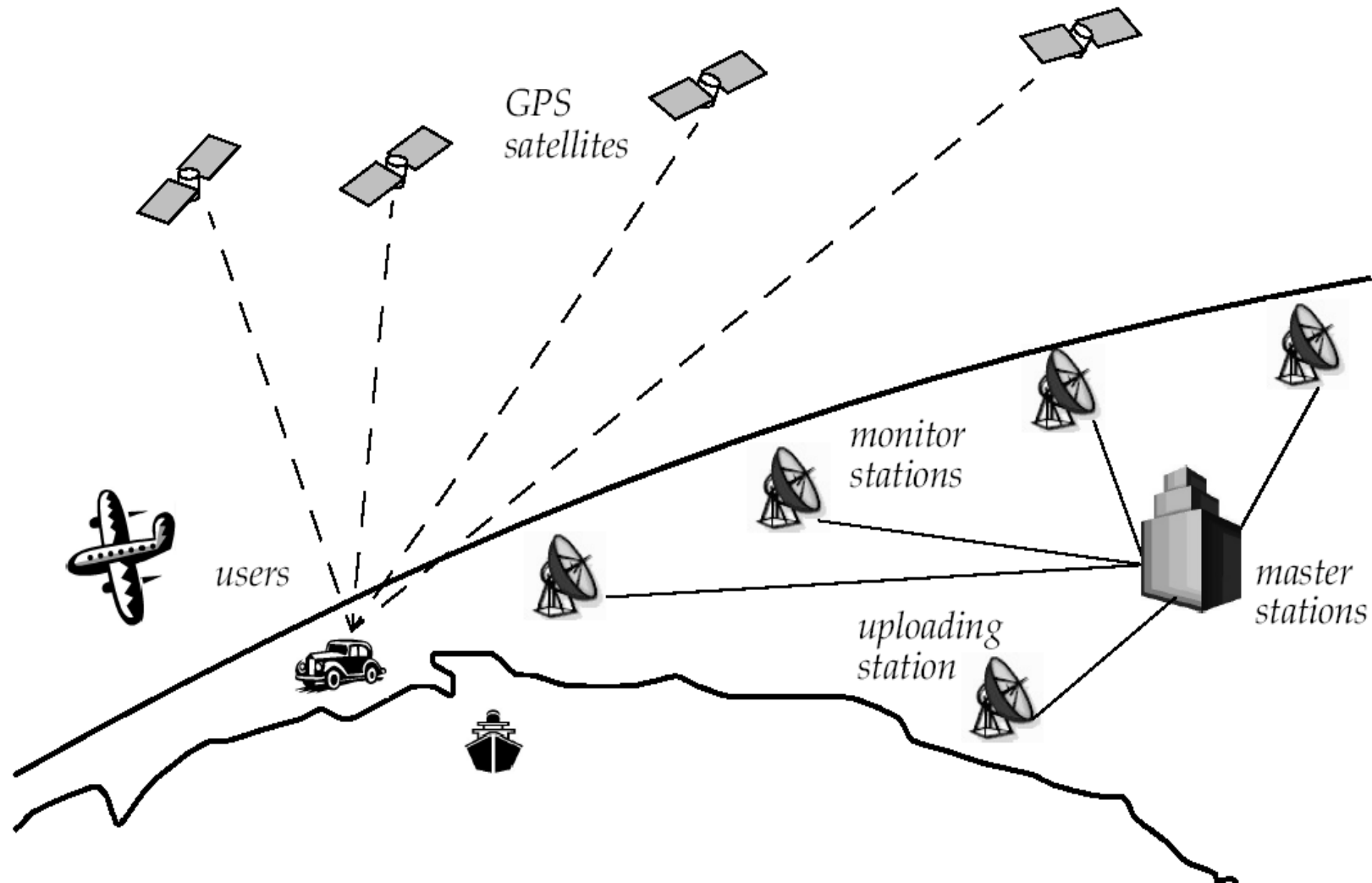
SENSORS CONTINUED

Global Positioning System (GPS)

- Developed for military use
- 1995 it became accessible for commercial applications
- 24 satellites orbiting the earth every 12 hours at a height of 20.190 km.
- 4 satellites are located in each of 6 orbits with 60 degrees orientation between each other. The orbital planes do not rotate with respect to stars. Orbits arranged so that at least 6 satellites are always within line of sight from any point on Earth's surface.
- From 2008: 32 satellites to improve localization accuracy through redundancy
- Location of any GPS receiver is determined through a time of flight measurement (satellites send orbital location (*ephemeris*) plus time; the receiver computes its location through **trilateration** and **time correction**)
- Technical challenges:
 - Time synchronization between the individual satellites and the GPS receiver
 - Real time update of the exact location of the satellites
 - Precise measurement of the time of flight
 - Interferences with other signals



Global Positioning System (GPS) (2)



Global Positioning System (GPS) (3)

- Time synchronization:
 - atomic clocks on each satellite
 - monitoring them from different ground stations.
- Ultra-precision time synchronization is extremely important
 - electromagnetic radiation propagates at light speed
- Roughly 0.3 m per nanosecond
 - position accuracy proportional to precision of time measurement
- Real time update of the exact location of the satellites:
 - monitoring the satellites from a number of widely distributed ground stations
 - master station analyses all the measurements and transmits the actual position to each of the satellites
- Exact measurement of the time of flight
 - the receiver correlates a pseudocode with the same code coming from the satellite
 - The delay time for best correlation represents the time of flight.
 - quartz clock on the GPS receivers are not very precise
 - the range measurement with four satellite allows to identify the three values (x, y, z) for the position and the clock correction ΔT
- Recent commercial GPS receiver devices allows position accuracies down to a couple meters.

GPS Error Sources

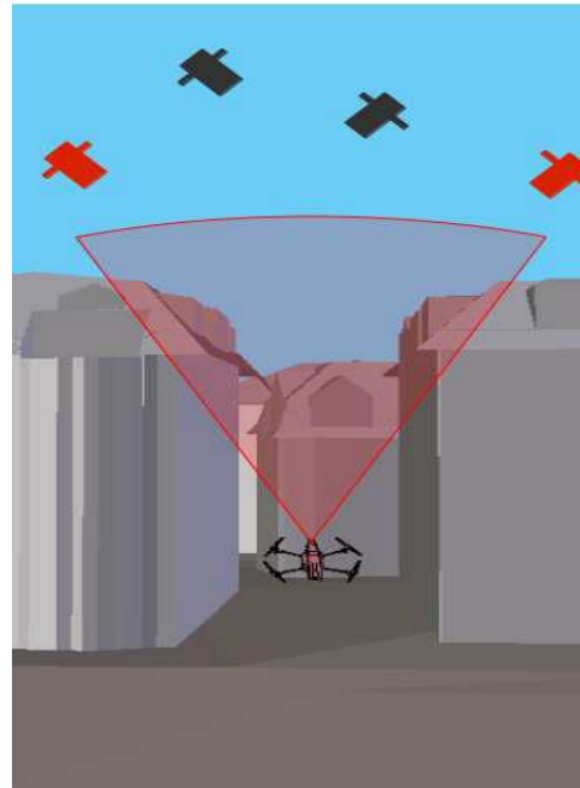
Satellite clock errors uncorrected by monitor stations may result in one meter errors:

- Ephemeris data errors: 1 meter
- Tropospheric delays: 1 meter.
 - The troposphere is the lower part (ground level to from 8 to 13 km) of the atmosphere that experiences the changes in temperature, pressure, and humidity associated with weather changes. Complex models of tropospheric delay require estimates or measurements of these parameters.
- Unmodeled ionosphere delays: 10 meters.
 - The ionosphere is the layer of the atmosphere from 50 to 500 km that consists of ionized air. The transmitted model can only remove about half of the possible 70 ns of delay leaving a ten meter unmodeled residual.
- Number of satellites under line of sight

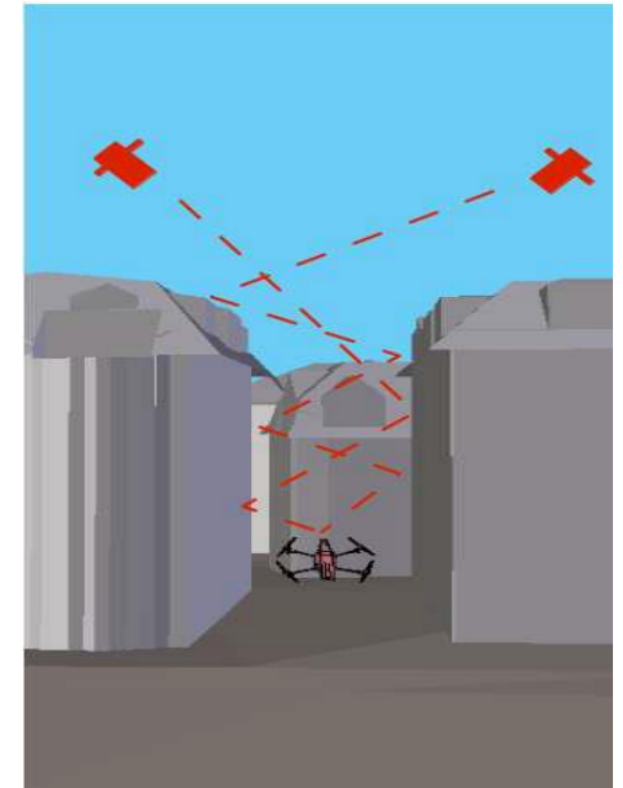
GPS Multipath

- Multipath: 0.5 meters
 - Multipath is caused by reflected signals from surfaces near the receiver that can either interfere with or be mistaken for the signal that follows the straight line path from the satellite. Multipath is difficult to detect and sometime hard to avoid.

Satellite coverage

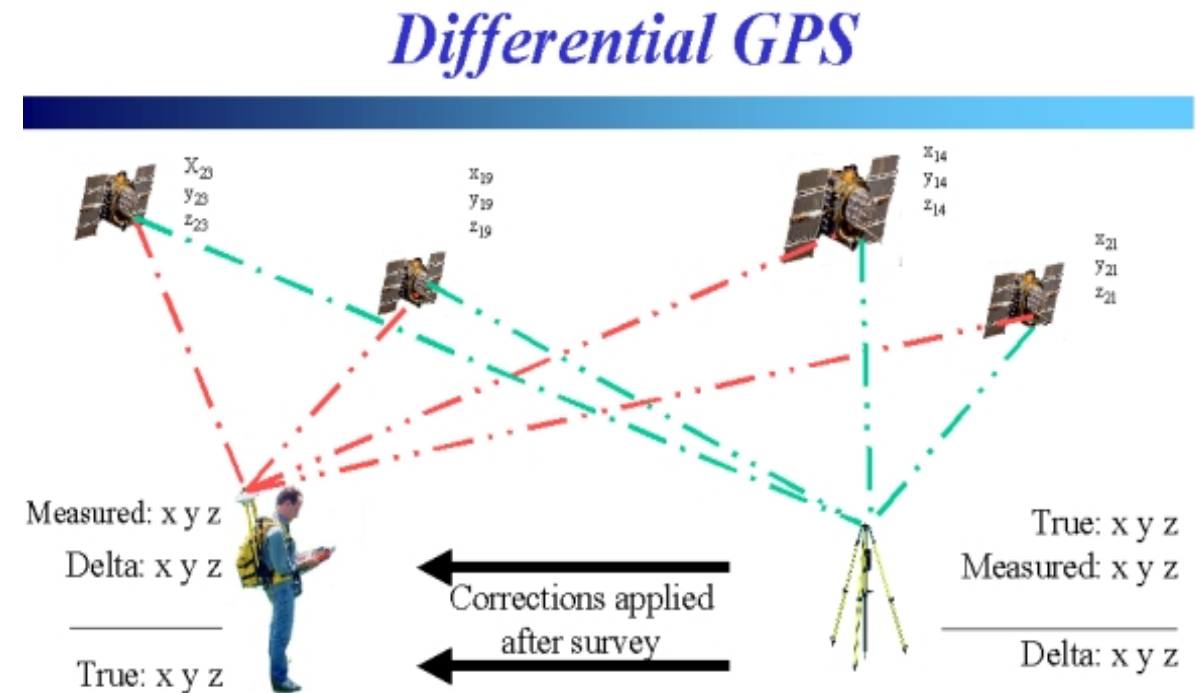


Multipath problem



Differential Global Positioning System (dGPS)

- Base station GPS receiver: set up on a precisely known location
- Base station receiver calculates its position based on satellite signals
- Compares this location to the known location
- Difference is applied to the GPS data recorded by the mobile GPS receivers
- Position accuracies in sub-meter to cm range



Other Global Positioning Systems

- GLONASS
 - Russian GPS – developed since 1976
 - Full global coverage as of 2011 (24 satellites)
- Galileo
 - European GPS – initiated 2003
 - 22 operational satellites in orbit
- IRNSS (Indian Regional Navigation Satellite System)
 - Initiated 2010
 - 8 satellites for Indian Coverage in orbit
 - Full operation
- BeiDou Navigation Satellite System 北斗卫星导航系统
 - Chinese GPS – developed since 2003
 - BeiDou Satellite Navigation System (BDS)
 - 2011 full China coverage – 2020 global coverage
 - 37 satellites system

RANGE SENSING

Range Sensing

- Color/ gray scale cameras: do NOT measure the distance to the object
- Range sensing: get the distance to the object
- Basic principles:
 - Time of flight
 - Sound/ Ultrasound (in air, underwater)
 - Light (Based on Phase or based on time)
 - Single rotating laser beam (LRF; e.g. Sick)
 - Multiple rotating laser beams (3D LRF; e.g. Velodyne)
 - Solid state laser (e.g. Intel RealSense L515)
 - LED light & imager (ToF camera, e.g. Kinect 2)
 - Radio Waves (Radar)
 - Projected Pattern
 - Single laser (Triangulation)
 - 2D pattern (e.g. Kinect 1)
 - Stereo Vision
 - Passive
 - Active with pattern (e.g. Intel RealSense D435)

RANGE SENSING: TIME OF FLIGHT

Range Sensors (time of flight) (1)

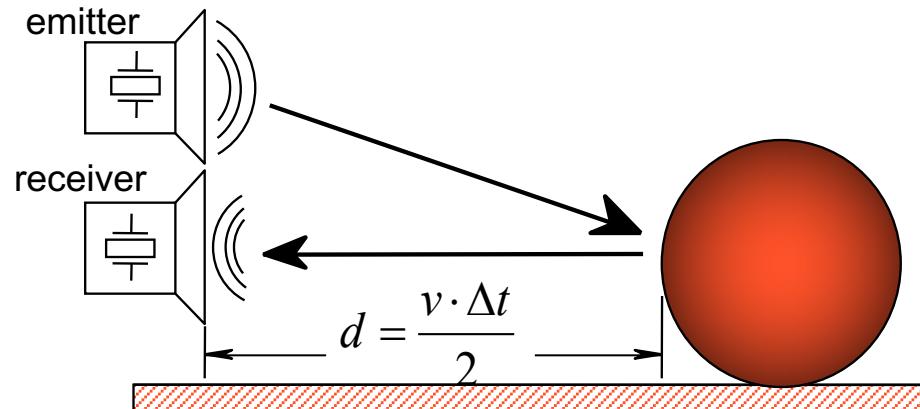
- Large range distance measurement -> called range sensors
- Range information:
 - key element for localization and environment modeling
- Ultrasonic sensors as well as laser range sensors make use of propagation speed of sound or electromagnetic waves respectively. The traveled distance of a sound or electromagnetic wave is given by
- Where
 - d = distance traveled (usually round-trip)
 - c = speed of wave propagation
 - t = time of flight.

$$d = c \cdot t$$

Range Sensors (time of flight) (2)

- It is important to point out
 - Propagation speed v of sound in air:
0.3 m/ms - 300 m/s
 - Propagation speed v of sound in water:
1.5 m/ms - 1,500 m/s
 - Propagation speed v of electromagnetic signals: 0.3 m/ns,
 - one million times faster.
 - 3 meters
 - is 10 ms for an ultrasonic system
 - only 10 ns for a laser range sensor
 - time of flight with electromagnetic signals is not an easy task
 - laser range sensors expensive and delicate
- The quality of time of flight range sensors mainly depends on:
 - Inaccuracies in the time of flight measure (laser range sensors)
 - Opening angle of transmitted beam (especially ultrasonic range sensors)
 - Interaction with the target (surface, specular reflections)
 - Variation of propagation speed (sound)
 - Speed of mobile robot and target (if not at stand still)

Factsheet: Ultrasonic Range Sensor (1)



1. Operational Principle

An ultrasonic pulse is generated by a piezo-electric emitter, reflected by an object in its path, and sensed by a piezo-electric receiver. Based on the speed of sound in air and the elapsed time from emission to reception, the distance between the sensor and the object is easily calculated.

2. Main Characteristics

- Precision influenced by angle to object (as illustrated on the next slide)
- Useful in ranges from several cm to several meters
- Typically relatively inexpensive

3. Applications

- Distance measurement (also for transparent surfaces)
- Collision detection

Ultrasonic Sensor (time of flight, sound) (1)

- transmit a packet of (ultrasonic) pressure waves
- distance d of the echoing object can be calculated based on the propagation speed of sound c and the time of flight t .

$$d = \frac{c \cdot t}{2}$$

- The speed of sound c (340 m/s) in air is given by $\sqrt{\gamma \cdot R \cdot T}$

Where

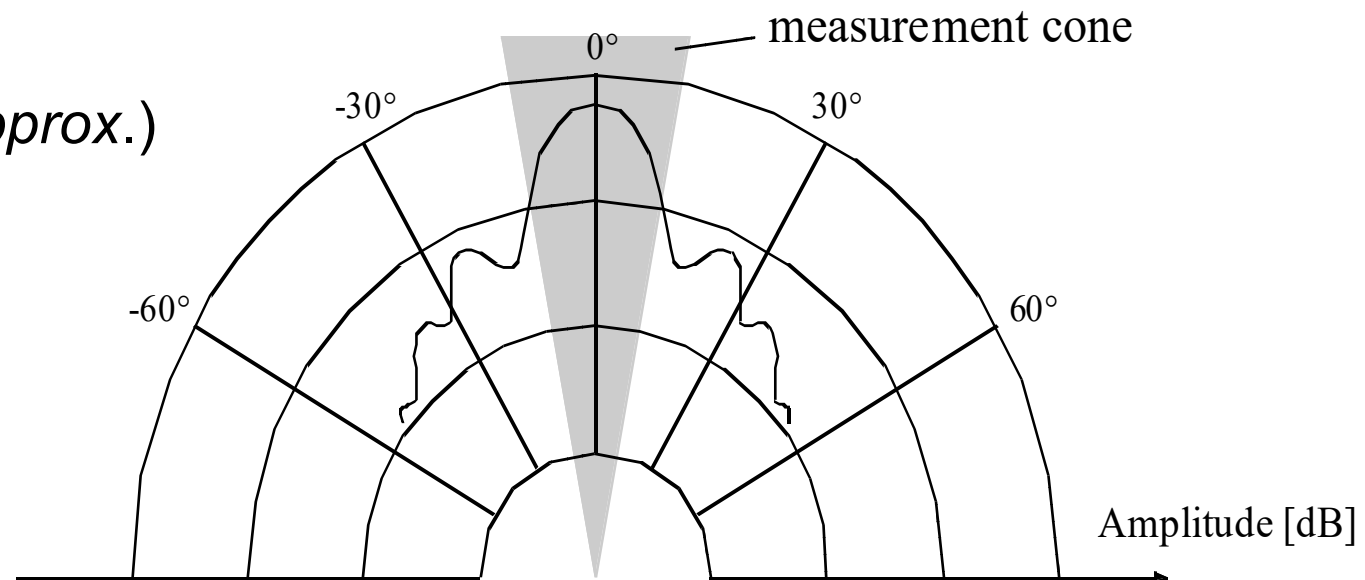
γ : adiabatic index (isentropic expansion factor) - ratio of specific heats of a gas

R : gas constant

T : temperature in degree Kelvin

Ultrasonic Sensor (time of flight, sound) (2)

- typical frequency: 40kHz - 180 kHz
 - Lower frequencies correspond to longer range
- generation of sound wave: piezo transducer
 - transmitter and receiver separated or not separated
- Range between 12 cm up to 5 m
- Resolution of ~ 2 cm
- Accuracy 98% \Rightarrow relative error 2%
- sound beam propagates in a cone (*approx.*)
 - opening angles around 20 to 40 degrees
 - regions of constant depth
 - segments of an arc (sphere for 3D)



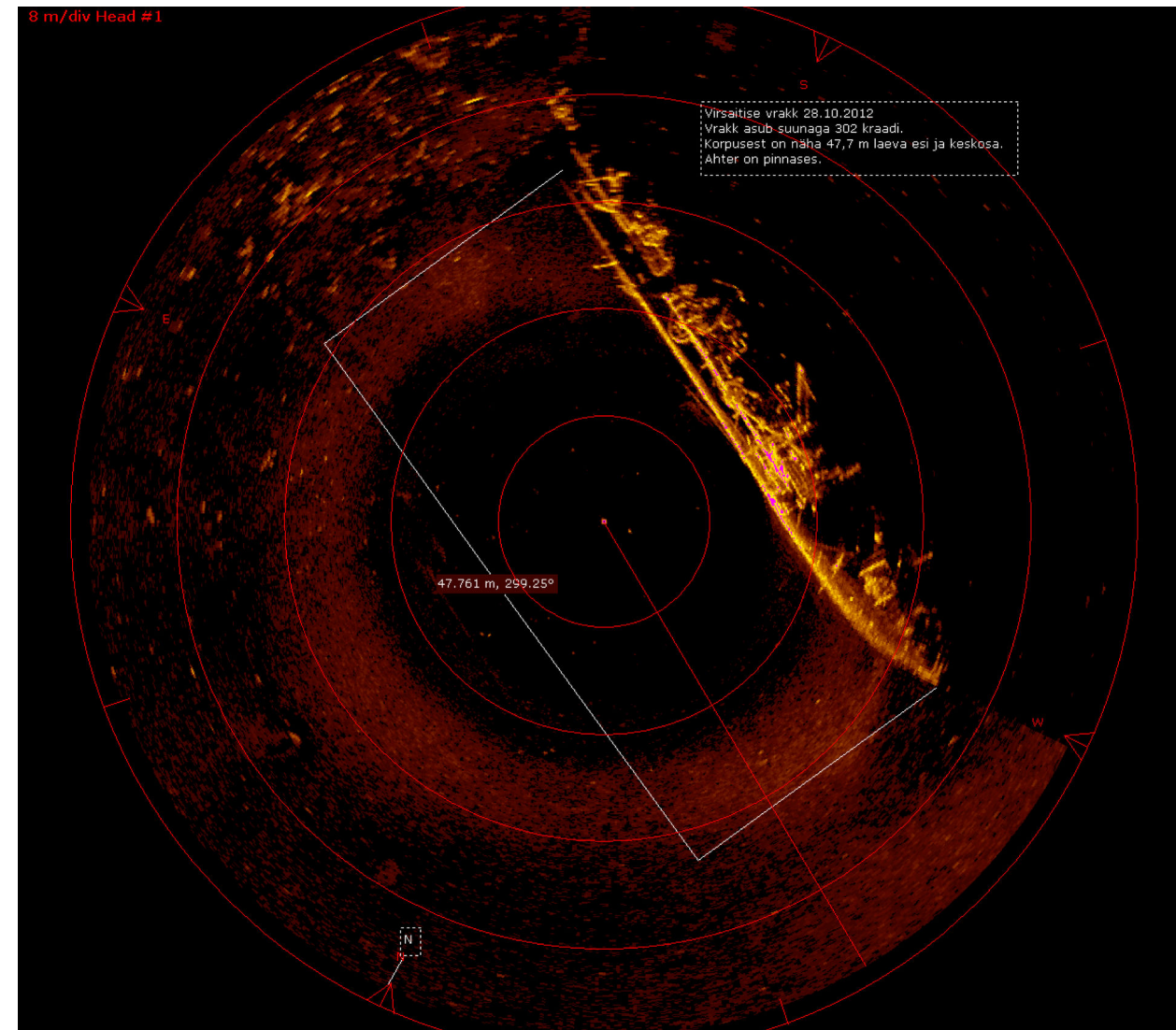
Typical intensity distribution of a ultrasonic sensor

Ultrasonic Sensor (time of flight, sound) (4)

- Bandwidth
 - measuring the distance to an object that is 3 m away will take such a sensor 20 ms, limiting its operating speed to 50 Hz. But if the robot has a ring of 20 ultrasonic sensors, each firing sequentially and measuring to minimize interference between the sensors, then the ring's cycle time becomes 0.4 seconds => frequency of each one sensor = 2.5 Hz.
 - This update rate can have a measurable impact on the maximum speed possible while still sensing and avoiding obstacles safely.

Underwater Sonar

- Light visibility very low => often sonar the only/ best sensor available
- Types:
 - Sonar
 - Side-scanning
 - Synthetic aperture sonar
- Problems:
 - Absorption
 - Reflections:
 - Layers of different water temperature
 - Layers of different salinity
 - ...



Laser Range Sensor (time of flight, electromagnetic)

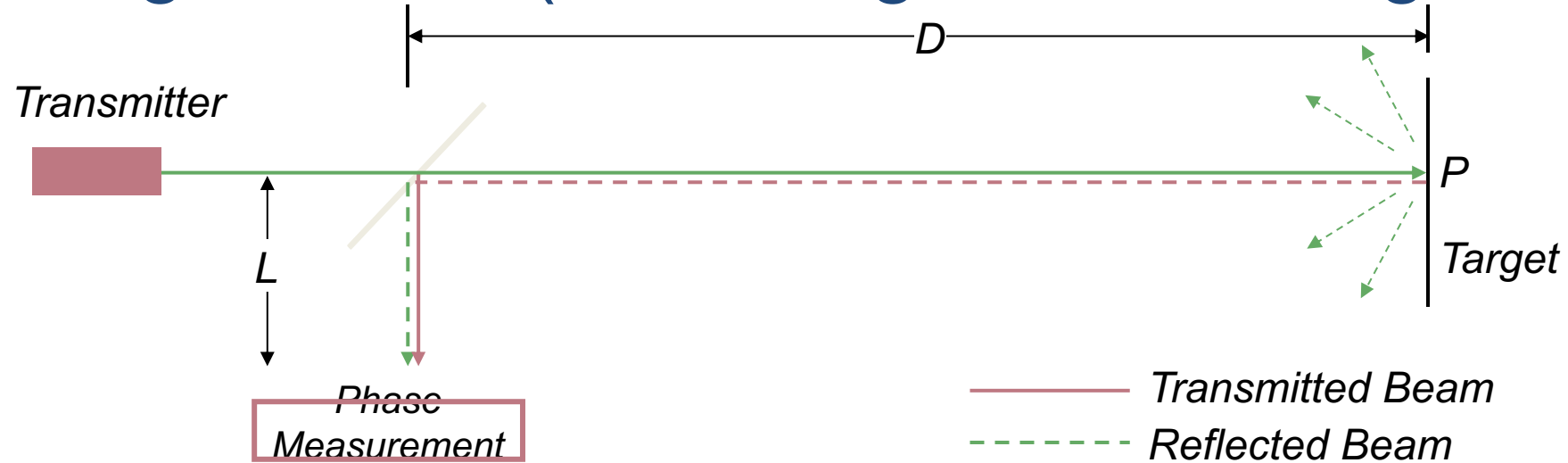
- Is called Laser range finder or Lidar (Light Detection And Ranging)



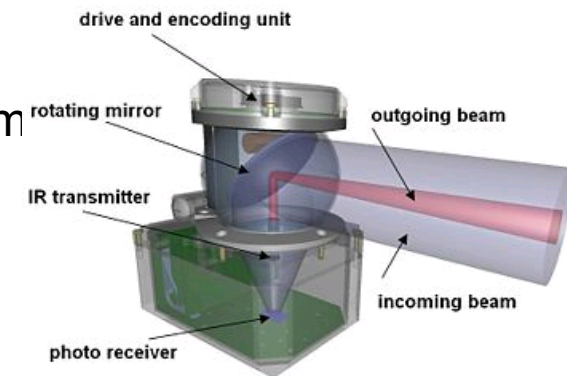
Autonomous Driving will rely heavily on range sensing =>
Many 3D range sensing companies emerge!

E.g. RoboSense (China)

Laser Range Sensor (time of flight, electromagnetic) (1)



- Transmitted and received beams coaxial
- Transmitter illuminates a target with a collimated laser beam
- Receiver detects the time needed for round-trip
- A mechanical mechanism with a mirror sweeps
 - 2D or 3D measurement



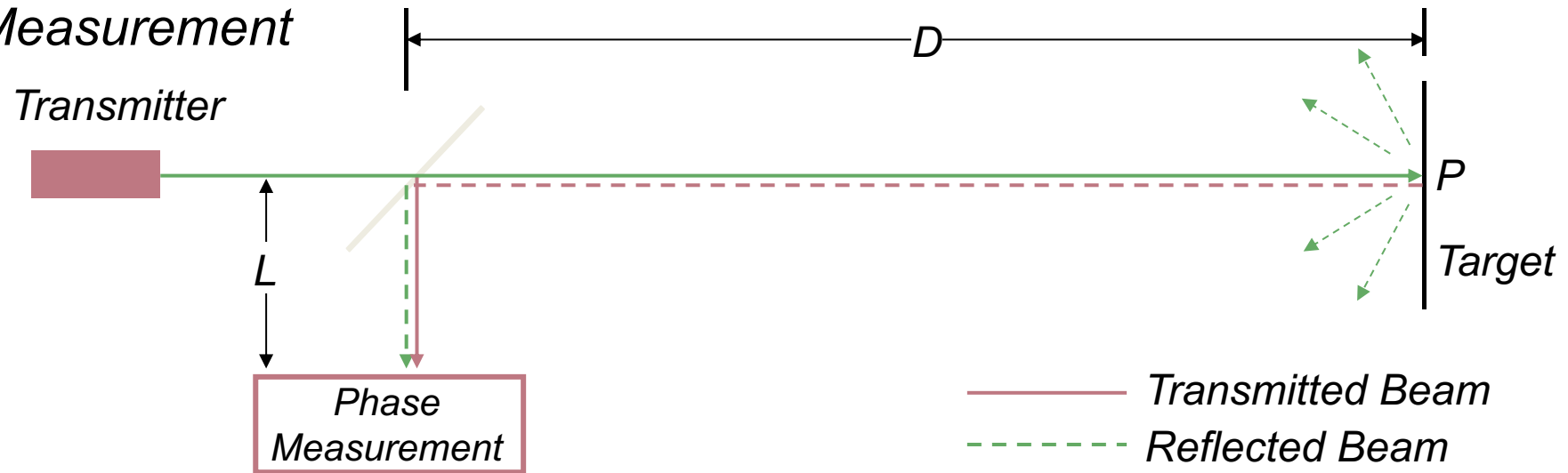
Laser Range Sensor (time of flight, electromagnetic) (2)

Time of flight measurement

- Pulsed laser (today the standard)
 - measurement of elapsed time directly
 - resolving picoseconds
- Phase shift measurement to produce range estimation
 - technically easier than the above method
- (3D) Laser Scanner == Lidar (Light detection and ranging)

Laser Range Sensor (time of flight, electromagnetic) (3)

- *Phase-Shift Measurement*



$$D' = L + 2D = L + \frac{\theta}{2\pi} \lambda$$

$$\lambda = \frac{c}{f}$$

Where:

c : is the speed of light; f the modulating frequency; D' the distance covered by the emitted light.

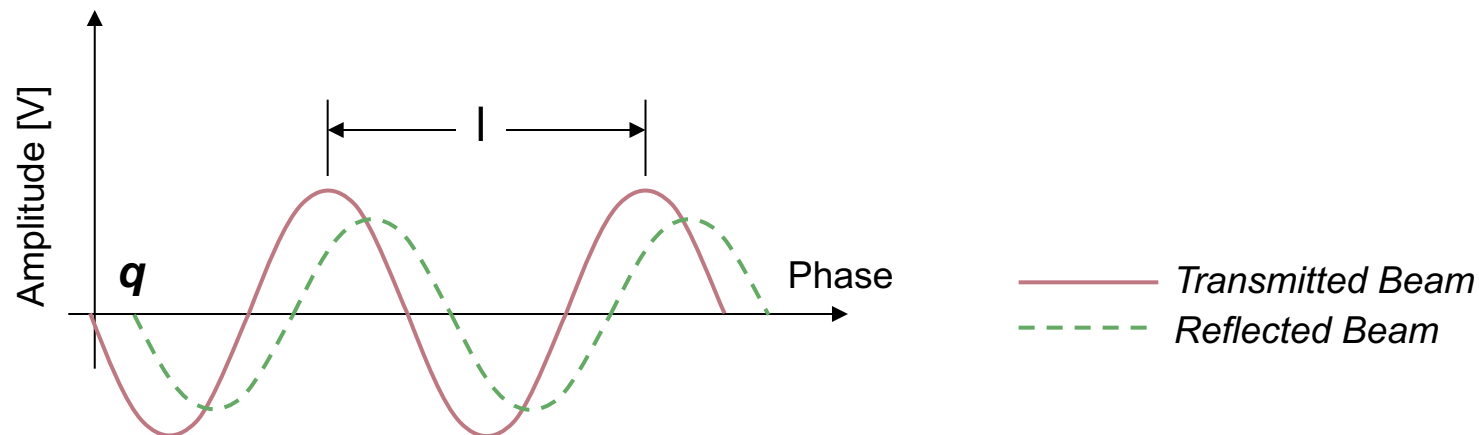
- for $f = 5 \text{ MHz}$ $\lambda = 60 \text{ meters}$

Laser Range Sensor (time of flight, electromagnetic) (4)

- Distance D , between the beam splitter and the target

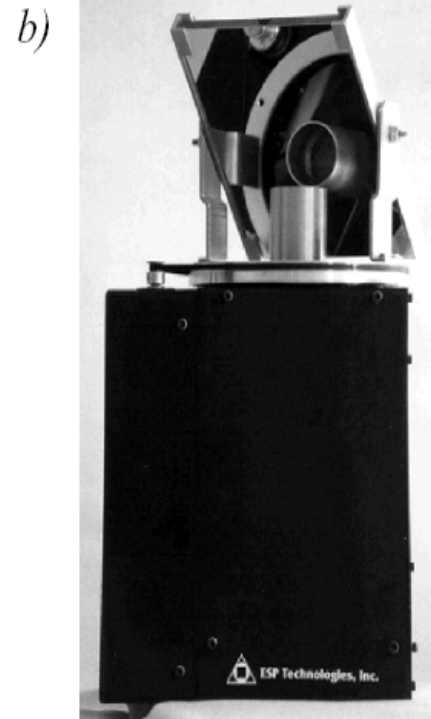
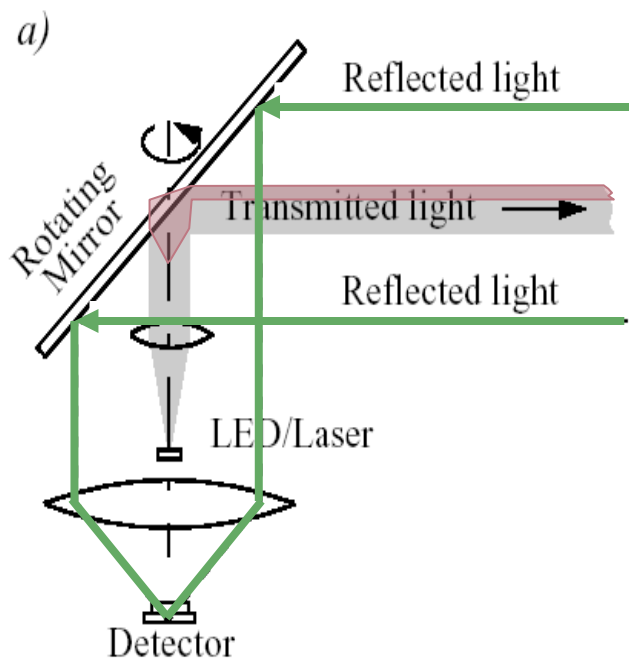
$$D = \frac{\lambda}{4\pi} \theta$$

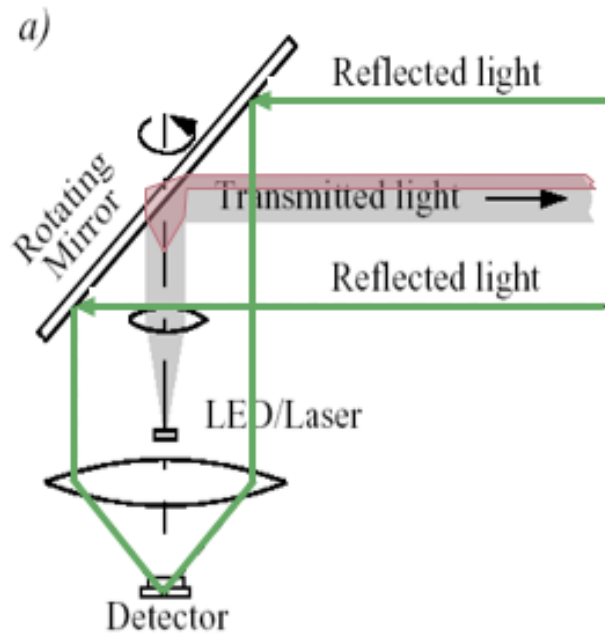
- where
 - θ : phase difference between transmitted and reflected beam
- Theoretically ambiguous range estimates
 - since for example if $\lambda = 60$ meters, a target at a range of 5 meters = target at 35 meters



Laser Range Sensor (time of flight, electromagnetic) (5)

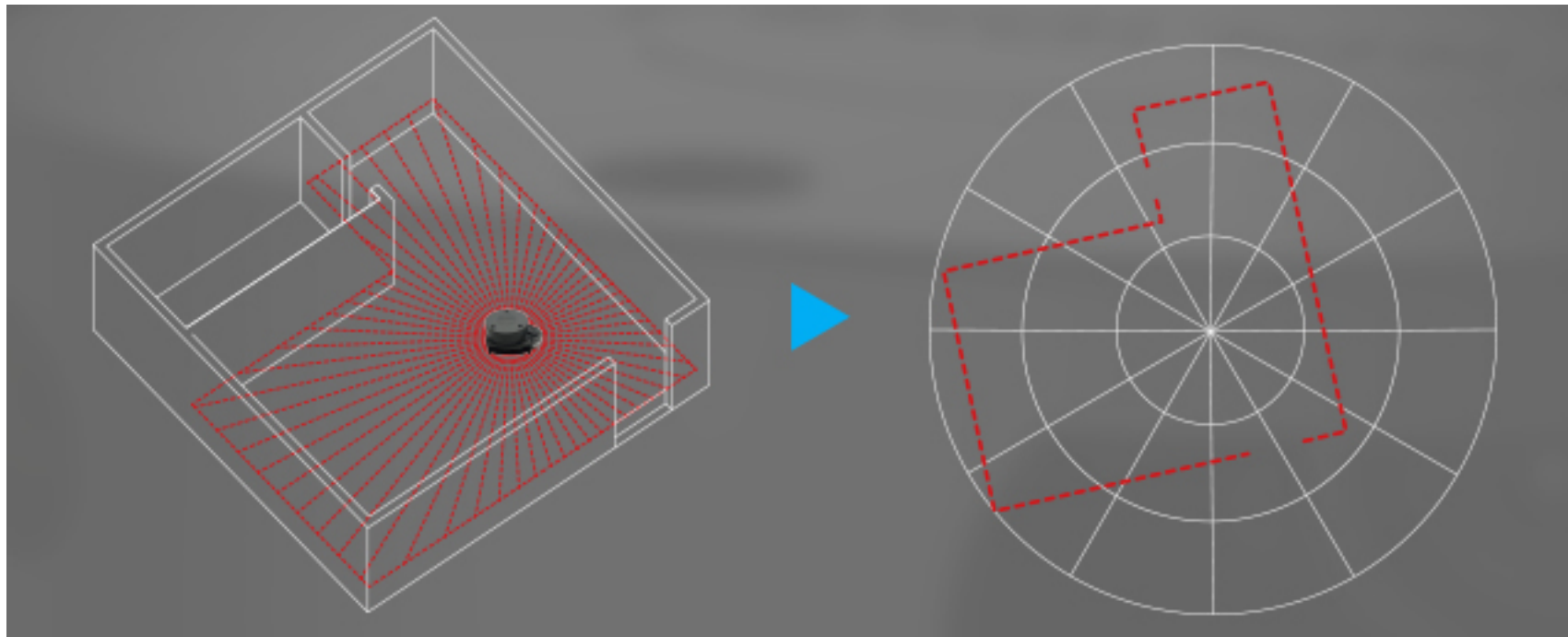
- Uncertainty of the range (phase/time estimate) is inversely proportional to the square of the received signal amplitude.
 - Hence dark, distant objects will not produce such good range estimated as closer brighter objects ...





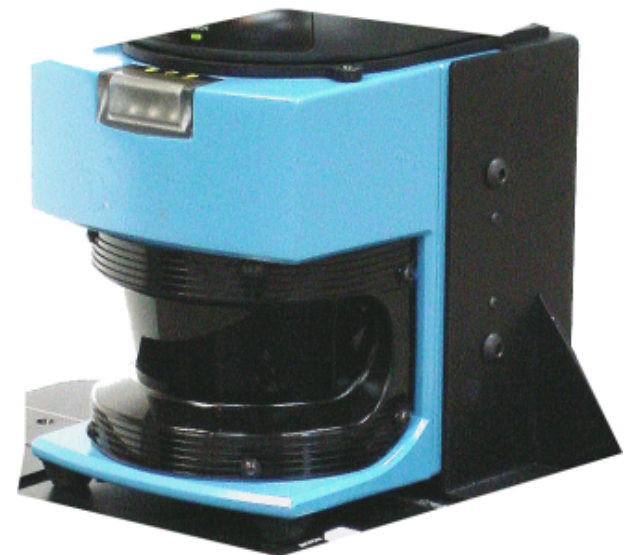
Laser Range Sensor (time of flight, electromagnetic)

- Typical range image of a 2D laser range sensor with a rotating mirror



The SICK LMS 200 Laser Scanner

- Angular resolution 0.25 deg
- Depth resolution ranges between 10 and 15 mm and the typical accuracy is 35 mm, over a range from 5 cm up to 20 m or more (up to 80 m)
 - depending on the reflectivity of the object being ranged.
- This device performs seventy five 180-degrees per sec.
- Dimensions: W155 x D155 x H210mm,
- Weight: 1,2 kg
- Cost: about RMB 22,000



Hokuyo UTM-30LX

- Long Detection range: 30m
- 0.1 to 10m: $\pm 30\text{mm}$, 1
- 0 to 30m: $\pm 50\text{mm}^*1$
- Field of View: 270°
- 40Hz
- Outdoor Environment
- Dimensions: 60 x 60 x 87 mm
- Weight: 370g
- Cost: about 35,000 RMB



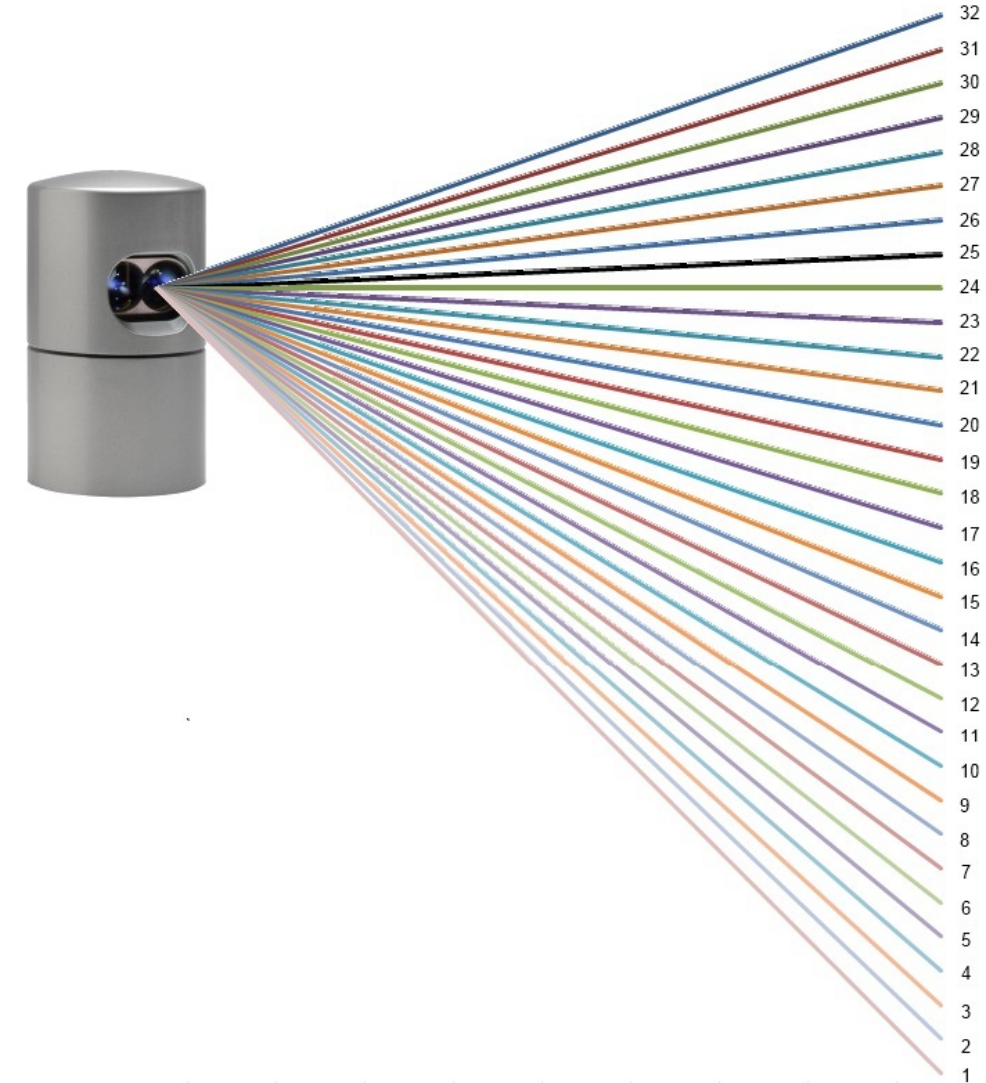
URG-04LX-UG01

- Low-power consumption (2.5W)
- Wide-range (5600mm×240°).
- 60 to 1,000mm : $\pm 30\text{mm}$,
- 1,000 to 4,095mm : $\pm 3\%$ of measurement
- 10Hz
- Dimensions: 50 x 50 x 70 mm
- Weight: 160g
- Cost: about 6,500 RMB



Velodyne hdl-32e

- 32 beams
- Range: up to 80 – 100 m
- +10.67 to -30.67 degrees field of view (vertical)
- 360° field of view (horizontal)
- 10 Hz frame rate
- Accuracy: <2 cm (one sigma at 25 m)
- Angular resolution (vertical) 1.33°
- 700,000 points per second
- internal MEMS accelerometers and gyros for six-axis motion correction
- Dimensions:
 - Diameter: 85mm,
 - Height: 144 mm
- Weight: 1kg
- Cost: about 220,000 RMB

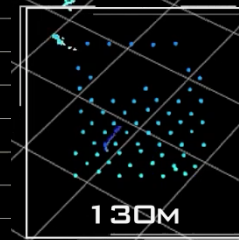
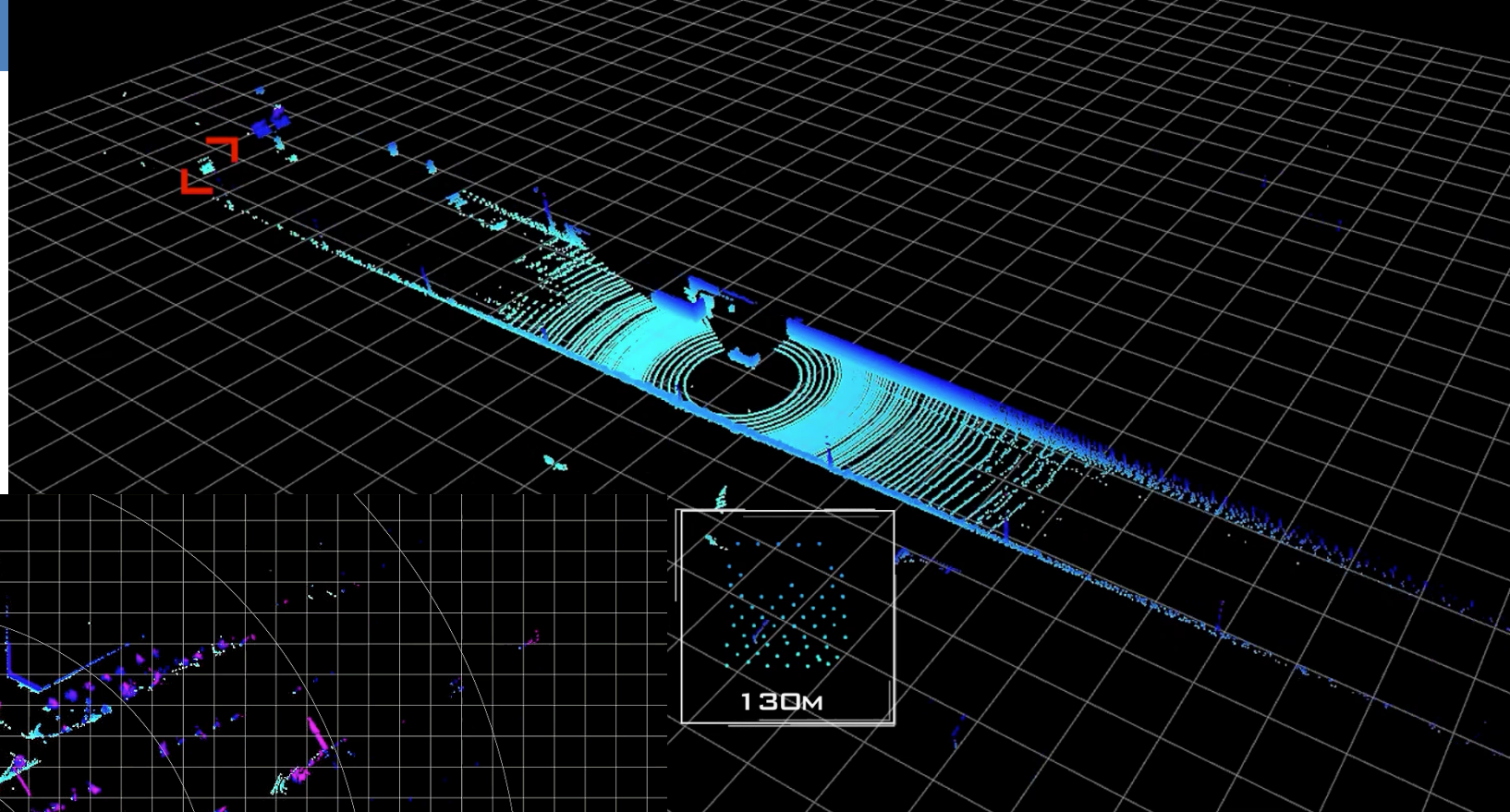
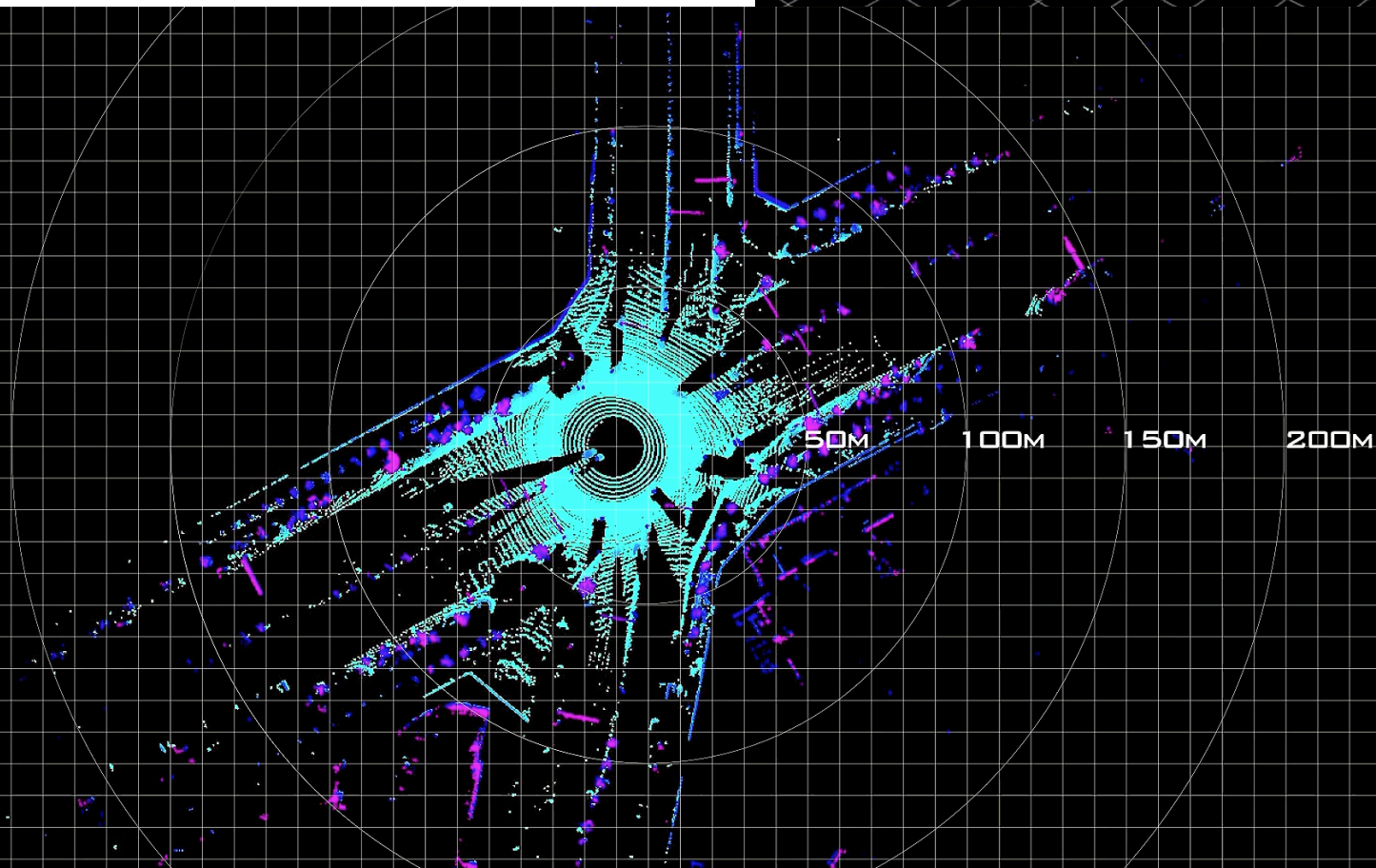


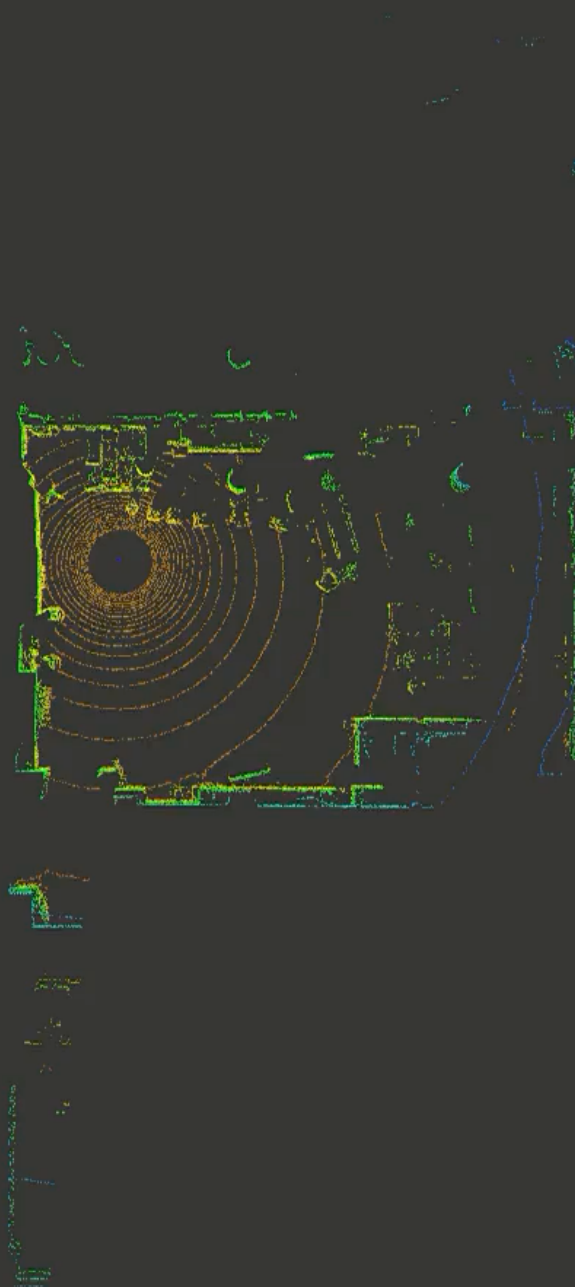
RoboSense Ruby

- 128 beams
- Range: 250m (200m@10% NIST)
- Range Accuracy (Typical): Up to $\pm 3\text{cm}$
- Vertical FOV: 40°
- Horizontal Resolution: $0.1^\circ / 0.2^\circ / 0.4^\circ$
- Vertical Resolution: Up to 0.1°
- Frame Rate: 5Hz/10Hz/20Hz
- Points Per Second: 2,304,000pts/s (Single return Mode)
- Points Per Second: 4,608,000pts/s (Dual return Mode)
- Operating Voltage: 19V - 32V
- Power Consumption: 45W
- Weight (without cabling): $\sim 3.75\text{ kg}$
- Cost: about RMB 500,000



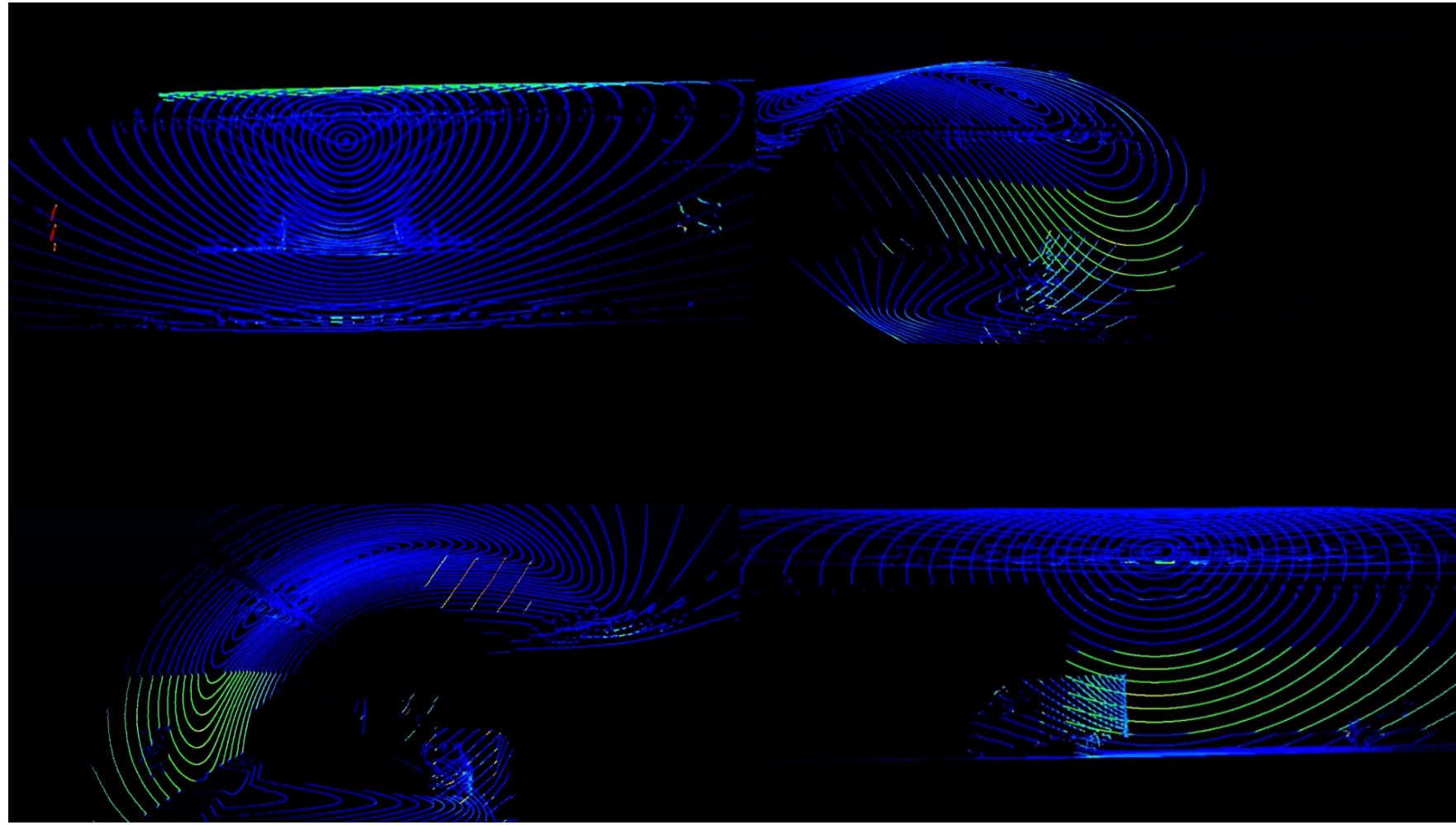
RoboSense Ruby on Road





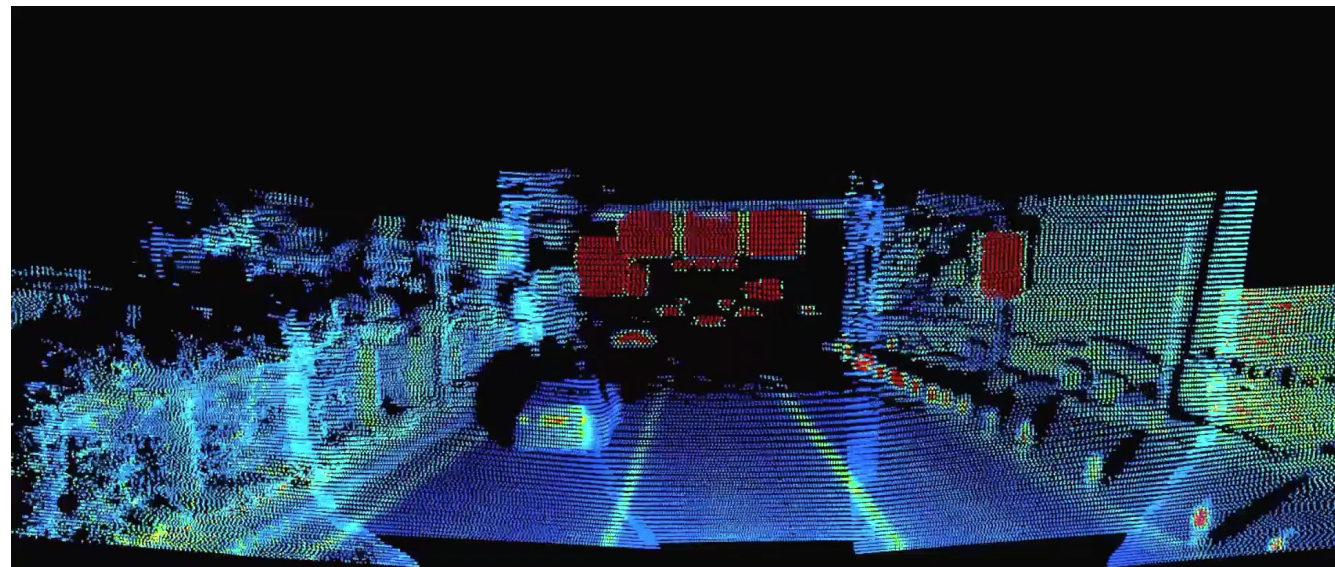
RoboSense Bpearl

- Hemispherical Lidar
- 32 beams
- Range: 100m (30m@10% NIST)
- Range Accuracy (Typical): Up to $\pm 3\text{cm}$
- Frame Rate: 10Hz/20Hz
- Points Per Second: 576,000pts/s (Single return Mode)
- Points Per Second: 1,152,000pts/s (Dual return Mode)
- Operating Voltage: 9V - 32V
- Power Consumption: 13W
- Weight: $\sim 0.92\text{ kg}$



RoboSense Lidar M1

- Solid state Lidar (300 x 125 pixel)
- Accuracy: Up to $\pm 5\text{cm}$
- Range: 150m on 10% NIST
- Vertical FOV: 25° ($-12.5^\circ \sim +12.5^\circ$)
- Vertical angular resolution: 0.2°
- Horizontal FOV: 120° ($-60.0^\circ \sim +60.0^\circ$)
- Horizontal angular resolution: 0.2°
- Refresh Rate: 15 Hz
- Data rate: 1,125,000pts/s (single return)
- Power consumption: 25w
- Weight: $\sim 800\text{g}$



Intel RealSense L515

- 9m distance
- Depth: 1024 x 786 pixel @ 30Hz => 23mill pts per second
- RGB: 1920 x 1080 @ 30Hz
- Depth FOV: 70° x 55° ($\pm 2^\circ$)
- Weight: 100g
- With IMU
- Solid state laser with RGB camera
- Sensitive to ambient infrared light =>
Does NOT work outdoors!



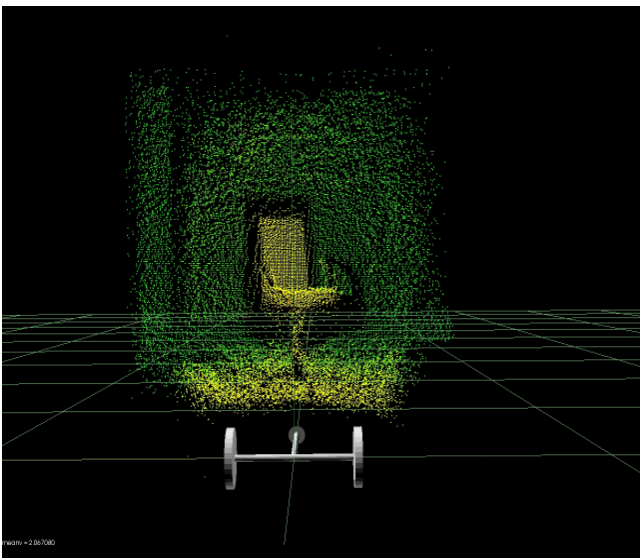
Cubemos skeletal tracking with
the Intel® RealSense™ LiDAR Camera L515

3D Range Sensor: Time Of Flight (TOF) camera

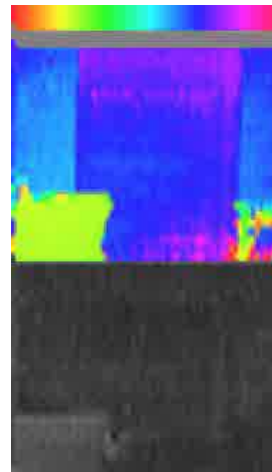
- 3D information with high data rate (100 Hz)
- Compact and easy to manage
- High, non-uniform measurement noise
- High outlier rate at jump edges
- Wrap-around error (phase ranging)
- Sensitive to ambient infrared light =>
Does NOT work outdoors!



- Kinect 2
- Resolution 1920x1080 pixels
- Field of view: 70 deg (H), 60 deg (V)
- Claimed accuracy: 1 mm
- Claimed max range: 6 meters

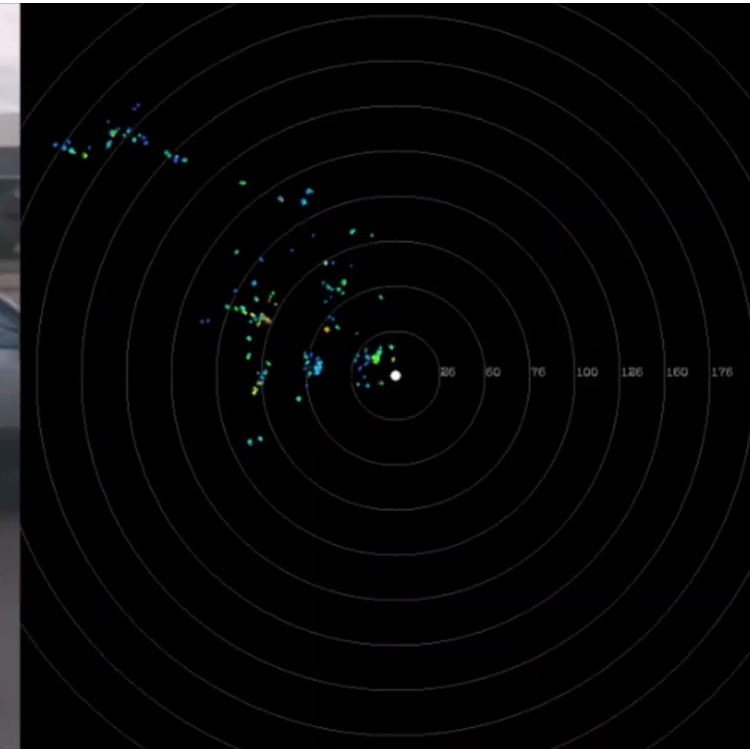


Swiss Ranger 3000
(produced by MESA)



Radar, 4D imaging radar (e.g. Oculii)

- Works in various weather and environment conditions:
 - fog, heavy rain, pitch darkness, air pollution
- High range (300+ meters)
- Capture doppler shifts (speed of other objects in a single scan) – this is the 4th dimension
- 250M+
- $<1^\circ$ Resolution
- 120° FOV

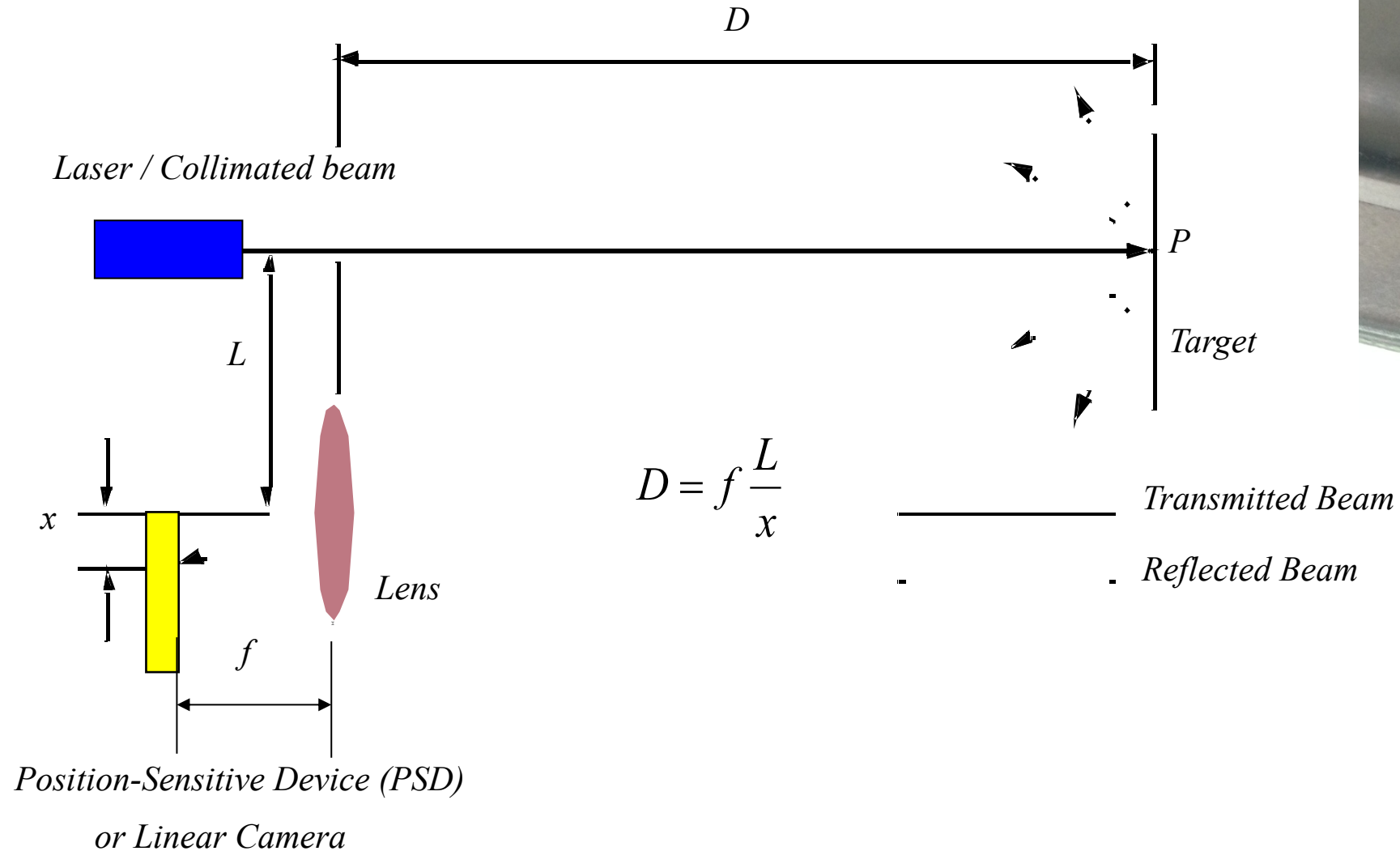


RANGE SENSING: PROJECTED PATTERN

Triangulation Ranging

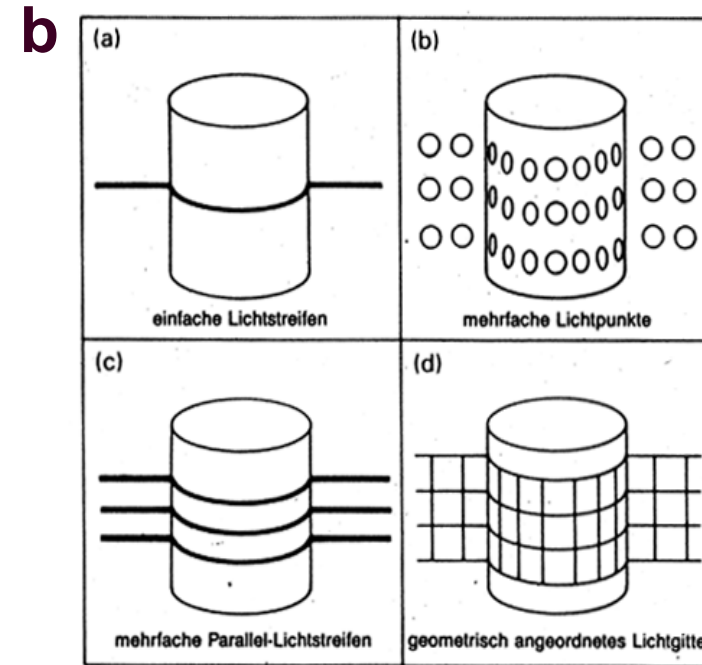
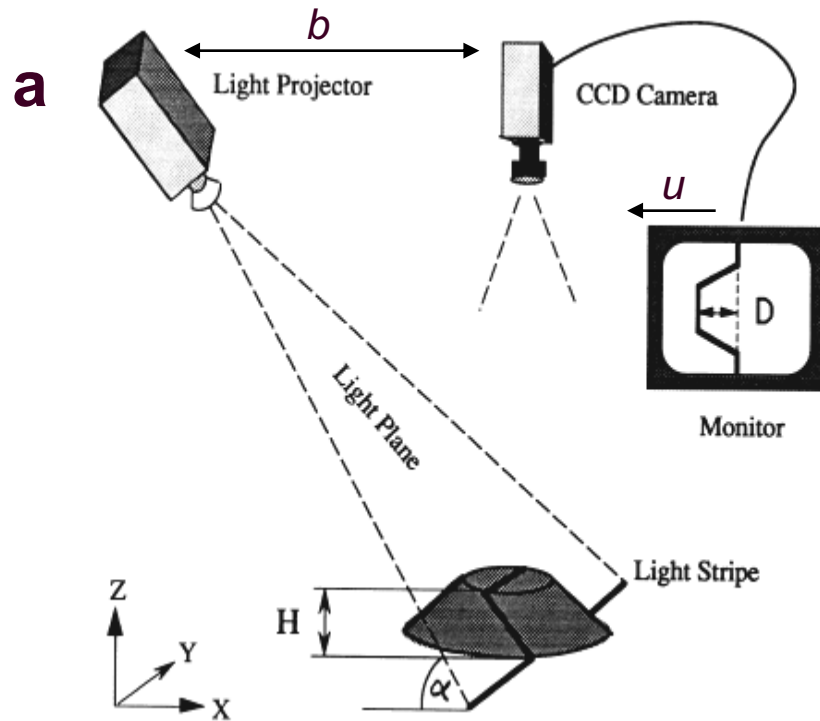
- Geometrical properties of the image to establish a distance measurement
- e.g. project a well defined light pattern (e.g. point, line) onto the environment.
 - reflected light is then captured by a photo-sensitive line or matrix (camera) sensor device
 - simple triangulation allows to establish a distance.
- e.g. size of an captured object is precisely known
 - triangulation without light projecting

Laser Triangulation (1D)



Cliff sensor on Turtle-bot

Structured Light (vision, 2 or 3D): Structured Light



- Eliminate the correspondence problem by projecting structured light on the scene.
- Slits of light or emit collimated light (possibly laser) by means of a rotating mirror.
- Light perceived by camera
- Range to an illuminated point can then be determined from simple geometry.

Structured Light (vision, 2 or 3D)

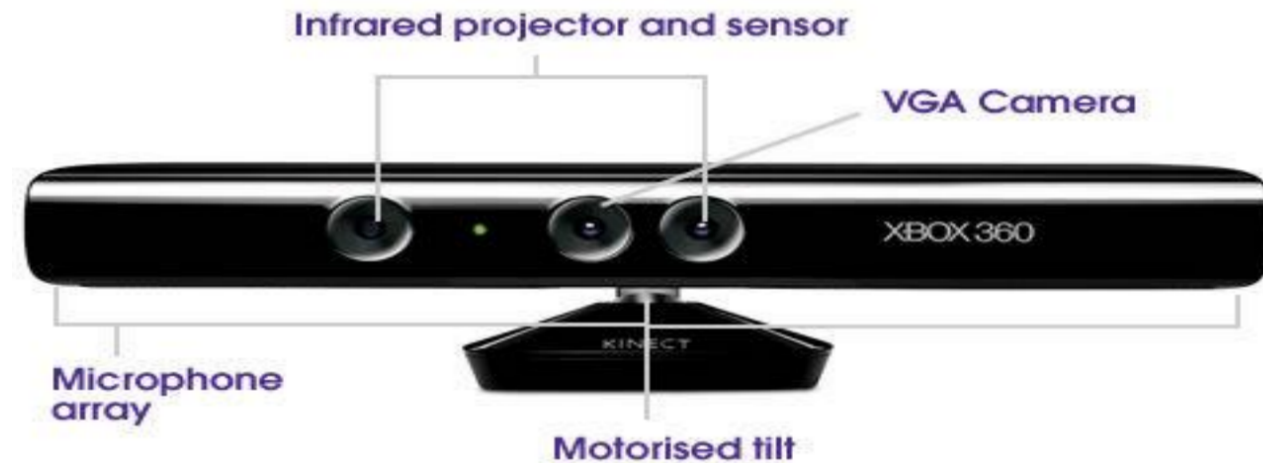
- Baseline length b :
 - the smaller b is the more compact the sensor can be.
 - the larger b is the better the range resolution is.

Note: for large b , the chance that an illuminated point is not visible to the receiver increases.

- Focal length f :
 - larger focal length f can provide
 - either a larger field of view
 - or an improved range resolution
 - however, large focal length means a larger sensor head

PrimeSense Cameras

- Devices: Microsoft Kinect and Asus Xtion
- Developed by Israeli company PrimeSense in 2010
- Components:
 - IR camera (640 x 480 pixel)
 - IR Laser projector
 - RGB camera (640 x 480 or 1280 x 1024)
 - Field of View (FoV):
 - 57.5 degrees horizontally,
 - 43.5 degrees vertically



IR Pattern

Sensitive to infrared light

=>

Does NOT work outdoors!



Depth Map



Microsoft Kinect: Depth Computation (1)

- **Depth from Stereo**

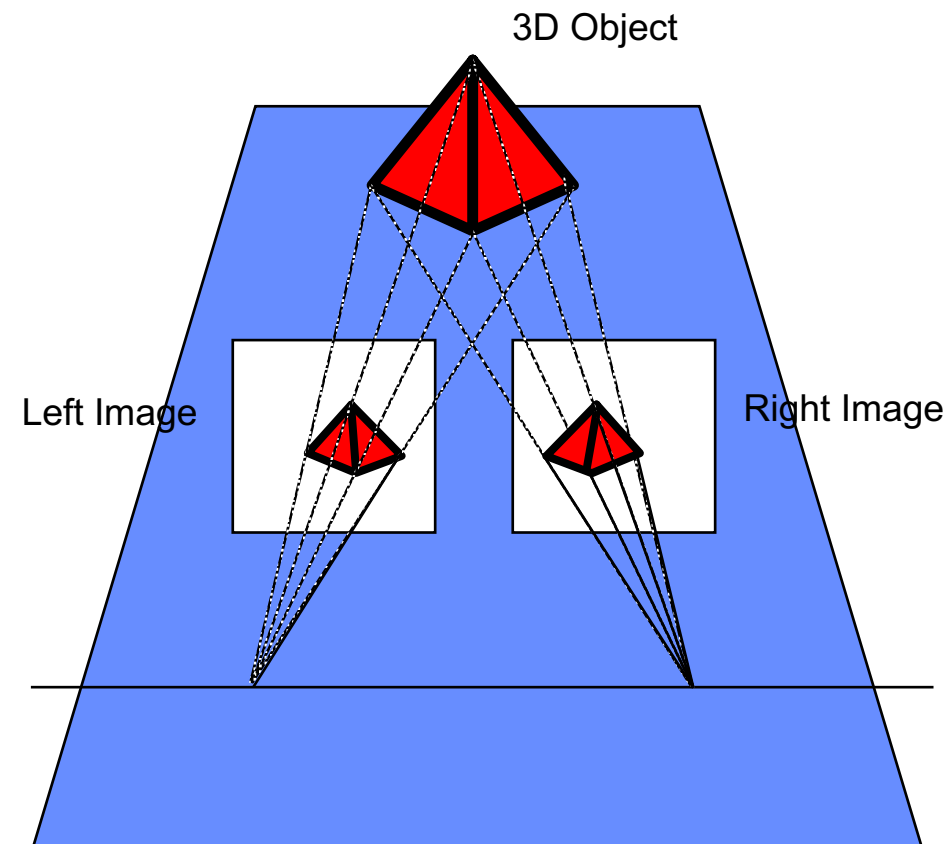
- The Kinect uses an infrared projector and an infrared sensor; it does not use its RGB camera for depth computation
- The technique of analyzing a known pattern is structured light
- The IR projector projects a pseudo-random pattern across the surface of the room.
- The direction of each speckle of the pattern is known (from pre calibration during manufacturing) and is hardcoded into the memory of the Kinect
- By measuring the position of each speckle in the IR image, its depth can be computed



RANGE SENSING: STEREO VISION

Stereo Cameras

- Theory will be covered in detail in Vision Lecture
- Estimate depth by using 2 cameras



Stereo example: ZED camera

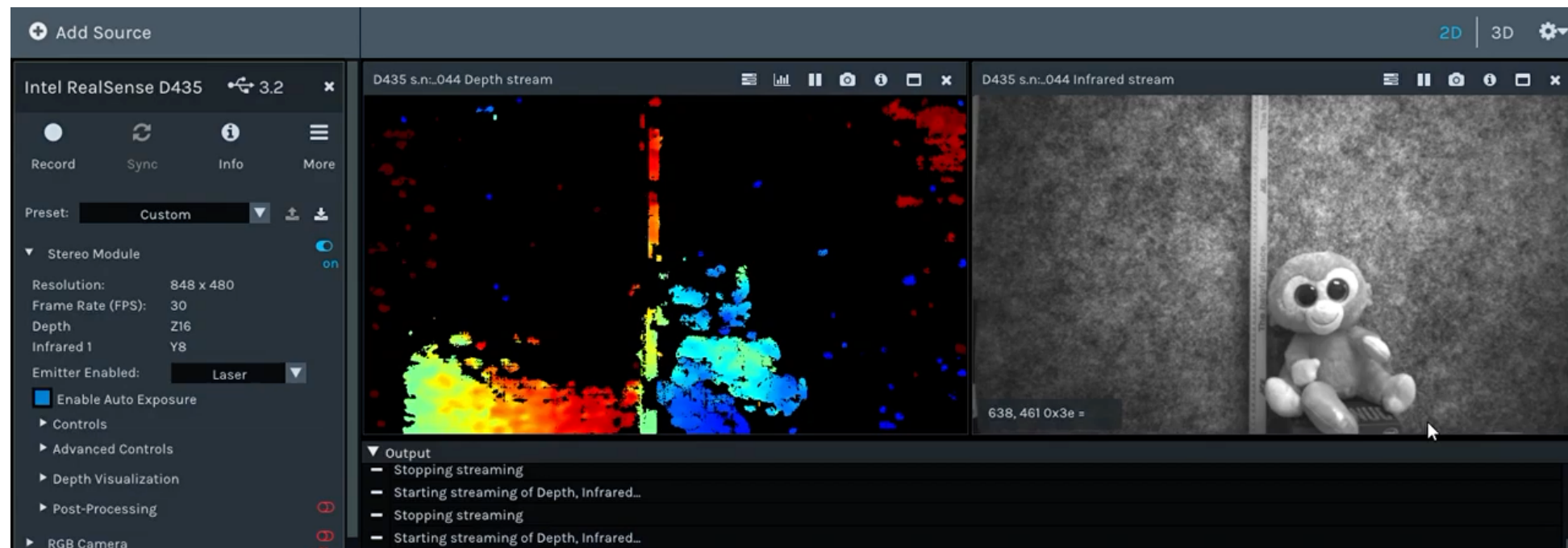


- Dual 4MP Camera @15Hz (lower resolution => higher fps)
- Up to 20m distance
- Passive Sensor
- Doesn't work on single color surfaces (e.g. white wall)!



Intel RealSense D435

- Stereo Infrared – works indoors and outdoors
- Active pattern (e.g. for white wall) – only works indoors!
- Depth resolution: 1280×720
- Depth Field of View (FOV): $86^\circ \times 57^\circ (\pm 3^\circ)$
- RGB: 1920×1080
- Small and lightweight
- With IMU

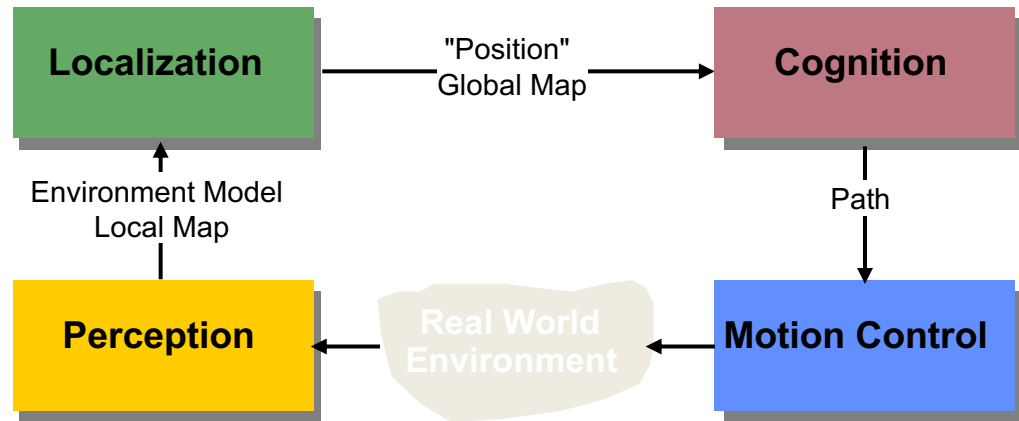


ADMIN

Admin

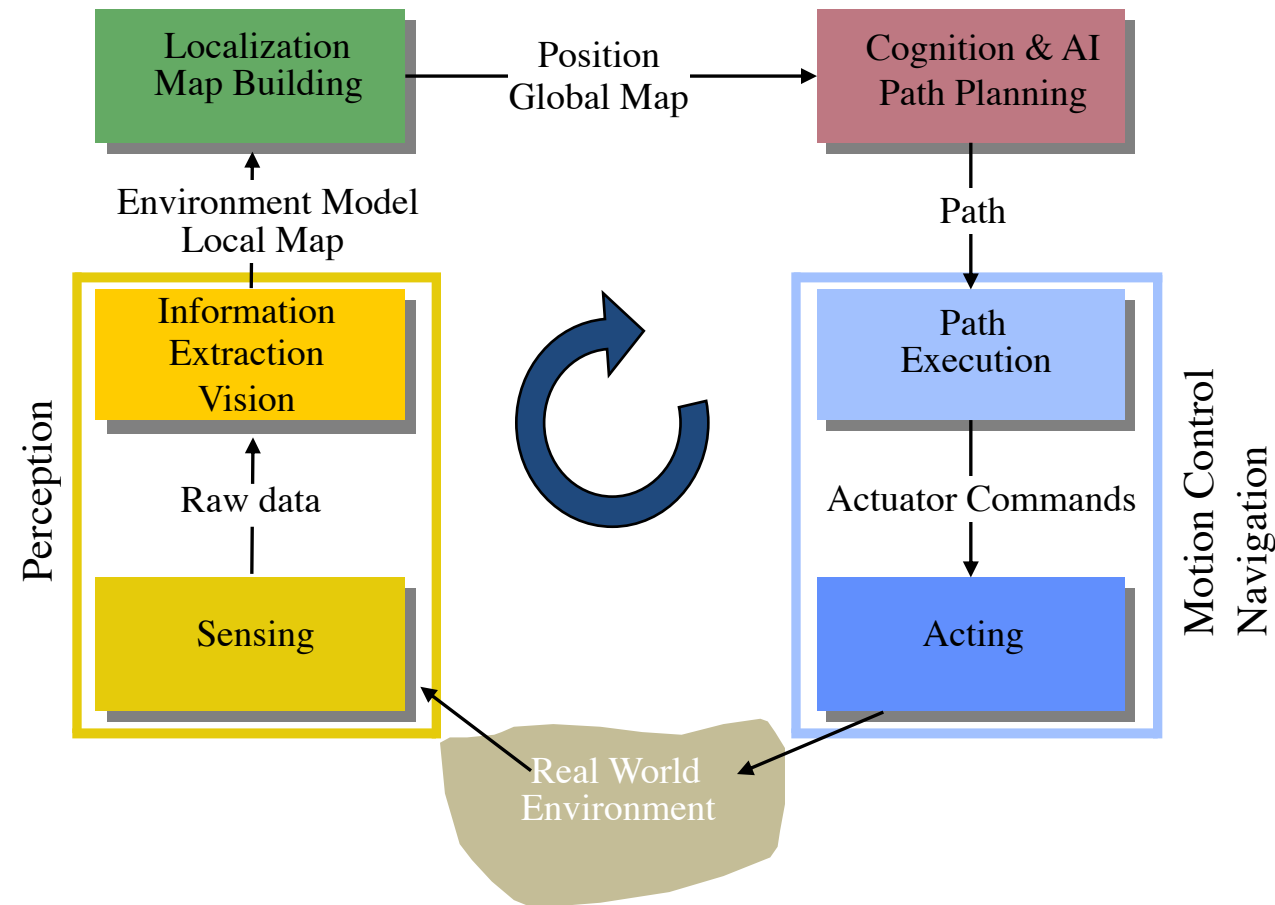
- HW 1 is due today.
- HW 2 will be published today
- Project:
 - Selection due today!
 - Make an appointment with the supervising MARS Lab graduate and Prof. Schwertfeger:
 - Next week
 - Discuss the details
 - You will need to write a proposal (details follow soon).
 - You will need to do a literature research about your topic.
 - You will need to present one of those papers “as if they were your own”. 10 minutes!

PERCEPTION



Line extraction from laser scans
Vision

General Control Scheme for Mobile Robot Systems

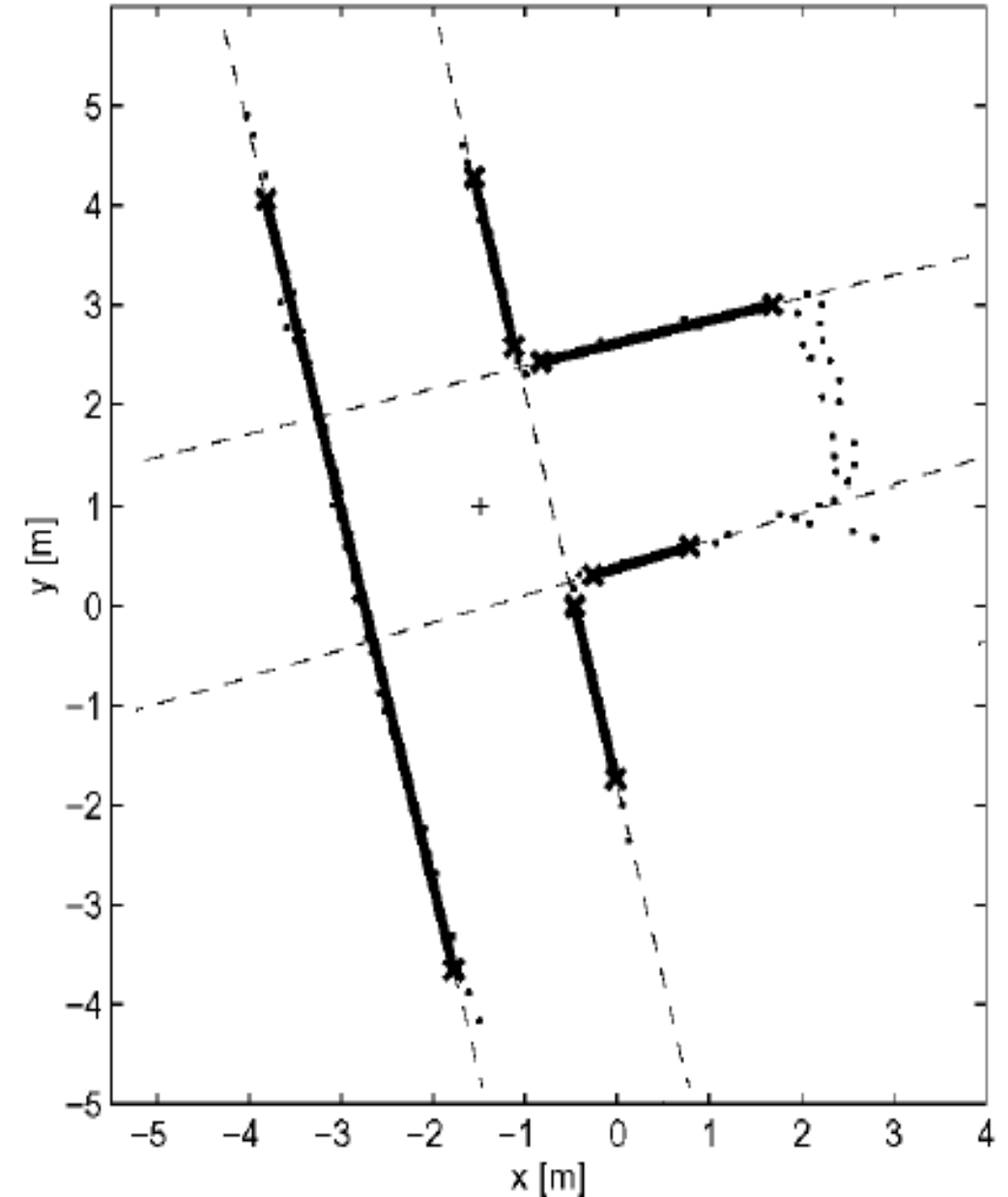


LINE EXTRACTION

Split and merge
Linear regression
RANSAC
Hough-Transform

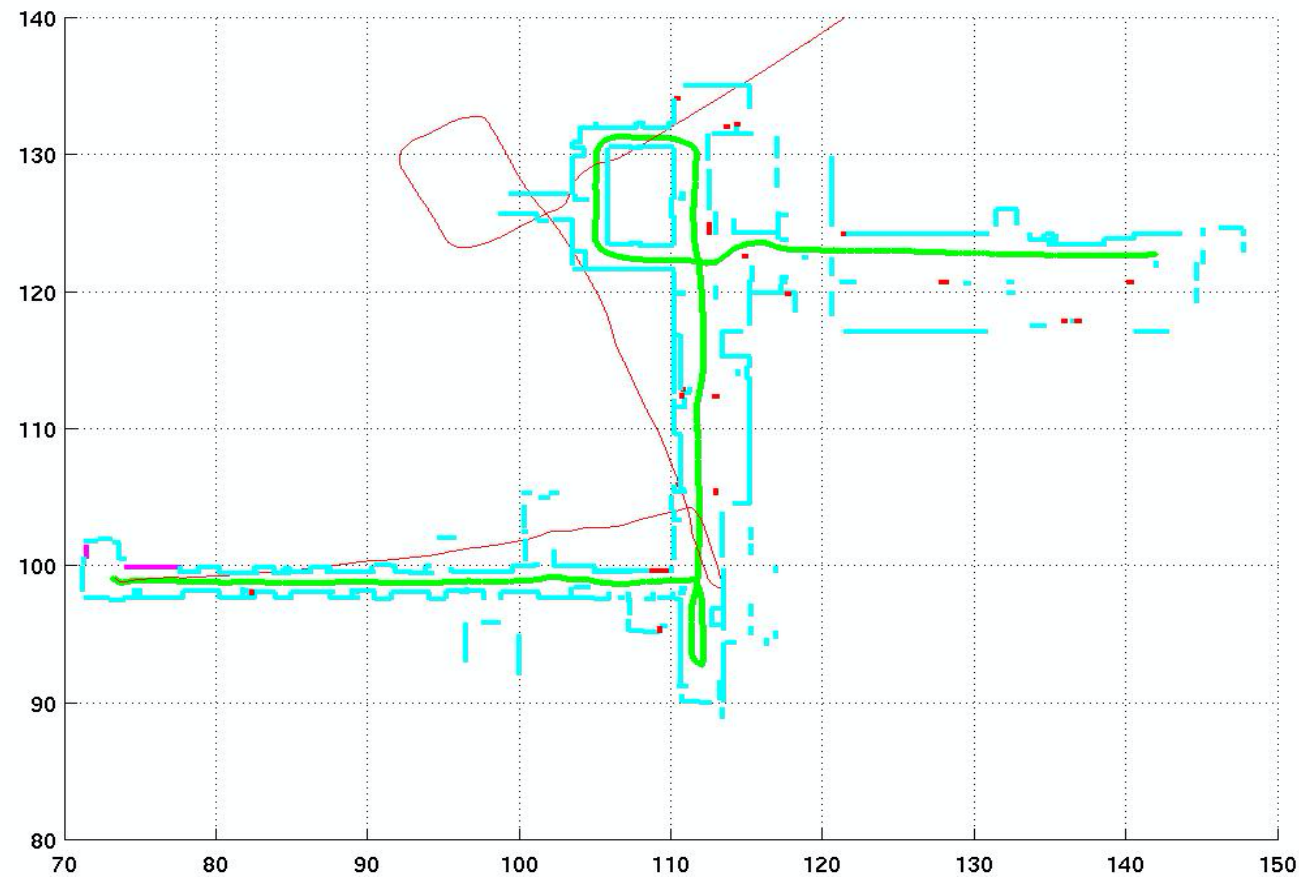
Line Extraction: Motivation

- Laser Range Scan
 - Example: 360 deg – black points
 - Example: dashed lines: desired line extractions
 - Use detected lines for:
 - Scan registration (find out transform between frames of two consecutive LRF scans – change due to robot motion)
- OR
- Mapping using line representation



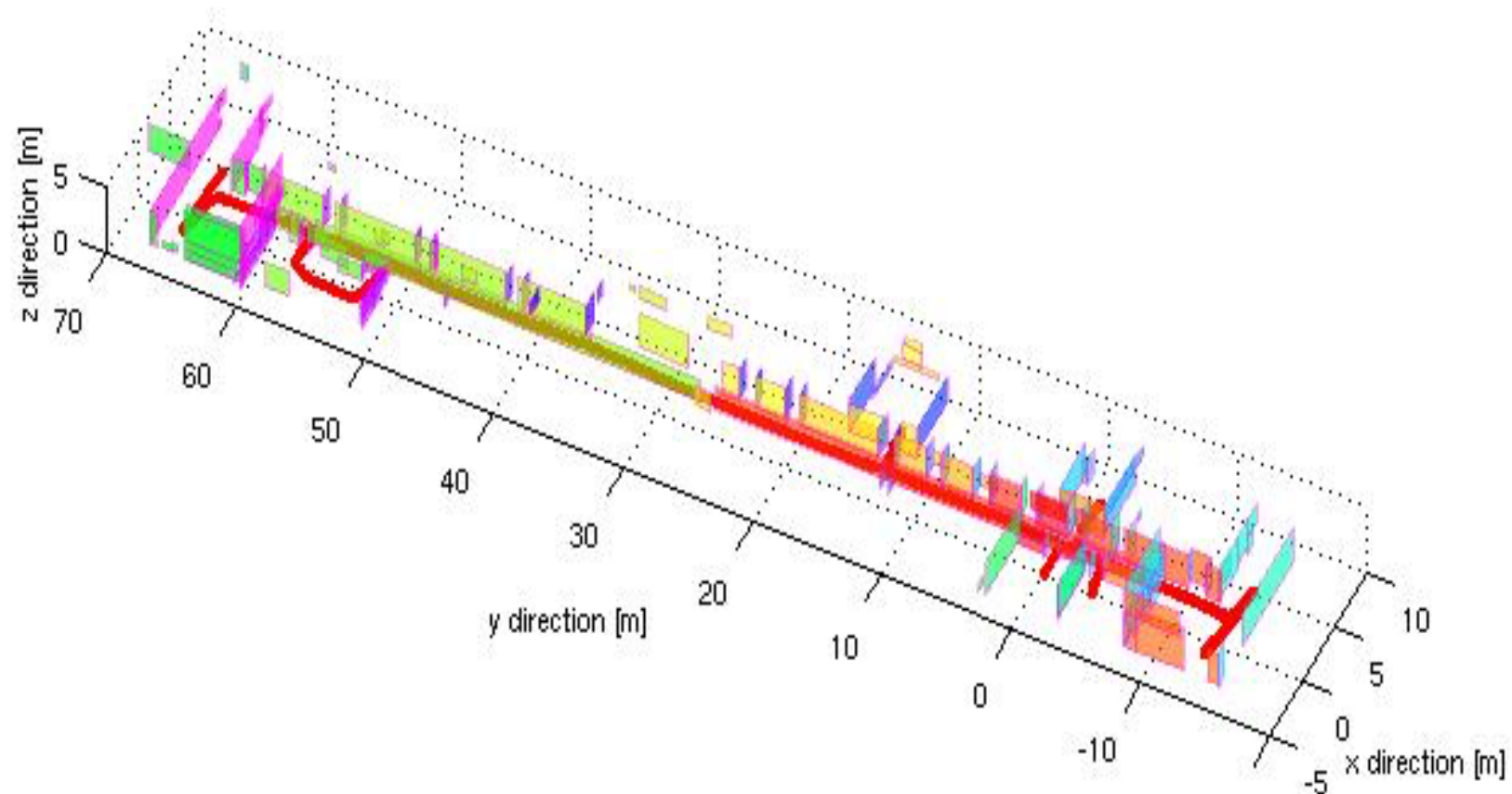
Line Extraction: Motivation

- Map of hallway built using line segments



Line Extraction: Motivation

- Map of the hallway built using orthogonal planes constructed from line segments

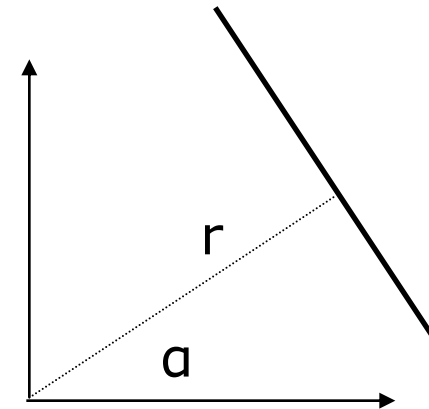
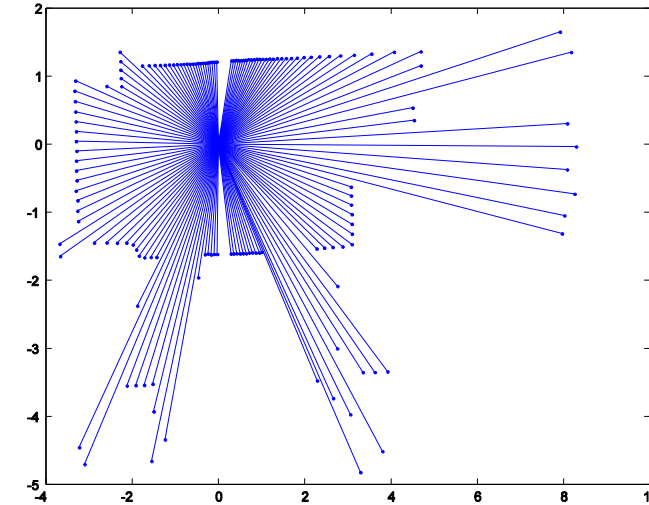


Line Extraction: Motivation

- Why laser scanner:
 - Dense and accurate range measurements
 - High sampling rate, high angular resolution
 - Good range distance and resolution.
- Why line segment:
 - The simplest geometric primitive
 - Compact, requires less storage
 - Provides rich and accurate information
 - Represents most office-like environment.

Line Extraction: The Problem

- Scan point in polar form: (ρ_i, θ_i)
- Assumptions:
 - Gaussian noise
 - Negligible angular uncertainty
- Line model in polar form:
 - $x \cos \alpha + y \sin \alpha = r$
 - $-\pi < \alpha \leq \pi$
 - $r \geq 0$

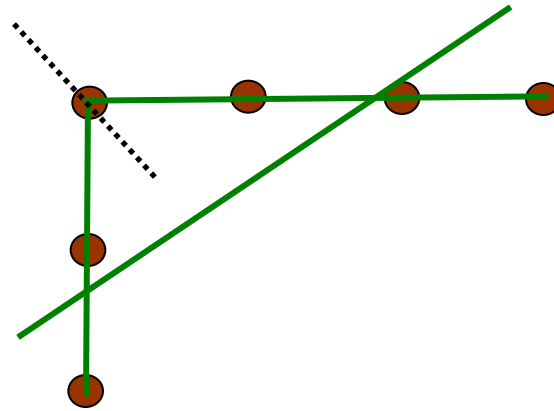


Line Extraction: The Problem (2)

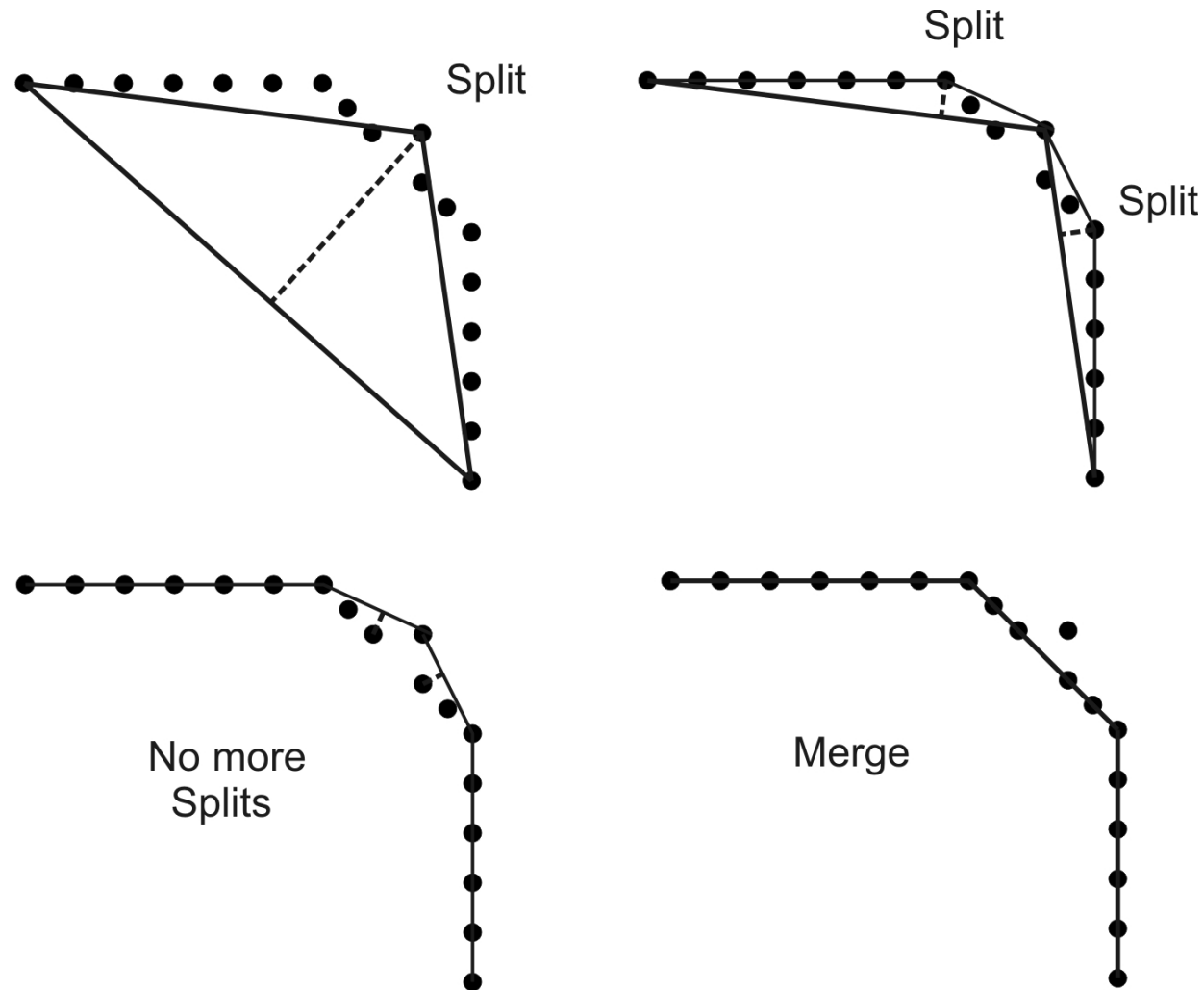
- Three main problems:
 - How many lines ?
 - Which points belong to which line ?
 - This problem is called SEGMENTATION
 - Given points that belong to a line, how to estimate the line parameters ?
 - This problem is called LINE FITTING
- The Algorithms we will see:
 - 1.Split and merge
 - 2.Linear regression
 - 3.RANSAC
 - 4.Hough-Transform

Algorithm 1: Split-and-Merge (standard)

- The most popular algorithm which is originated from computer vision.
- A recursive procedure of fitting and splitting.
- A slightly different version, called Iterative-End-Point-Fit, simply connects the end points for line fitting.



Algorithm 1: Split-and-Merge (Iterative-End-Point-Fit)

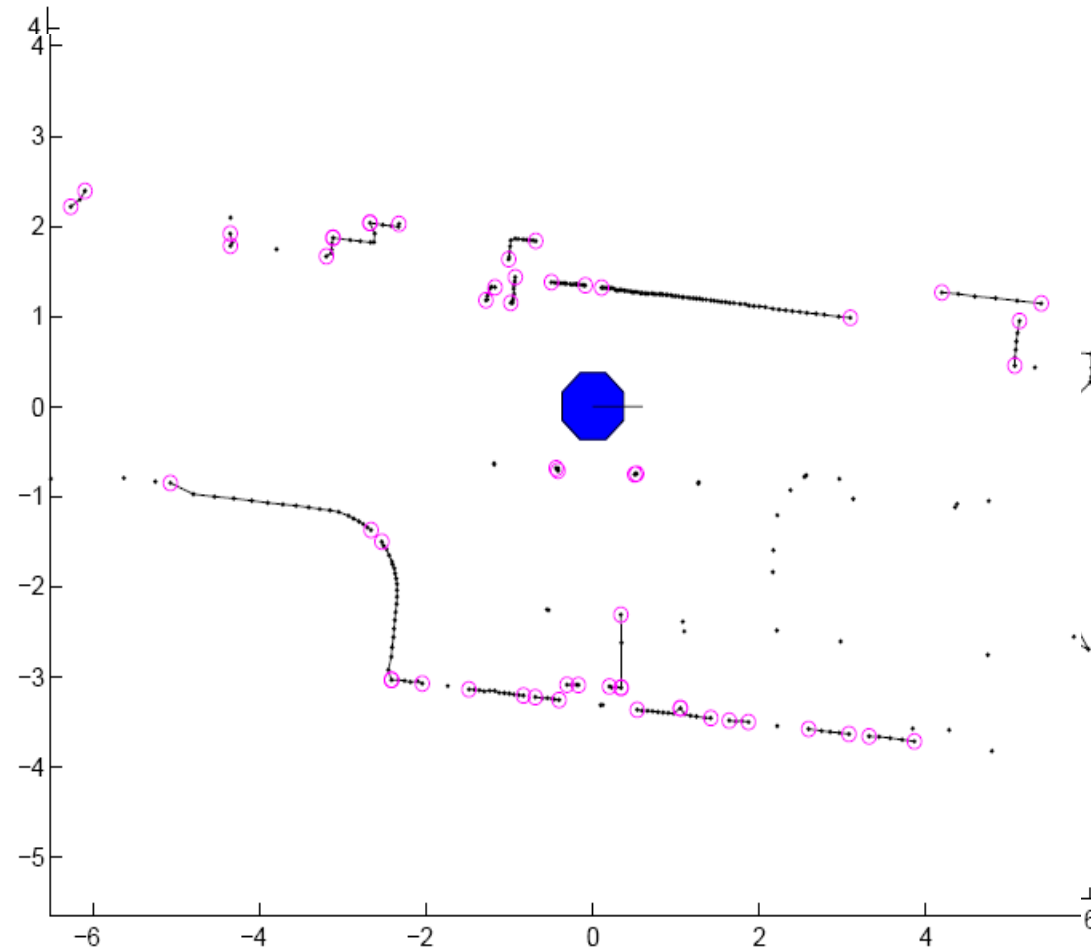


Algorithm 1: Split-and-Merge

Algorithm 1: *Split-and-Merge*

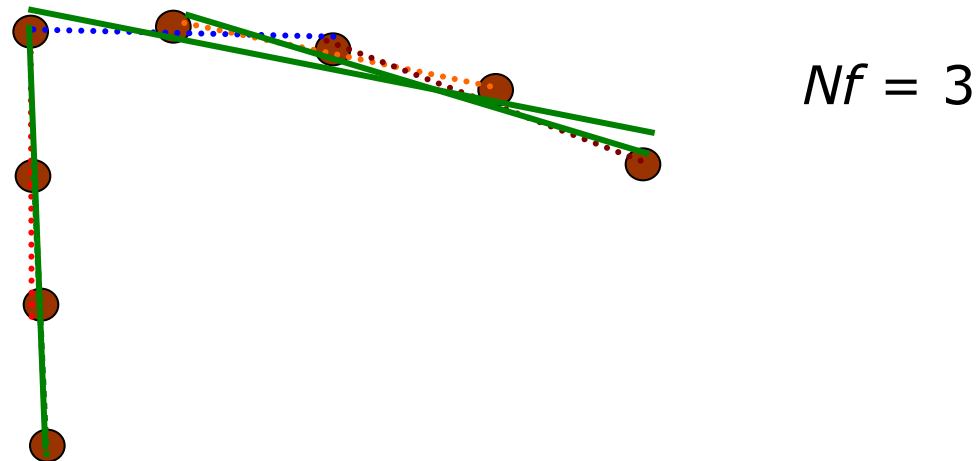
1. Initial: set s_1 consists of N points. Put s_1 in a list L
 2. Fit a line to the next set s_i in L
 3. Detect point P with maximum distance d_P to the line
 4. If d_P is less than a threshold, continue (go to step 2)
 5. Otherwise, split s_i at P into s_{i1} and s_{i2} , replace s_i in L by s_{i1} and s_{i2} , continue (go to 2)
 6. When all sets (segments) in L have been checked, merge collinear segments.
-

Algorithm 1: Split-and-Merge: Example application



Algorithm 2: Line-Regression

- Uses a “sliding window” of size Nf
- The points within each “sliding window” are fitted by a segment
- Then adjacent segments are merged if their line parameters are close



Algorithm 2: Line-Regression

Algorithm 2: *Line-Regression*

1. Initialize sliding window size N_f
 2. Fit a line to every N_f consecutive points (a window)
 3. Compute a line fidelity array, each is the sum of Mahalanobis distances between every three adjacent windows
 4. Construct line segments by scanning the fidelity array for consecutive elements having values less than a threshold, using an AHC algorithm
 5. Merge overlapped line segments and recompute line parameters for each segment
-

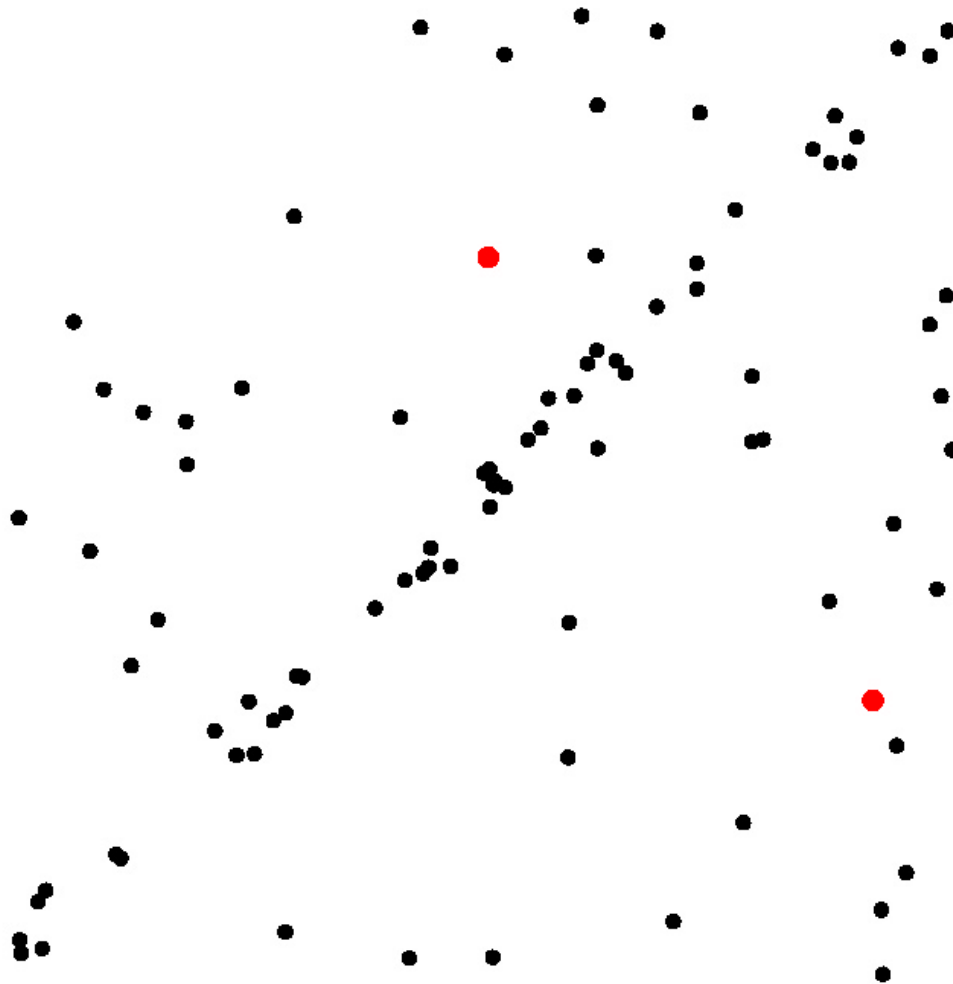
Algorithm 3: RANSAC

- Acronym: **Random Sample Consensus.**
- Generic & robust fitting algorithm of models with outliers
 - Outliers: points which do not satisfy a model
- RANSAC: apply to any problem where:
 - identify the inliers
 - which satisfy a predefined mathematical model.
- Typical robotics applications:
 - line extraction from 2D range data (sonar or laser);
 - plane extraction from 3D range data
 - structure from motion
- RANSAC:
 - iterative method & non-deterministic
- Drawback: A nondeterministic method, results are different between runs.

Algorithm 3: RANSAC

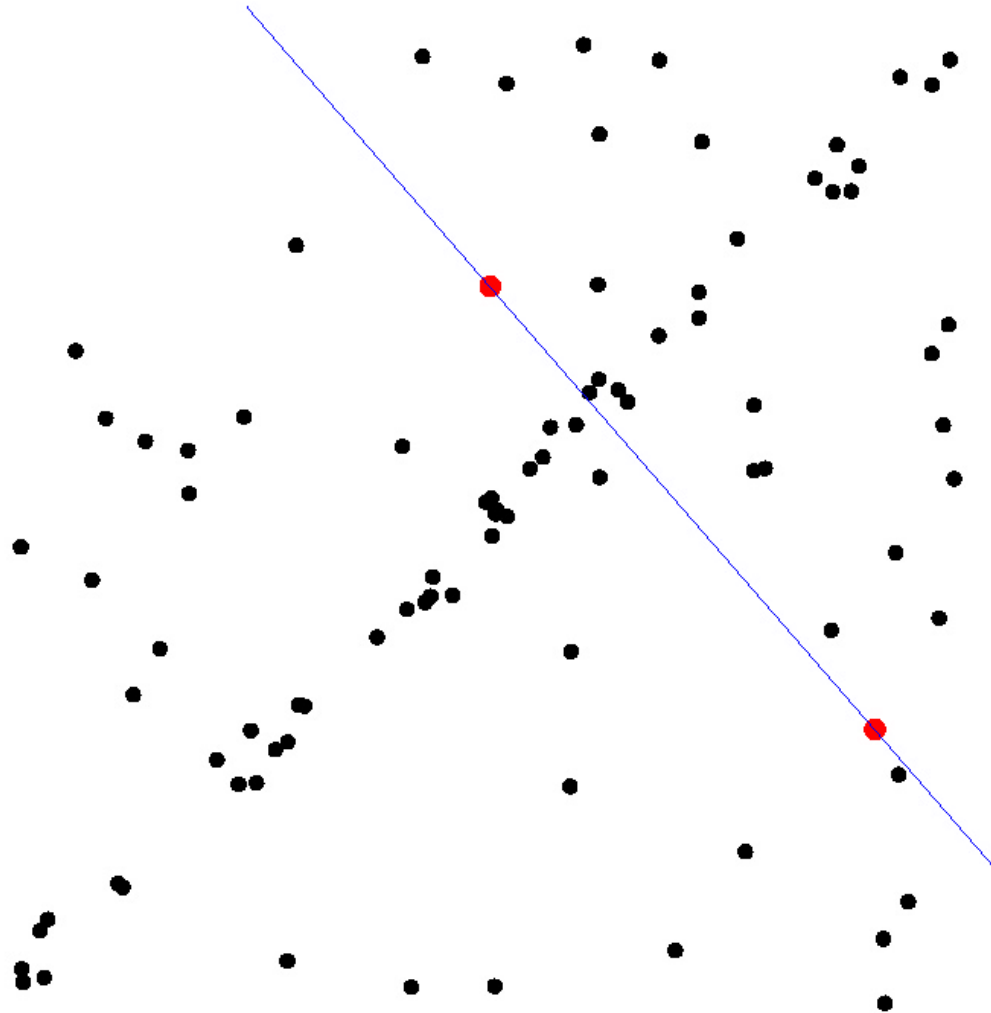


Algorithm 3: RANSAC



- **Select sample of 2 points at random**

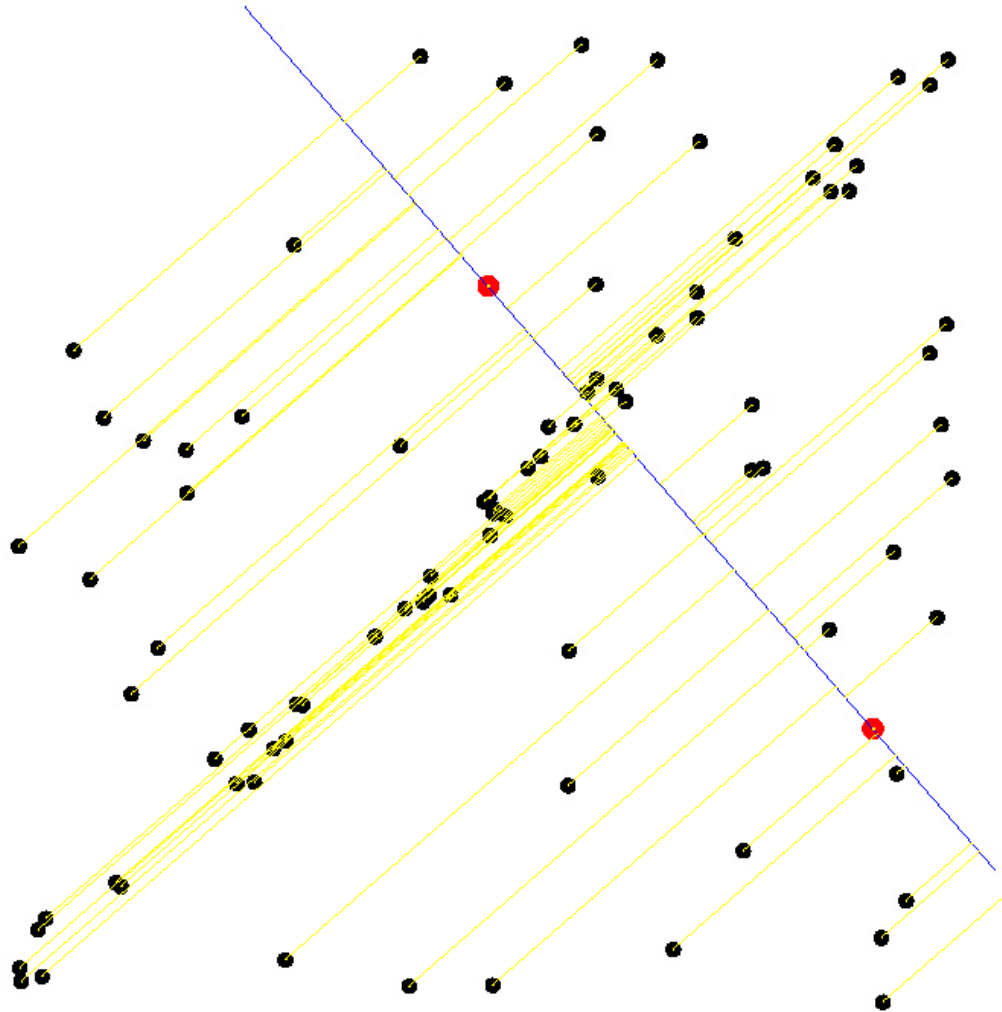
Algorithm 3: RANSAC



- Select sample of 2 points at random

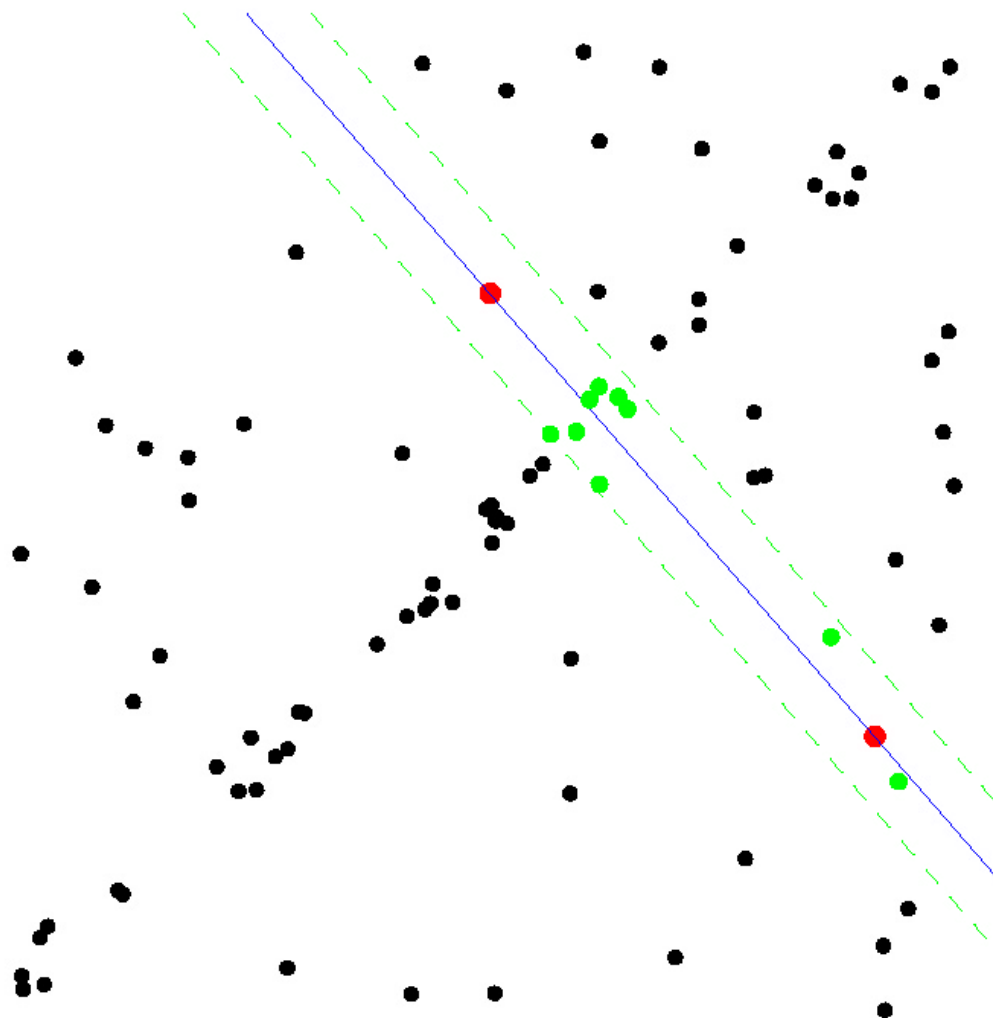
- **Calculate model parameters that fit the data in the sample**

RANSAC



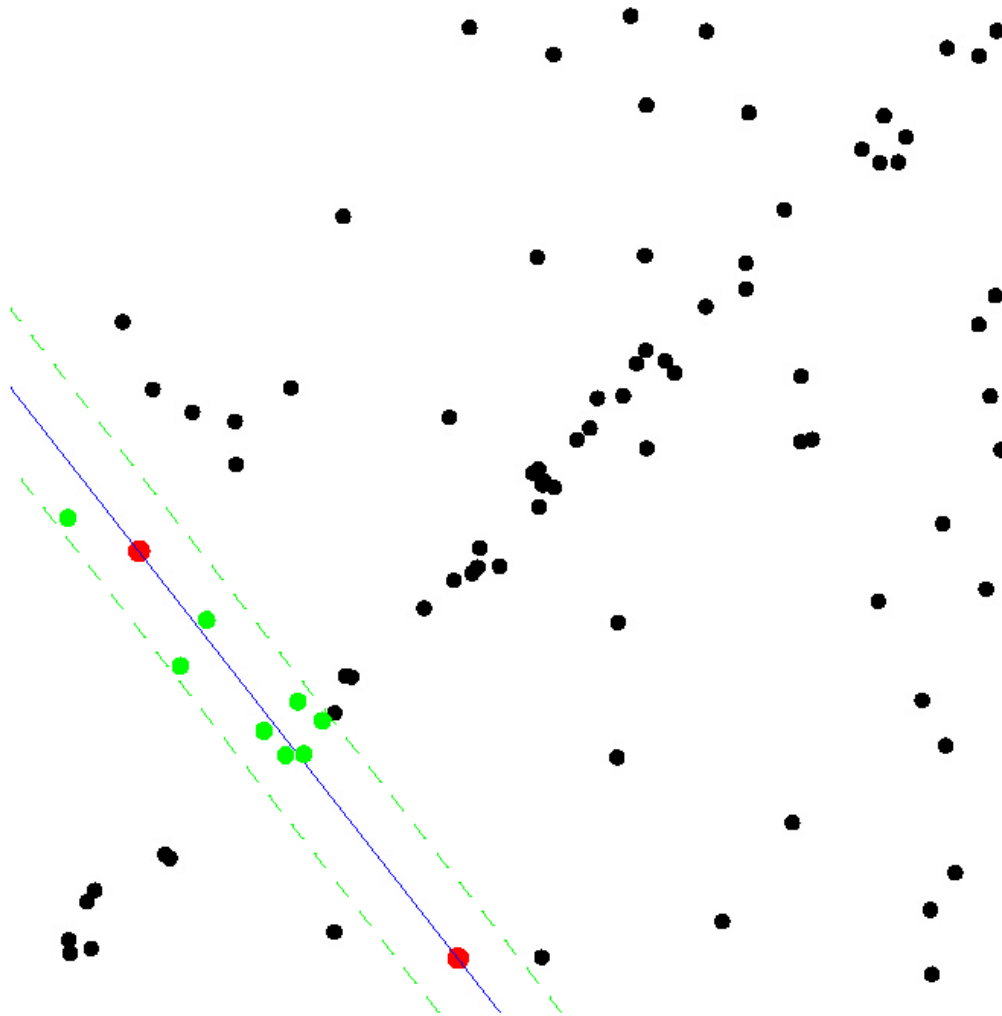
- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- **Calculate error function for each data point**

Algorithm 3: RANSAC



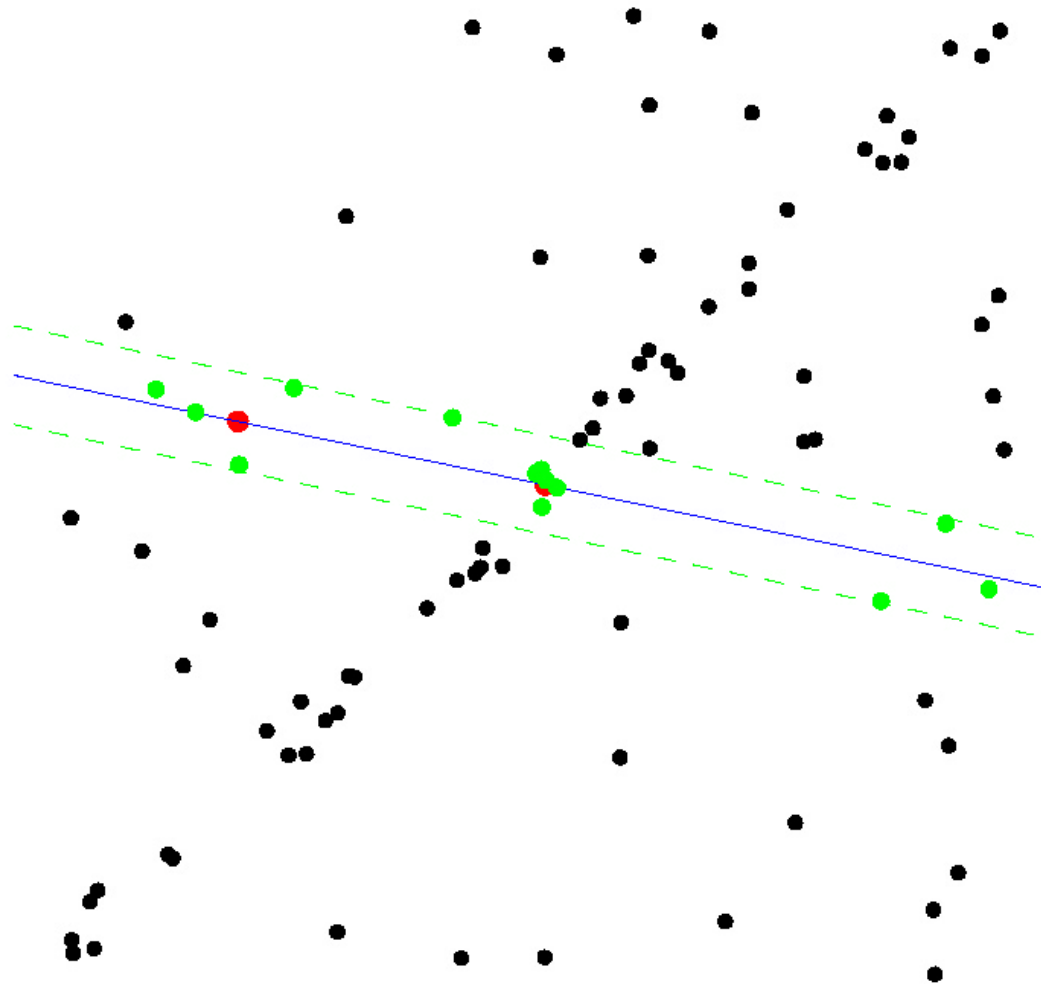
- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- **Select data that support current hypothesis**

Algorithm 3: RANSAC



- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

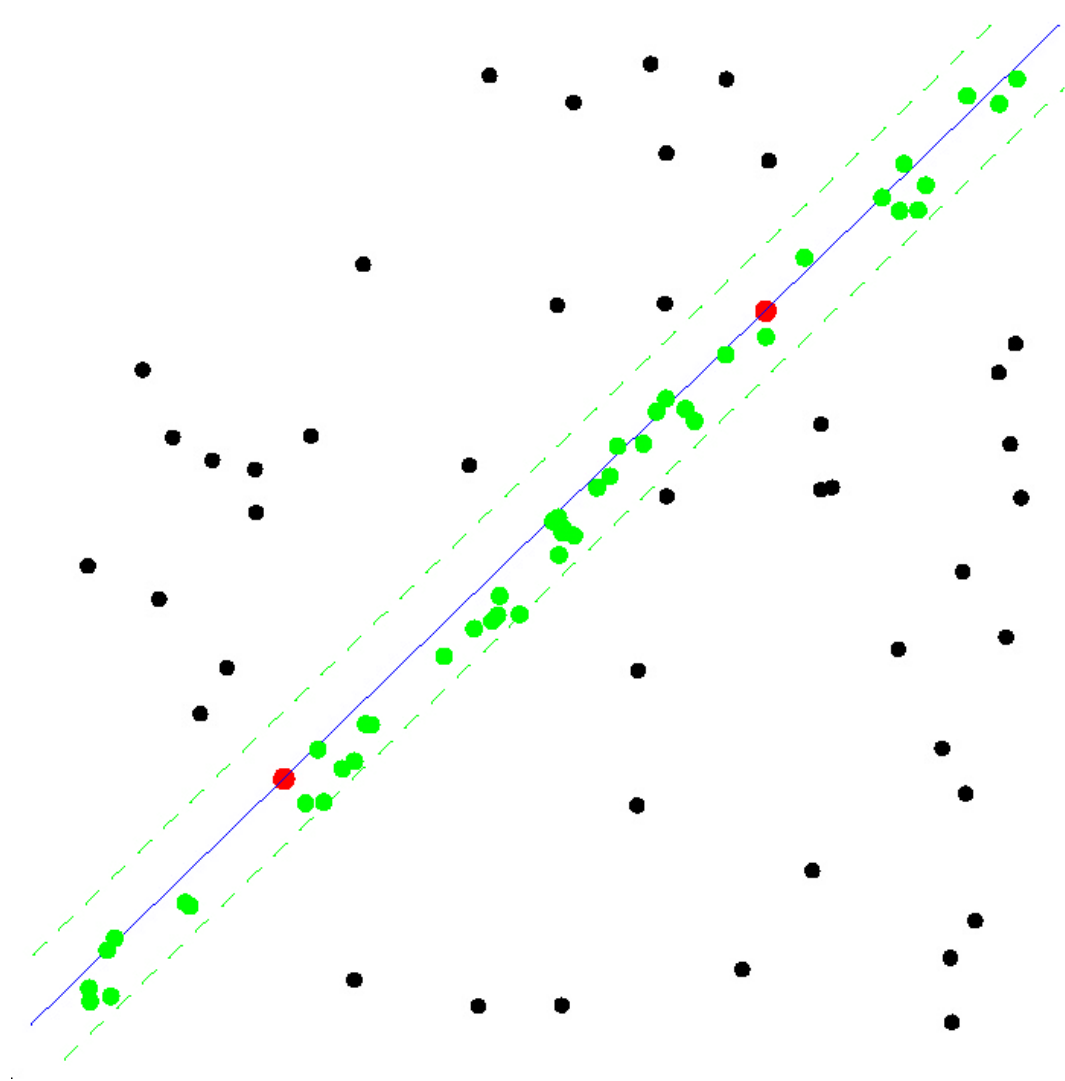
Algorithm 3: RANSAC



- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

Algorithm 3: RANSAC

ALL-INLIER SAMPLE



Algorithm 3: RANSAC

Algorithm 4: *RANSAC*

1. Initial: let A be a set of N points
 2. **repeat**
 3. Randomly select a sample of 2 points from A
 4. Fit a line through the 2 points
 5. Compute the distances of all other points to this line
 6. Construct the inlier set (i.e. count the number of points with distance to the line $< d$)
 7. Store these inliers
 8. **until** Maximum number of iterations k reached
 9. The set with the maximum number of inliers is chosen as a solution to the problem
-

Algorithm 3: RANSAC

How many iterations does RANSAC need?

- Because we cannot know in advance if the observed set contains the maximum number of inliers, the ideal would be to check all possible combinations of 2 points in a dataset of N points.
- The number of combinations is given by $N(N-1)/2$, which makes it computationally unfeasible if N is too large. For example, in a laser scan of 360 points we would need to check all $360 \cdot 359 / 2 = 64,620$ possibilities!
- Do we really need to check all possibilities or can we stop RANSAC after iterations? The answer is that indeed we do not need to check all combinations but just a subset of them if we have a rough estimate of the percentage of inliers in our dataset
- This can be done in a probabilistic way