Introduction to Signal Processing

by Xiliang Luo, Zhi Ding

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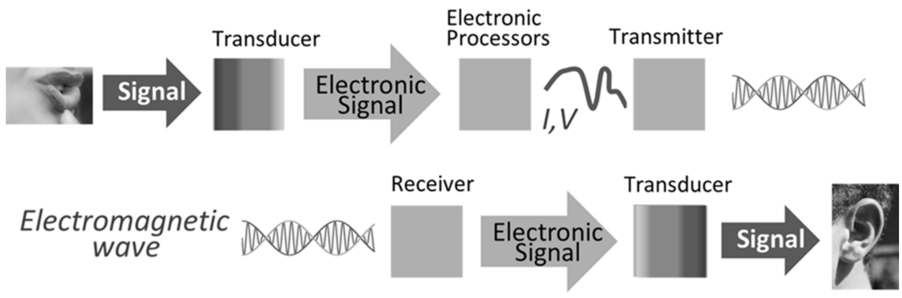
What is Signal Processing



From Wikipedia

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Applications

❑ **Application fields of signal processing**

- **Audio/Speech signal processing**
- **Image processing**
- **Wireless communication**

- Control systems
- Array processing
- Seismology
- Financial signal processing

- **Feature extraction, Quality improvement, Compression**

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References

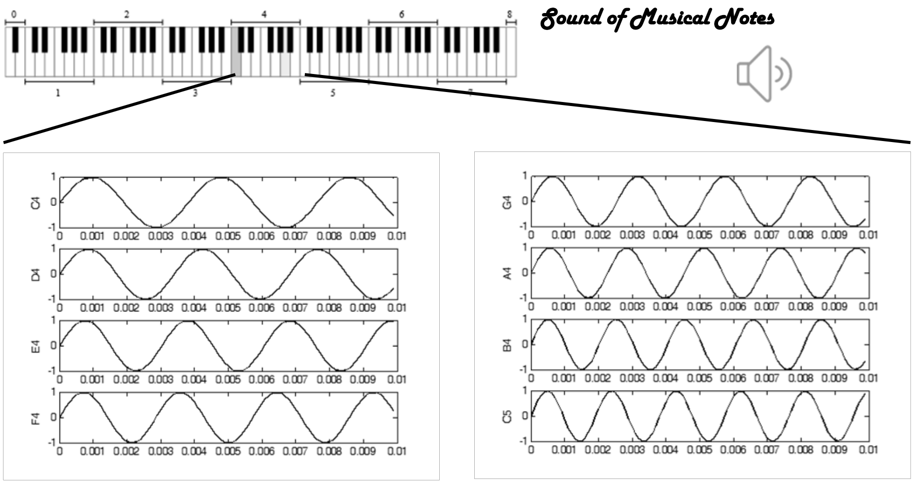
1. *A. V. Oppenheim, A. S. Willsky and S. Hamid Nawab, Signals and Systems, Pearson Education Ltd., 2nd Edition, 2014.*
2. *EE16A in Berkeley: http://inst.eecs.berkeley.edu/~ee16a/sp15/*
3. *Single-Pixel Camera: http://dsp.rice.edu/cscamera*
4. *Emmanuel Candes, Compressive Sensing – A 25 Minute Tour, First EU-US Frontiers of Engineering Symposium, Cambridge, September, 2010*
5. *Xuedong Huang and Li Deng, An Overview of Modern Speech Recognition*
6. *Wikipedia on various topics: compressive sensing, speech recognition, noise cancellation*
7. *E. Perahia and R. Stacey, Next Generation Wireless LANs, 2nd Ed., Cambridge*

Part I:
Fourier Transform
& Filtering

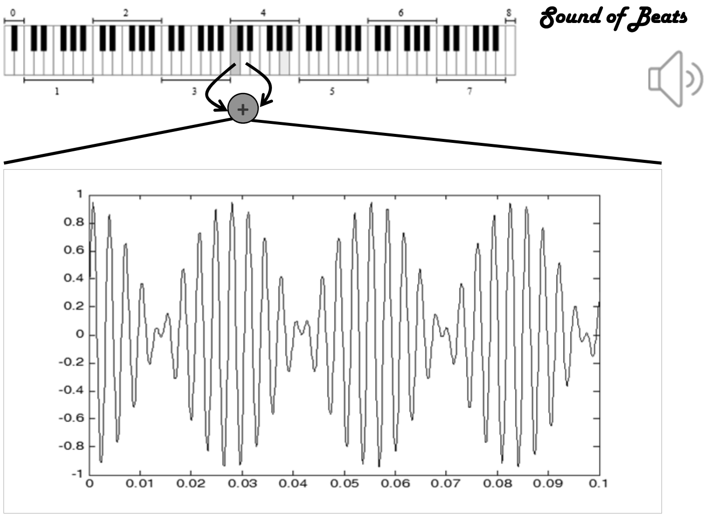
What is frequency?

How to characterize frequency?

Signals in Time Domain



Signals in Time Domain



Question:

How to define and visualize the frequency in a signal?

Answer:

Fourier Transform

/ˈfʊəriˌeɪ, -iər/

1768-1830
French Mathematician, Physicist, Historian

Frequency Domain

Fourier Transform of a signal is defined as:

Signal analysis: $$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt$$

f: is the Frequency in Hertz = 1/second

Correspondingly, $X(f)$ can be used to recover the signal as:

Signal synthesis: $$x(t) = \int_{-\infty}^{+\infty} X(f)e^{+j2\pi ft}d\omega$$

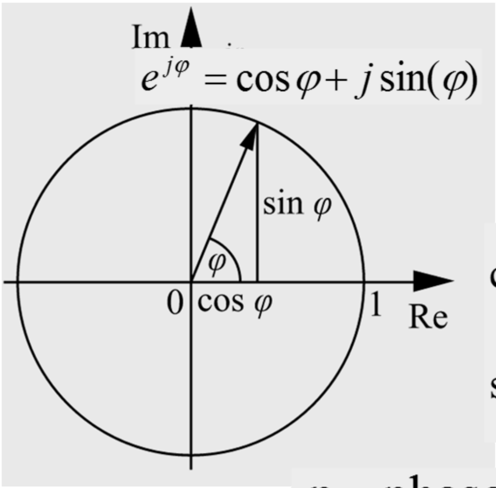
Some notations we OFTEN use

u(t) = step function = 1 only for positive t.

δ[n] = delta function (known as Kronecker Delta)
= impulse function = 1 only for n=0.

f= frequency = tells you how fast signals change
= not just how fast

Euler's Formula (frequency)



$$\cos\varphi = \frac{1}{2}(e^{j\varphi} + e^{-j\varphi})$$

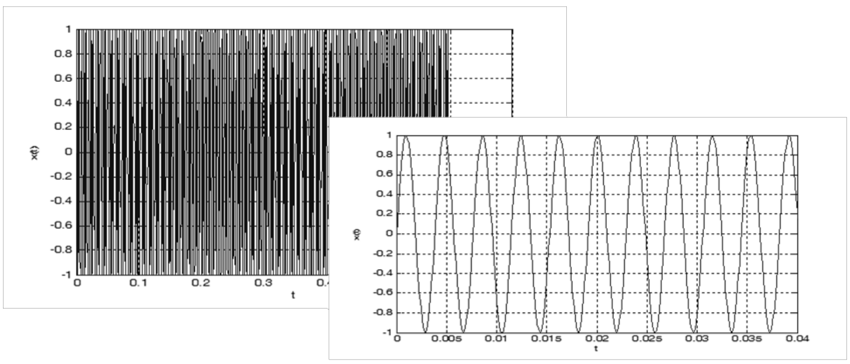
$$\sin\varphi = \frac{1}{2j}(e^{j\varphi} - e^{-j\varphi})$$

$$\varphi = \text{phase}$$

Frequency Domain

Example 1: Fourier Transform of the C4 tone

$$x(t) = \begin{cases} \sin(2\pi \cdot f_0\ t), & t \in [-1,1] \\ 0, & o.w. \end{cases} \quad f_0 = 261.626\text{Hz}$$



Frequency Domain

Example 1: Fourier Transform of the C4 tone

$$X(f) = \int_{-1}^{1} \sin(2\pi f_0 t) \cdot \exp[-j2\pi f t] dt$$

$$= \frac{1}{2j}\begin{bmatrix} \int_{-1}^{+1} \exp[j2\pi(f_0 - f)t]\, dt\ - \\ \int_{-1}^{+1} \exp[j2\pi(-f_0 - f)t]\, dt \end{bmatrix}$$

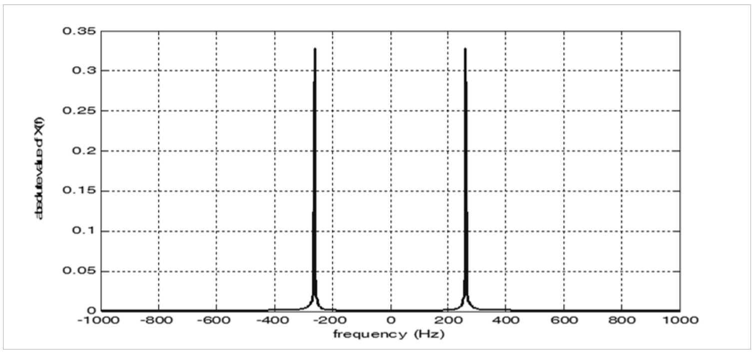
$$= \frac{\sin(2\pi(f - f_0))}{j2\pi(f - f_0)} - \frac{\sin(2\pi(f + f_0))}{j2\pi(f + f_0)}$$

Frequency Domain

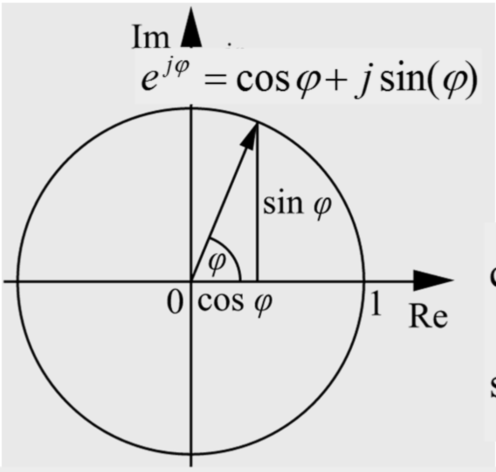
Example 1: Fourier Transform of the C4 tone

$$x(t) = \begin{cases} \sin(2\pi \cdot f_0\, t)\,, & t \in [0,1] \\ 0, & o.w. \end{cases}$$

$f_0 = 261.626\text{Hz}$



Euler’s Formula (frequency)



$$e^{j\varphi} = \cos\varphi + j\sin(\varphi)$$

$$\cos\varphi = \frac{1}{2}(e^{j\varphi} + e^{-j\varphi})$$

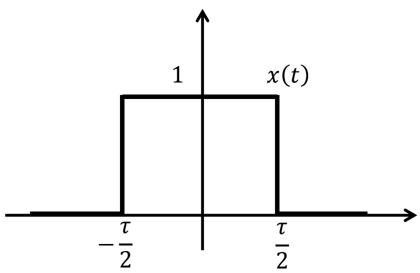
$$\sin\varphi = \frac{1}{2j}(e^{j\varphi} - e^{-j\varphi})$$

$$\varphi = 2\pi f_0 \bullet t = \text{linear phase change in } t$$

Frequency Domain

Example 2: Fourier Transform of the rectangular pulse

$$x(t) = \begin{cases} 1, & t \in \left[-\frac{\tau}{2}, \frac{\tau}{2}\right] \\ 0, & o.w. \end{cases}$$



Frequency Domain

Example 2: Fourier Transform of the rectangular pulse

$$X(f) = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} 1 \cdot \exp[-j2\pi f t] dt$$

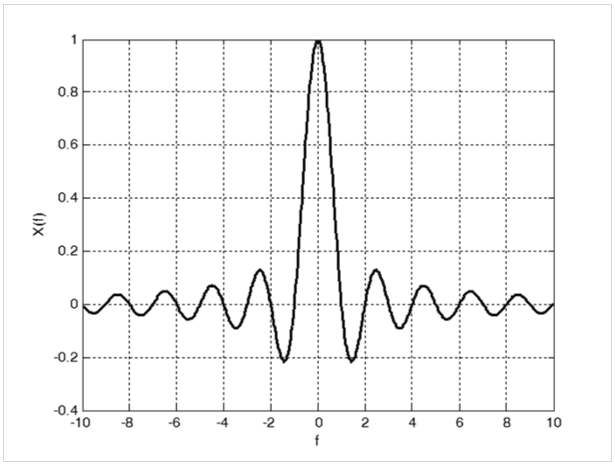
$$= \frac{1}{-j2\pi f}\left[\exp\left(\frac{-j2\pi f\tau}{2}\right) - \exp\left(\frac{+j2\pi f\tau}{2}\right)\right]$$

$$= \frac{\sin(\pi f\tau)}{\pi f}$$

Frequency Domain

Example 2: Fourier Transform of the rectangular pulse

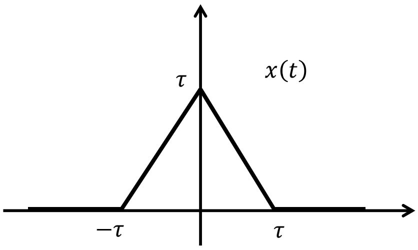
$\tau = 1$



Frequency Domain

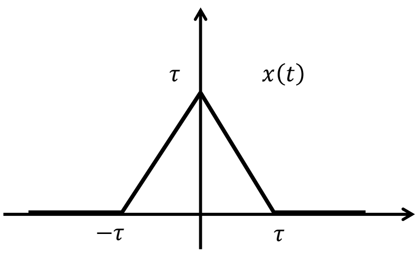
Example 3: Fourier Transform of the triangular pulse

$$x(t) = \begin{cases} \tau - |t|, & t \in [-\tau, \tau] \\ 0, & o.w. \end{cases}$$



Frequency Domain

Example 3: Fourier Transform of the triangular pulse



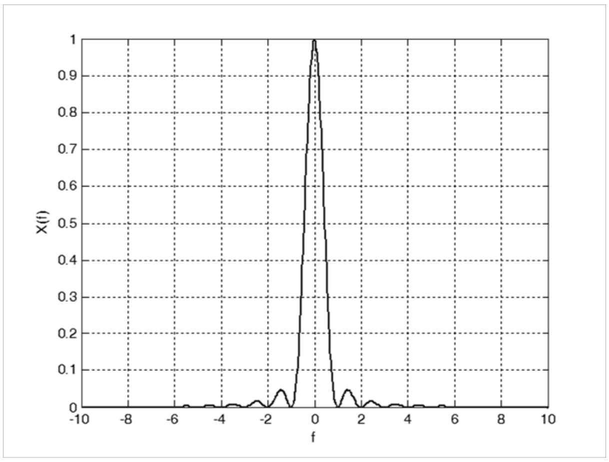
$$X(\omega) = \int_{-\infty}^{+\infty} x(t) \cdot \exp[-j2\pi f t] dt$$

$$= 2 \int_0^{\tau} (\tau - t) \cos(2\pi f t)\, dt$$

$$= \left(\frac{\sin(\pi f \tau)}{\pi f}\right)^2$$

Frequency Domain

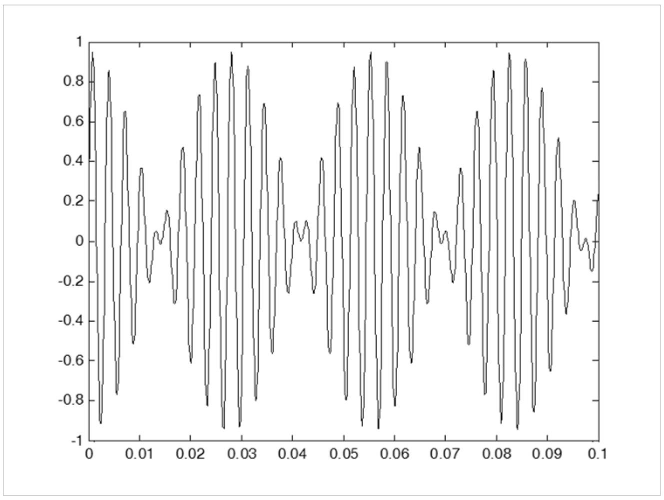
Example 3: Fourier Transform of the triangular pulse



$\tau = 1$

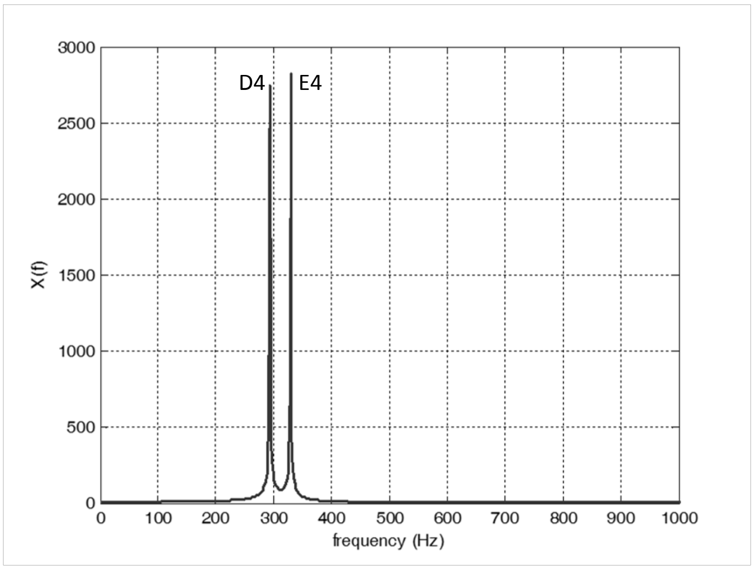
Time Domain

Back to where we begin …



Frequency Domain

Back to where we begin …



Define discrete time signals

x(t) is continuous time signal;

x[n] = x(nT) discrete signal sampled periodically

If your T is small enough, no information loss:
Nyquist sampling theorem says T < 1/(2W).

W=Bandwidth (in frequency) of a signal

Digital signal processing only deal with discrete signals

Some notations we OFTEN use

u(t) = step function = 1 only for positive t.

$\delta[n]$ = delta function (known as Kronecker Delta)
= impulse function = 1 only for n=0.

W=Bandwidth of x(t)
= maximum positive frequency for which X(f) is not zero.

Digital signal processing only deal with discrete signals

Discrete Fourier Transform

In computer, we have to discretize the signal and the following continuous Fourier Transform:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt$$

Sampling every T_s (F_s=1/T_s) seconds:

$$X(k) = \sum_{n=-\infty}^{+\infty} x(nT_s) e^{-j2\pi\left(\frac{kF_s}{N}\right)(nT_s)}$$
$$= \sum_{n=-\infty}^{+\infty} x(nT_s) e^{-j2\pi\left(\frac{kn}{N}\right)}$$

Fast Fourier Transform (FFT)

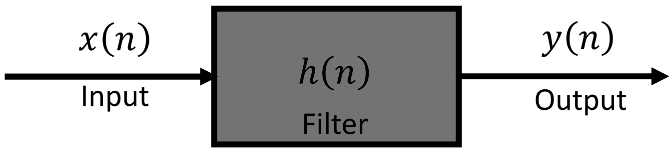
Discrete Fourier Transform:

$$X(k) = \sum_{n=-\infty}^{+\infty} x(nT_s)e^{-j2\pi\left(\frac{kn}{N}\right)}$$

- N is the power of 2 → very efficient algorithms
  - DFT is also called FFT!

Note: You will learn more during the practice session!

Filtering

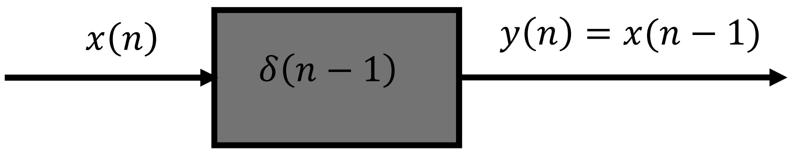


The role of a filter is to perform the following operation called "Convolution":

$$y(n) = \sum_{k=-\infty}^{+\infty} h(k)x(n-k)$$

Filtering

Example 1: Unit Delay

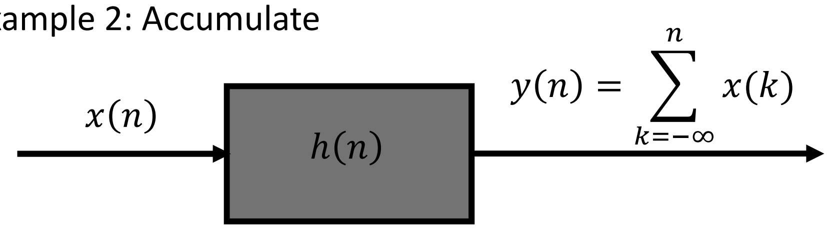


$$\delta(n-1) = \begin{cases} 1, & n = 1 \\ 0, & n \neq 1 \end{cases}$$

$$y(n) = \sum_{k=-\infty}^{+\infty} \delta(k-1)x(n-k) = x(n-1)$$

Filtering

Example 2: Accumulate

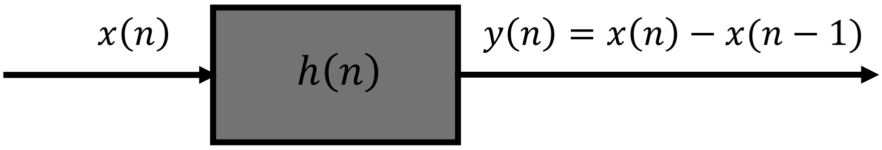


$$h(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$y(n) = \sum_{k=-\infty}^{+\infty} h(k)x(n-k) = \sum_{k=0}^{+\infty} x(n-k)$$

Filtering

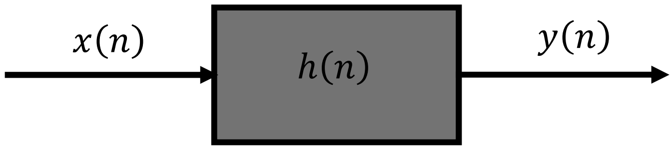
Example 3: Difference



$$h(n) = \begin{cases} 1, & n = 0 \\ -1, & n = 1 \\ 0, & o.w. \end{cases}$$

$$y(n) = \sum_{k=-\infty}^{+\infty} h(k)x(n-k) = x(n) - x(n-1)$$

Filtering Changes Fourier Transform



$$y(n) = \sum_{k=-\infty}^{+\infty} h(k)x(n-k)$$

Time Domain

$$Y(k) = H(k)X(k)$$

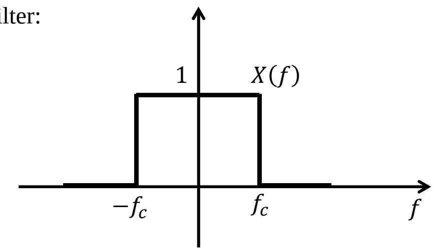
Frequency Domain

Note: You will learn more during the practice session!

Part II: Homework

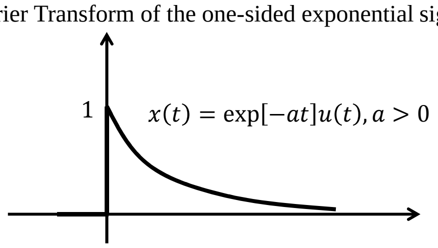
HW Problems

1. Find the time domain signal that has the following Fourier Transform, which is called Low-Pass Filter:



$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{+j2\pi f t} df$$

2. Find the Fourier Transform of the one-sided exponential signal.



HW Problems

3. Find the Discrete Fourier Transform of the following signals:

   3.a $x(n) = \delta(n-1), n = 0,1, \dots, N-1$

   3.b $x(n) = a^n, n = 0,1, \dots, N-1$

   3.c $x(n) = \sin(2\pi f_0 n)\,, n = 0,1, \dots, N-1$

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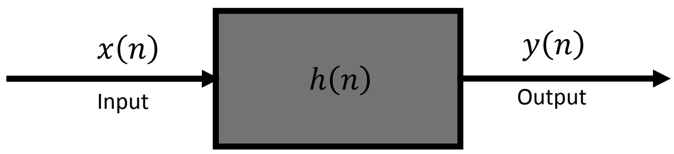
HW Problems

4. Let y(n) be the filtering output when inputting x(n), determine the output with the following inputs:



   4.a $x(n-n_0)$, n_0 is a fixed natural number

   4.b $ax(n)$, a is a fixed real number

   4.c $x(n)e^{j2\pi f_0 n}$, f_0 is a fixed real number

Part III:
Overview of Signal Processing Applications

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Example System: IPAD Air 2

- Great stuff running apps, but:
  - What makes the display tick?
  - How does the Wi-Fi work?
  - How does it sense touch on the screen?
  - How does it sense the motion?
  - How does Siri work?
  - ...

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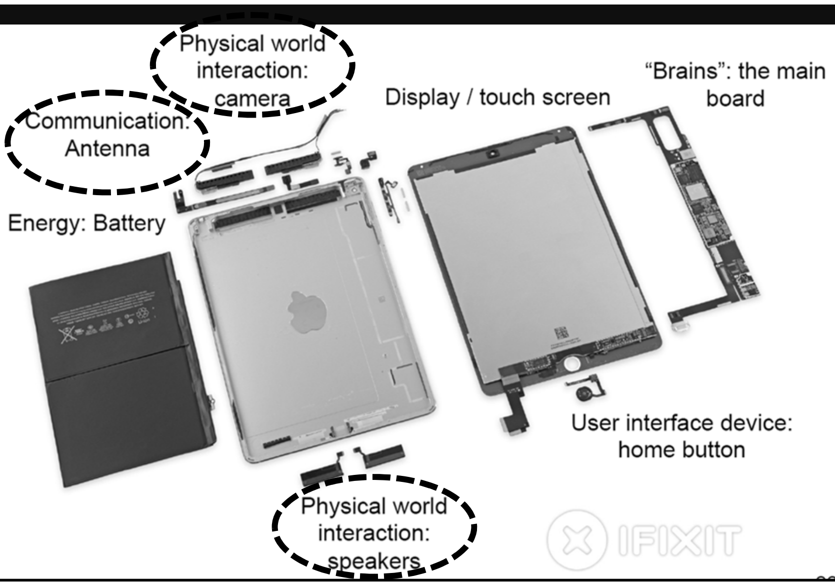
Inside an IPAD Air 2



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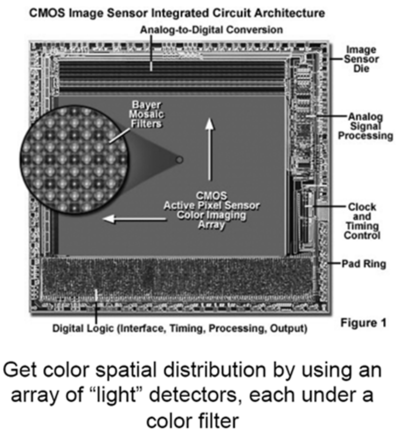
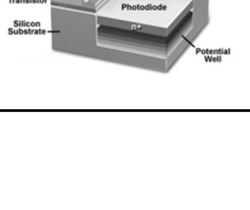
The Camera

Goal: Convert light into electrical signals

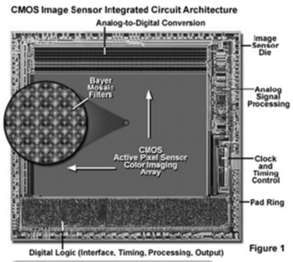




Get color spatial distribution by using an array of "light" detectors, each under a color filter

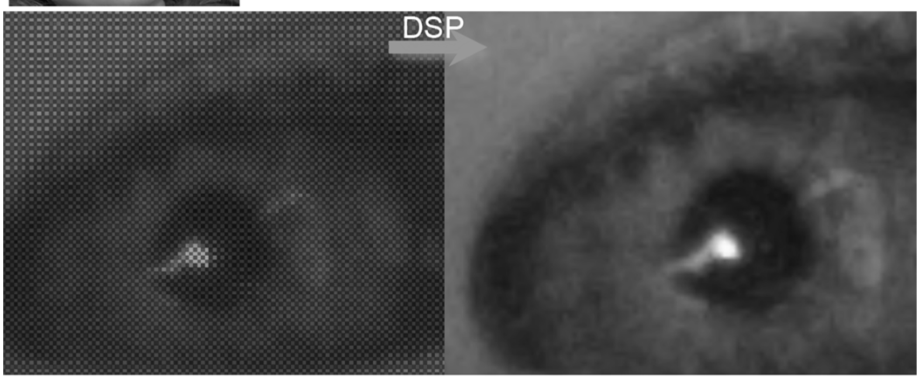
Each pixel takes the energy of incoming photons and produces charge, which is then read out by scanning through the rows and columns of the array

The Camera: Signal Processing



Focus/exposure Control → preprocessing → white-balancing → demosaic → Color transform → Post-processing → Compression

The Camera: Power of SP



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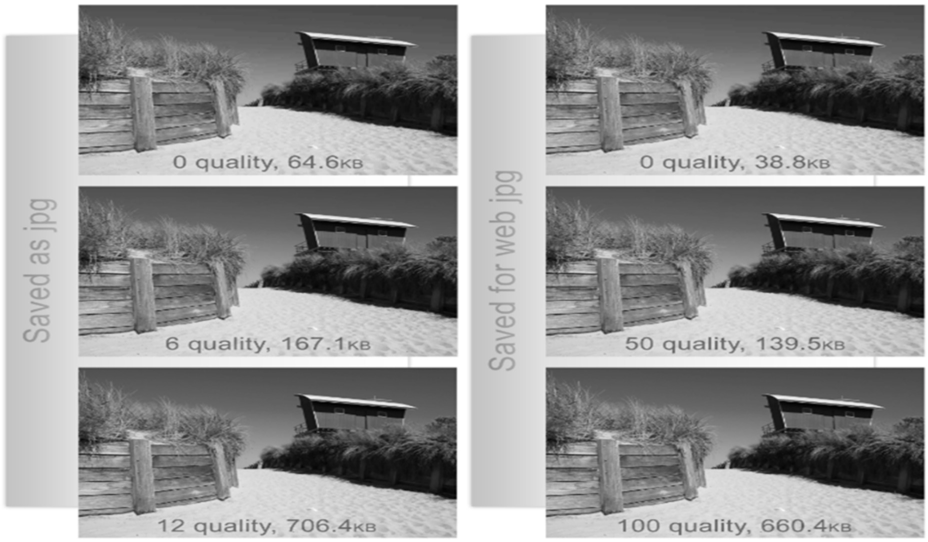
The Camera: SP for Compression



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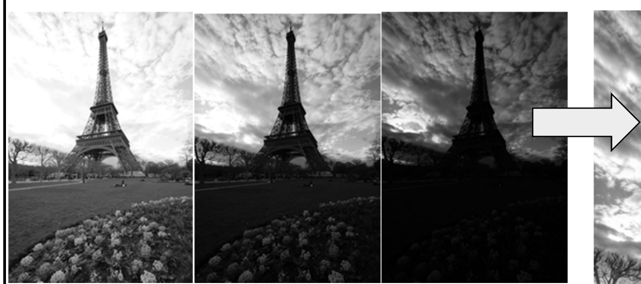
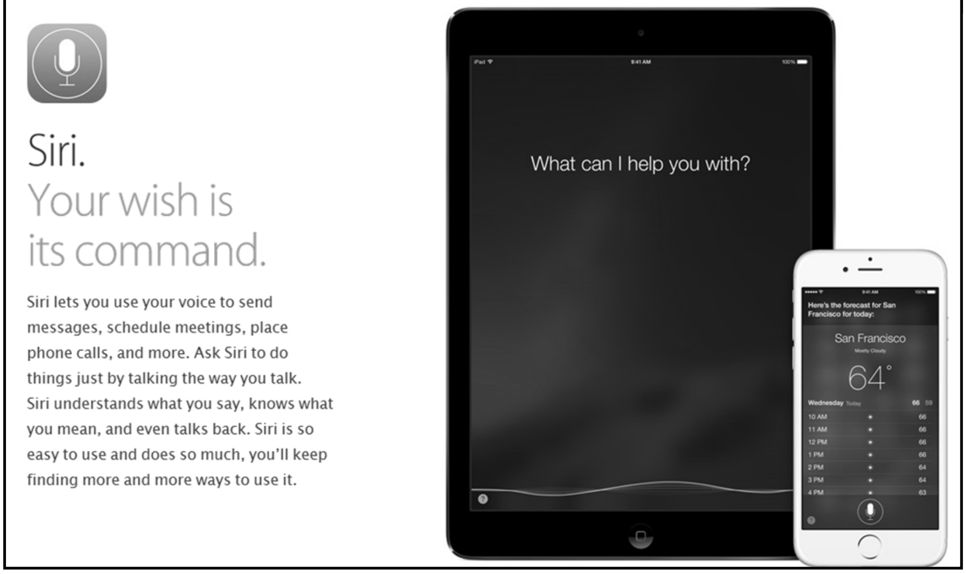
The Camera: Comp. Photography



High Dynamic Range(HDR)

The Microphone: Siri



Speech Recognition

- From the technology perspective, speech recognition has been going through several waves of major innovations since over some 50 years ago.
- The most recent wave of innovations since 2009 is based on **deep learning** concepts, architectures, methodologies, algorithms, and practical system implementations enabled by big training data and by GPU-based big compute
  - defines the current state of the art in speech recognition accuracy and has been in dominant use (e.g. Siri) since 2013 throughout the speech industry worldwide

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Speech Recognition

- Applications
  - Dictation
  - Telephone-based Information (directions, air travel, banking, etc)
  - Hands-free (in car)
  - Second language ('L2') (accent reduction)
  - Audio archive searching
  - Linguistic research
    - Automatically computing word durations, etc

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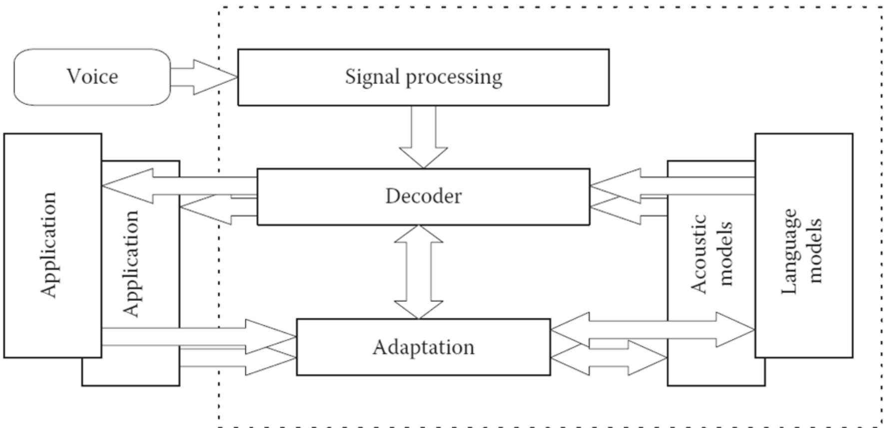
Speech Recognition



Basic System Architecture of a Speech-Recognition System

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Speech Recognition

- **Fundamental of statistical speech recognition:**

$$\hat{\mathbf{W}} = \arg\max_{\mathbf{W}} P(\mathbf{W}|\mathbf{A}) = \arg\max_{\mathbf{W}} \frac{P(\mathbf{W})P(\mathbf{A}|\mathbf{W})}{P(\mathbf{A})}$$

A: acoustic observation

W: word sequence

The goal of speech recognition is to find out the word sequence that has the max posterior probability.

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Speech Recognition

The New York Times

Scientists See Promise in Deep-Learning Programs

John Markoff

November 23, 2012

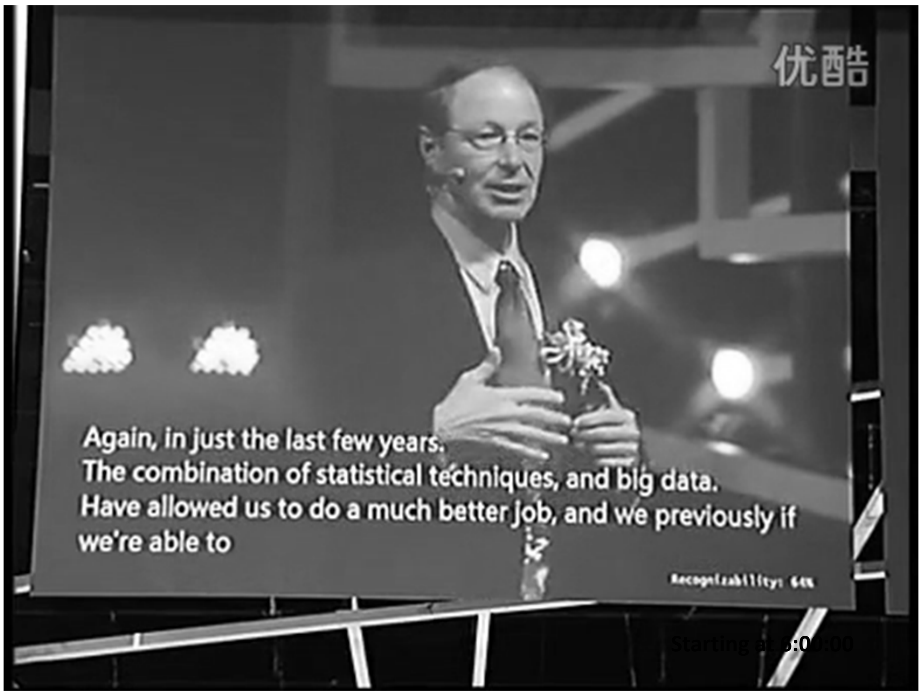
Richard F. Rashid

SVP, Microsoft

A voice recognition program translated the speech into Mandarin Chinese

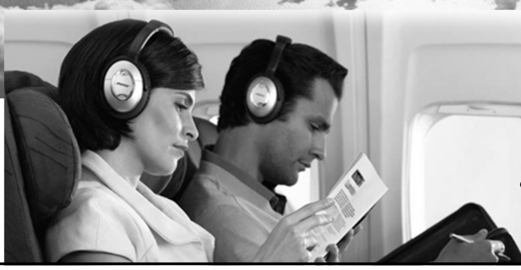
Tianjin, Oct. 25, 2012

<u>Deep learning</u> technology enabled speech-to-speech translation



The Headphone: ANC

Engine Noise



Noise-Cancelling Headphone

Before After

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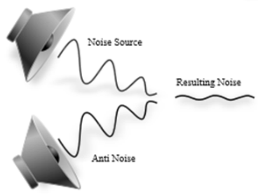
The Headphone: ANC

Bose QuietComfort 25 review:

The best noise-canceling headphones get better



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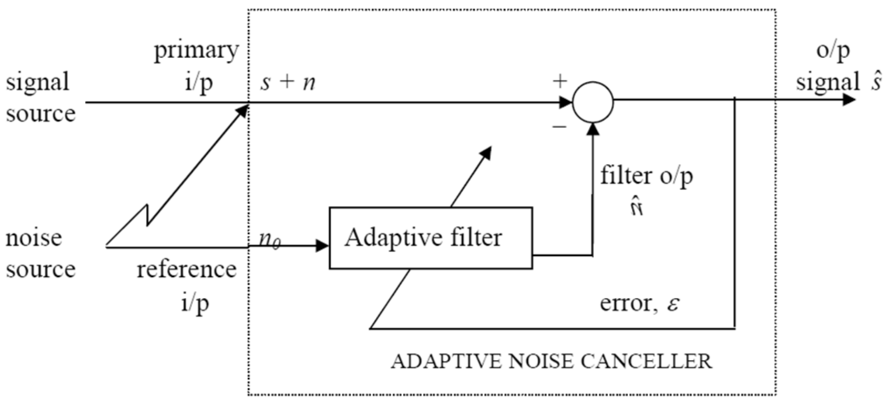
The Headphone: ANC

Adaptive Noise Canceller



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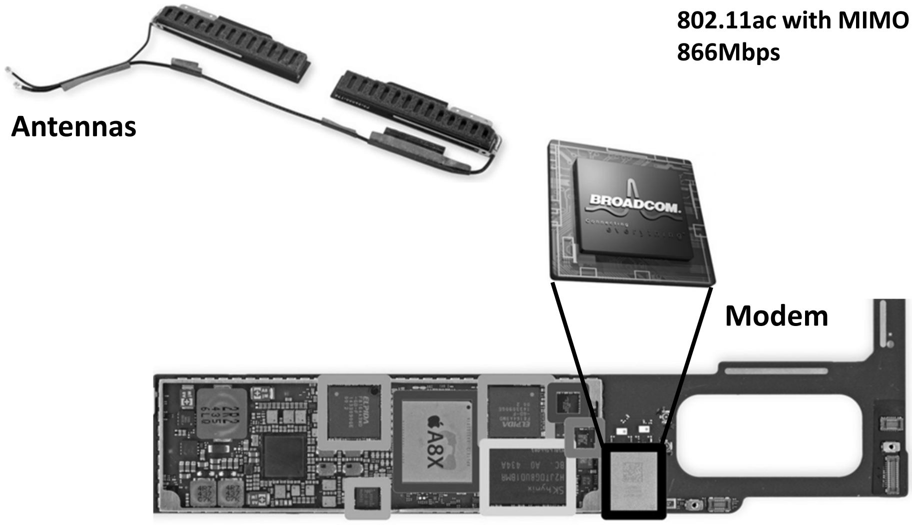
The WiFi Module



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The WiFi Module

- **A brief WiFi History**
  - 1997: 802.11 standard
    - IR
      clause 15
    - FHSS,
      clause 14
    - DSSS in 2.4GHz
      2Mbps
      clause 16
  - 1999: 802.11b
    - DSSS in 2.4GHz
      11Mbps, clase 17
  - 1999: 802.11a
    - OFDM in 5GHz
      54Mbps, clase 18
  - 2003: 802.11g
    - OFDM in 2.4GHz
      54Mbps
      clase 19
  - 2009: 802.11n
    - OFDM+MIMO SM
      20MHz BW → 300Mbps
      40MHz BW → 600Mbps
      clause 20 (HT PHY) in the 2012 revision of IEEE 802.11 standard
  - 2007~now: 802.11ac
    - VHT
    - enhancing 802.11n in 5GHz

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The WiFi Module

❑ A brief WiFi History

Spread spectrum → Multicarrier systems → Multiple antennas

Spread spectrum
- Mandated by regulators like FCC
- 802.11/11b DSSS and FHSS

Multicarrier systems
- OFDM is used for achieving high data rates
- First specified in 11a and then for 11g and 11n

Multiple antennas
- Enhanced capacity and reliability using MIMO-OFDM
- Main PHY feature of 11n

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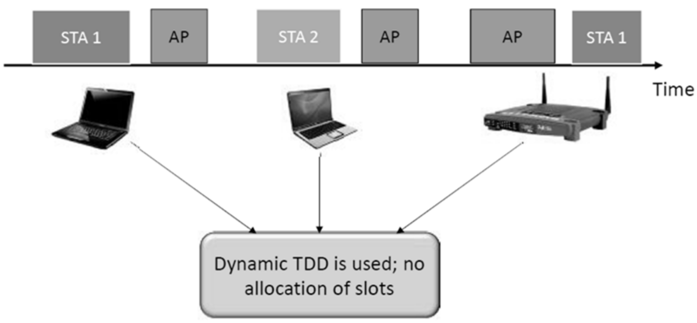
The WiFi Module

❑ Time-Division Duplexing



- All 802.11 transmissions in a given network are on the same RF channel
- AP and STAs are tuned to same RF channel

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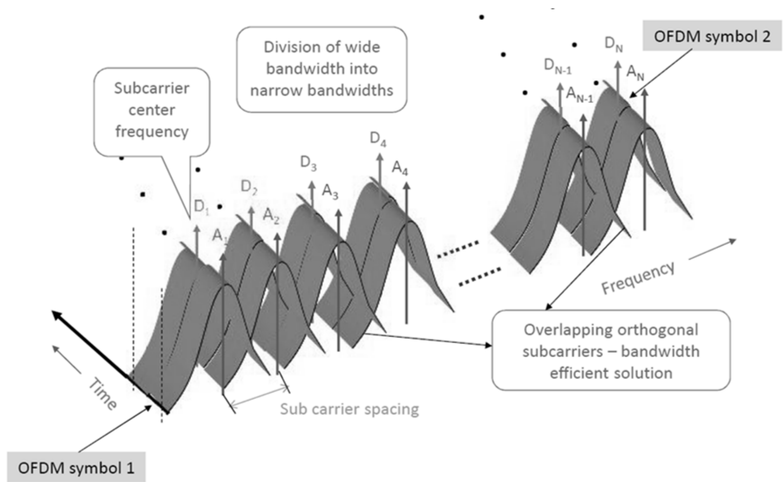
The WiFi Module

- OFDM



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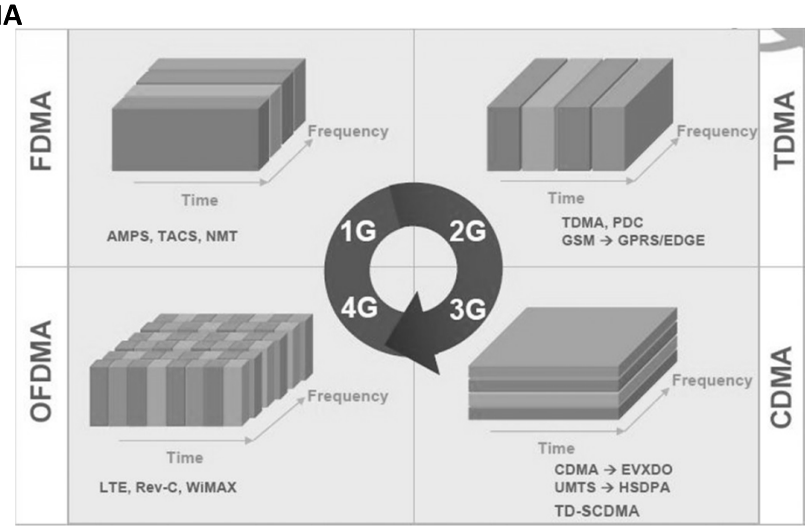
The WiFi Module

- OFDMA



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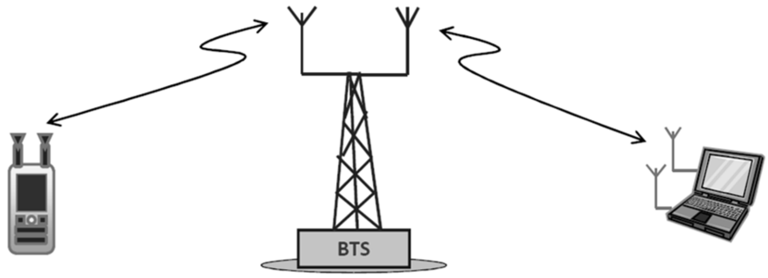
The WiFi Module

- MIMO



Smart signal processing/coding used at the TX./and or Rx.

Antennas are typically ordinary but DSP algorithms are important

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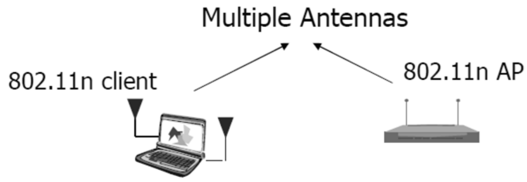
The WiFi Module

- MIMO



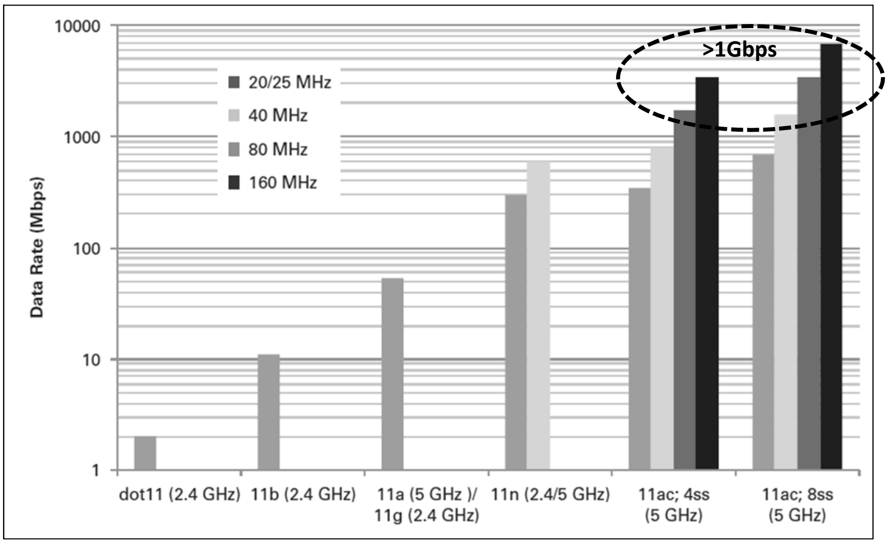
Using multiple antennas for multiplexing or diversity

Multiplexing ⇒ Higher rates
Highly correlated channels leads to poor performance

Diversity ⇒ Better performance/range
Indirectly lead to better rates

Combining spatial multiplexing and STBC can give advantages from both

Increase in WiFi PHY Rates



Quiz

- Find the Fourier transform of two signals

g(t)=A[u(t-1)-u(t-5)]

G(f) = integration g(t) exp(-j….) dt … … continue

p(n)=b^n u(n-1), n=0, 1,2,3,.. N-1, |b| < 1.

P(k)= sum p(n) exp(-j 2 pi nk/N) ? Continue…. 几何级数

Part VI: Advanced Reading

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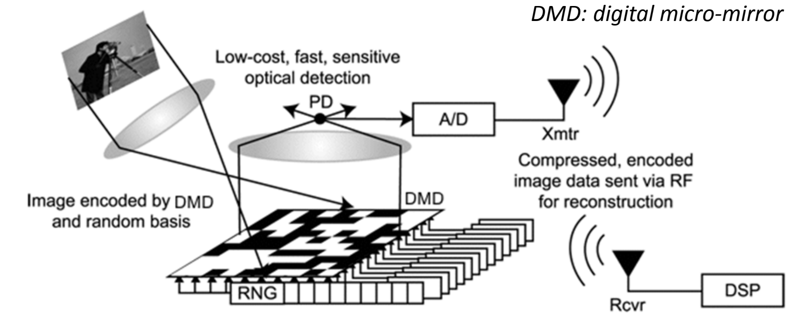
Single-Pixel Camera

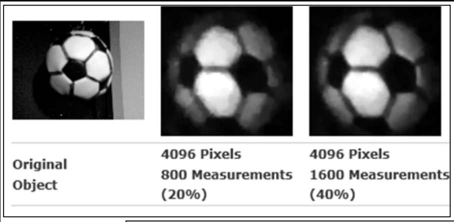
DMD: digital micro-mirror device

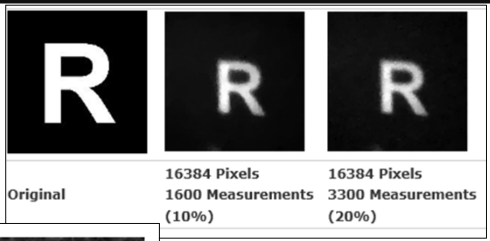


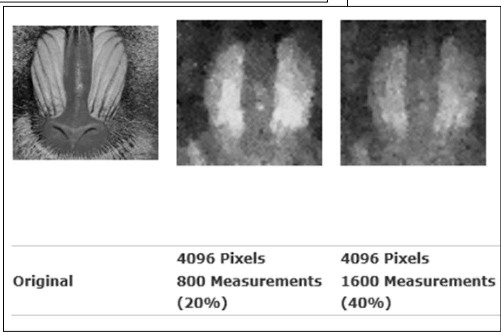
Camera directly acquires random projections of a scene without first collecting the pixels/voxels. The camera architecture employs a digital micromirror array to optically calculate linear projections of the scene onto pseudorandom binary patterns. Its key hallmark is its ability to obtain an image or video with a single detection element (the "single pixel") while measuring the scene fewer times than the number of pixels/voxels.

** Richard Baraniuk, Kevin Kelly, et al, http://dsp.rice.edu/cscamera*

Single-Pixel Camera







Compressive Sensing plays the trick!

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Compressive Sensing

Compressed sensing (also known as **compressive sensing**, **compressive sampling**, or **sparse sampling**) is a signal processing technique for efficiently acquiring and reconstructing a signal, by finding solutions to underdetermined linear systems

In a nutshell …

- ❑ Can obtain super-resolved signals from just a few sensors
- ❑ Sensing is nonadaptive: no effort to understand the signal
- ❑ Simple acquisition process followed by numerical optimization

First papers:

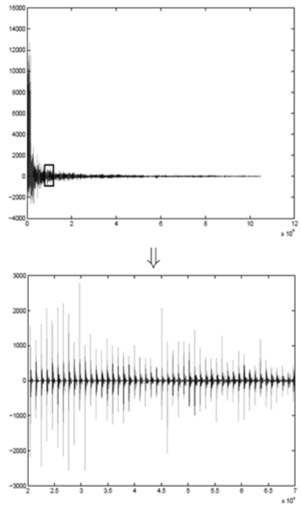
- ❑ Candes, Romberg and Tao, 2006
- ❑ Candels and Tao, 2006
- ❑ Donoho, 2006

** Emmanuel Candes, Compressive Sensing – A 25 Minute Tour*

Compressive Sensing

1 megapixel image

wavelet coefficients



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Compressive Sensing

1. Compute 1,000,000 wavelet coefficients of mega-pixel image
2. Set to zero all but the 25,000 largest coefficients
3. Invert the wavelet transform

original image

after zeroing out smallest coefficients

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Compressive Sensing

- Take 96K incoherent measurements of "compressed" image
- Compressed image is perfectly sparse (25K nonzero wavelet coeffs)
- Solve ℓ_1

original (25k wavelets)

perfect recovery

Introduction to Communications

by Yanlin Geng & Xiaojun Yuan

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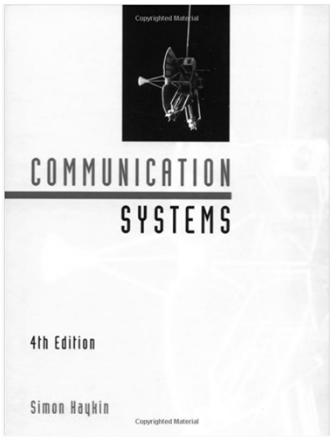
References

1. Simon Haykin, Communication Systems, John Wiley & Sons, 4th Edition, 2001.



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Word Counting

A sequence: "ACBACABA"

Q: How many A, B, C?
A: A = 4, B = 2, C = 2

Q: How frequently?
A: pA = 0.5, pB = 0.25, pC = 0.25

Word Counting

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Use {0, 1}'s to represent A, B, C:
    ? minimum length
    can recover "ACBACABA"

Observation:
    more frequently → shorter {0, 1} sequence

    Say: A → 0, B → 1, C → 01
    Then: "ACBACABA" → "0011001010"

Q: can we recover "ACBACABA" uniquely?

Word Counting

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A: no, since 001 → AC or AAB

Q: how to solve?

Huffman coding

A: Huffman coding

- A → 1
- B → 00
- C → 01

Q: any pattern?
A: prefix-free

- no codeword is prefix of others
- prefix code

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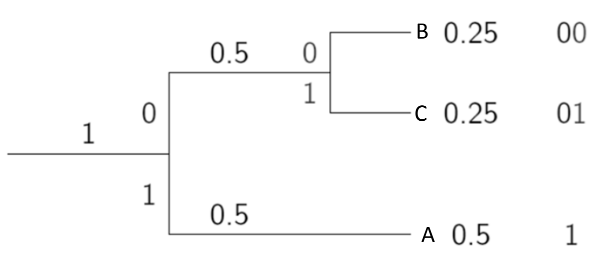
Huffman coding

Q: how Huffman coding works?
A: Keep merging two smallest frequencies:

A sequence: "ACBACABA" p_i codeword



Huffman coding

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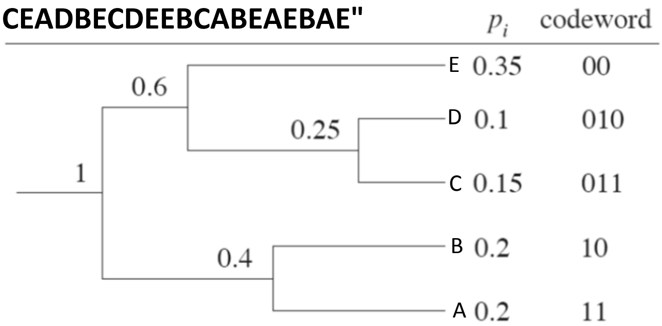
A slightly more complicated example.

“CEADBECDEEBCABEAEBAE"



Huffman coding

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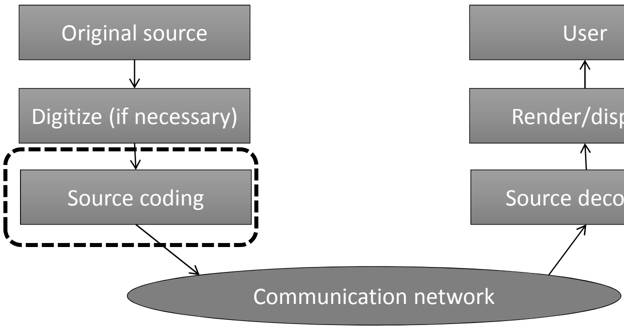
Q: codewords using {0, 1, 2}? find out

The End-to-End Commun. System



- The following lecture is about the oval.
- The simplest network is a single physical communication link.

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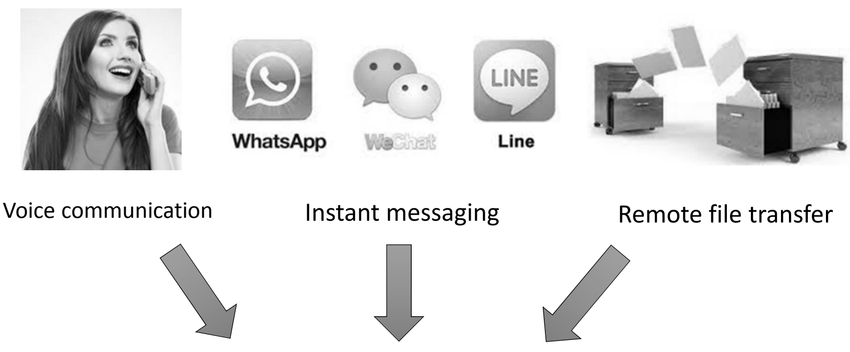
Modern Commun. is Digitalized



Voice communication
Instant messaging
Remote file transfer

010101000111……

Everything is just bits

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Physical Links Inherently Analog

Analog = continuous-valued and continuous-time
- Voltage waveform on a cable
- Light on a fiber, or in free space
- Radio waves through the air

Solution → Modulation

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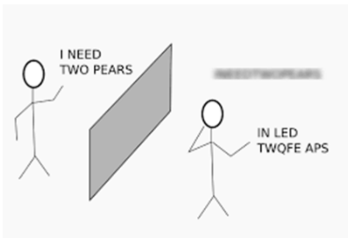
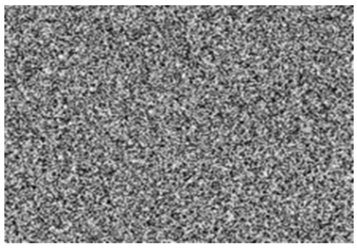
…and Physical Links Inherently Noisy



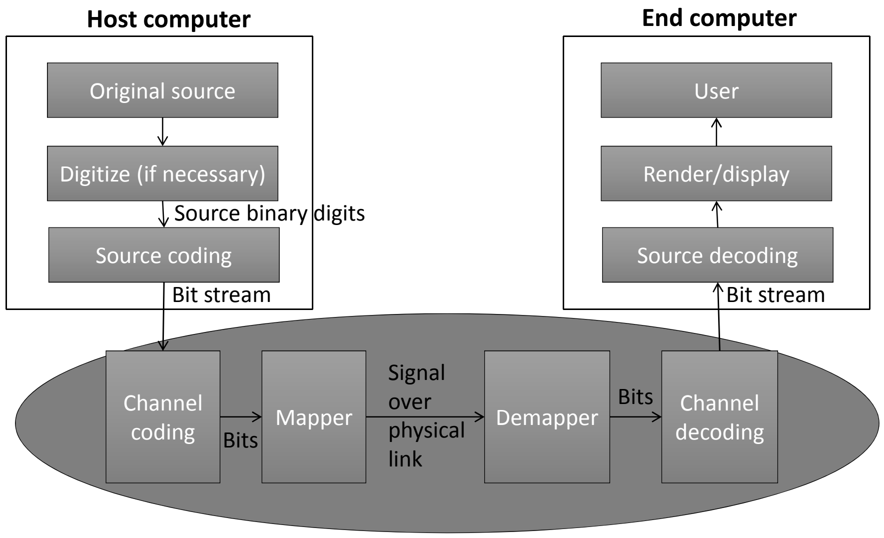
Obstacles



White noise

- Noise usually comes from the ambient environment and communication media.
- Communication errors occur due to channel noise.
- Solution → Channel coding

Single-Link Communication Model



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Modulation: Map Bits to Signals

Key Idea: Map or modulate the desired bit sequence onto a (continuous-time) analog signal

For ease of extracting the intended bits from the noisy received signals, we map bits to signals using a fixed set of discrete values.

For example, a two-level signaling scheme uses two “voltages”:

- V_0 is the binary value for “0”
- V_1 is the binary value for “1”

If $V_0 = -V_1$, we refer to this as bipolar signaling.

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Digital Signaling: Receiving

At the receiver, process and sample to get a “voltage”

- Voltages near V_0 would be interpreted as “0”
- Voltages near V_1 would be interpreted as “1”

If V_0 and V_1 are spaced far enough apart, we can tolerate some degree of noise – but there will be occasional errors!

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Digital Signaling: Receiving

We can specify the behavior of the receiver with the following decision rule (that shows how incoming voltages are mapped to “0” and “1”).

Decision Rule:

If the received voltage is below d, then “0” is transmitted; otherwise, “1” is transmitted.

In the above, d is called the threshold voltage.

The threshold voltage is usually chosen as the middle of the two voltage levels, i.e., $d = (V_0+V_1)/2$. Why?

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How to Reduce Decoding Error?

One simple way to reduce decoding error is by repetition.

Code: Bit b coded as bb…b (n times)

Exponential fall-off (log scale)

But huge overhead (low code rate)

We can do much better!

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Hamming Distance

The Hamming distance (HD) between two strings of equal length is the number of positions at which the corresponding symbols are different.

Examples:

- HD between "karolin" and "kerstin" is 3.
- HD between 1011101 and 1001001 is 2.

The HD between a valid binary codeword and the same codeword with e errors is e.

Hamming Distance

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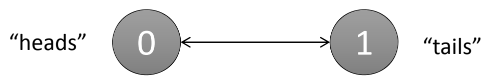
The Hamming Distance (HD) between a valid binary codeword and the same codeword with e errors is e.

The problem with no coding is that the two valid codewords (“0” and “1”) also have HD = 1. So a single-bit error changes a valid codeword into another valid codeword.



Q: What is the Hamming distance of a repetition code?

Hamming Distance and Coding

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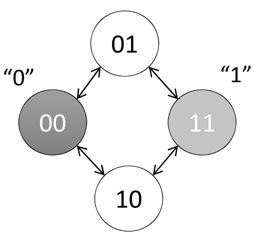
Encode so that the codewords are “far enough” from each other.

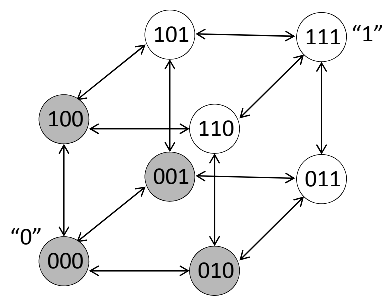
Likely error patterns shouldn’t transform one codeword to another.

Code: nodes chosen in hypercube + mapping of message bits to nodes

We choose 2k out of 2n nodes, meaning that we can map all k-bit message strings in a space of n-bit codewords. The code rate is k/n.



Can detect one-bit error



Can correct one-bit error

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Minimum HD vs. Detection & Correction Capabilities

If d is the minimum Hamming distance between codewords, we can detect all patterns of <= (d-1) bit errors

If d is the minimum Hamming distance between codewords, we can correct all patterns of (d-1)/2 or fewer bit errors

But how to construct codes with the above properties?

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A Simple Code: Parity Check

Add a parity check to message of length k to make the total number of “1” bits even.

If the number of “1”s in the received word is odd, then there has been an error.

- 010111100010 → original word with parity bit
- 010011100010 → single-bit error (detected)
- 010001100010 → two-bit error (not detected)

Minimum hamming distance of parity check code is 2.

- Can detect all single-bit errors
- Can detect all odd numbers of errors
- Cannot detect even numbers of errors
- Cannot correct any errors

Modulo-2 Algebra

Computations with binary numbers in code construction will involve Boolean algebra, or algebra in “GF(2)” (Galois field of order 2) or modulo-2 algebra:

0+0 = 0, 1+0 = 0+1 = 1, 1+1 = 0

0*0 = 0*1 = 1*0 = 0, 1*1 = 1

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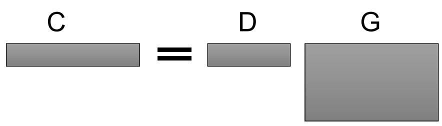
Linear Block Codes

Block Code: k message bits encoded to n code bits, i.e., each of 2k messages encoded into a unique n-bit combination via a linear transformation with GF(2) operations:



C is an n-element row vector containing the codeword

D is a k-element row vector containing the message

G is a k-by-n generation matrix

Each codeword bit is a specified linear combination of message bits.

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Minimum HD of Linear Code

- **Key property**: Sum of any two codewords is still a codeword → necessary and sufficient for code to be **linear**
  - So the all-zero codeword must be in any linear code --- why?
- (n, k) code has rate k/n
- Sometimes written as *(n, k, d)*, where d is the minimum Hamming distance of the code.
- The weight of a codeword is the number of "1"s in it.

- **The minimum HD of a linear code is the minimum weight found in its nonzero codewords. Why?**

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Examples: What are n, k, d?

{000, 111}	(3, 1, 3), rate = 1/3
{0000, 1100, 0011, 1111}	(4, 2, 2), rate = 1/2
{1111, 0000, 1100}	non-linear code
{0000, 1000, 0011, 1111}	non-linear code

0000000	1100001	1100110	0000111	**(7, 4, 3) code, rate = 4/7**
0101010	1001011	1001100	0101101	
1010010	0110011	0110100	1010101	
1111000	0011001	0011110	1111111	

The HD of a linear code is the minimum number of 1's in all codewords.

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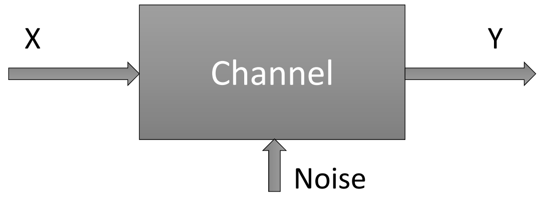
How Fast We can Deliver Bits

- **H(X):**
  - **Entropy** = the uncertainty in X
- **H(X|Y):**
  - **Uncertainty** about X reduced by knowing Y
- **I(X;Y) = H(X) – H(X|Y):**
  - **Mutual information** = **RATE** supported by the channel



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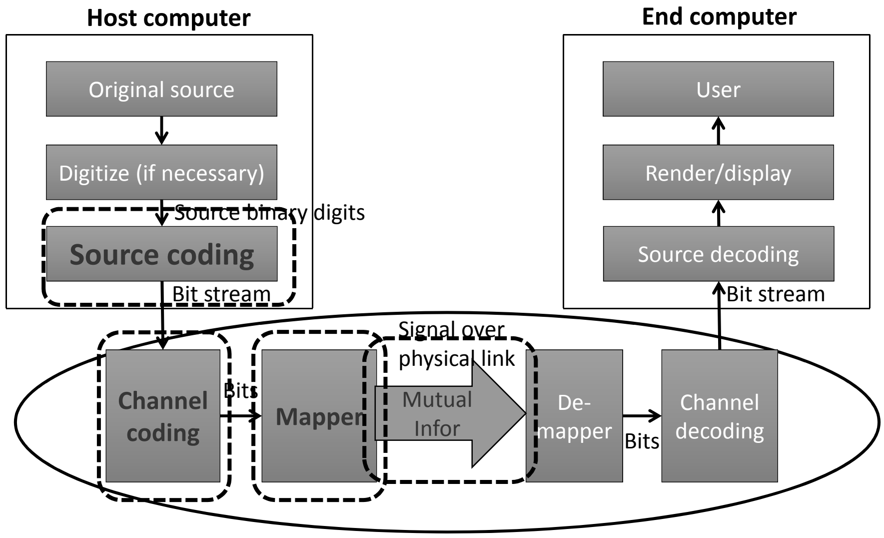
Single-Link Communication Model



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Homework

1. For the following text file, find out the number of occurrences of A, B, ..., Z and the space (‘a’ taken as ‘A’, ‘b’ as ‘B’, etc.)

a). do Huffman coding and calculate the number of bits needed to store this text file

b). how many bits needed is we simply apply the ASCII coding scheme?

```
Samsung still appears to be exclusively using its eight core
Exynos five four two zero as the S six app processor Though
the WSJ does not state whether Qualcomm will do so there is a
chance the company will also supply complementary RF transceiver
power management and envelope tracking ICs in units containing
its modems as is the case with the iPhone six which relies on a
Qualcomm modem and several complementary chips to go with Apple
A eight app processor
```

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Homework

2. Calculate the HD between the following two bit sequences:

a) 1001100111000

b) 0101110001001

3. Generate a code, i.e., a set of codewords, with the following property:

- **4-bit codewords with three information bits and with single error correction capability**

  i.e. min HD = 3

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Advanced Reading

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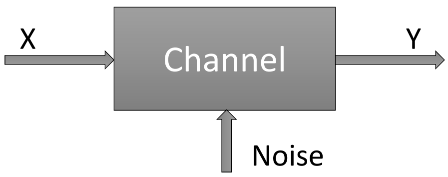
Channel Capacity



To characterize the channel, define

$$C = \max \{I(X; Y)\} = \max \{H(X) - H(X|Y)\}$$

where the maximization is over all possible distributions of X.

This is the most we can expect to reduce our uncertainty about X through the knowledge of Y, and so must be the most information we can expect to send through the channel.

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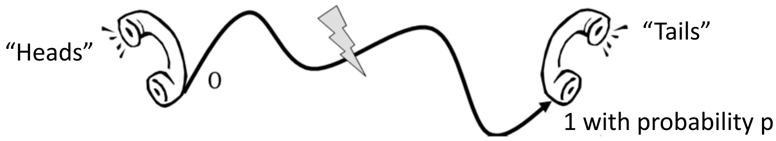
An Example: Binary Symmetric Channel

Suppose that during transmission a “0” is turned into a “1” or a “1” is turned into a “0” with probability *p*, independently of transmissions at other times.

This is a binary symmetric channel (BSC) – a useful and widely used abstraction.



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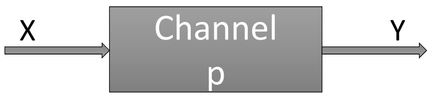
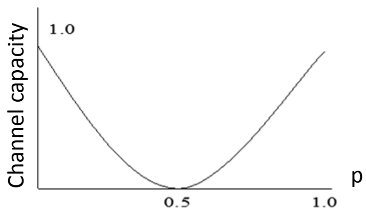
Capacity of the Binary Symmetric Channel

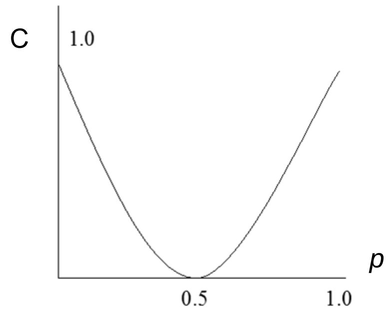


C = max {H(Y) – H(Y|X)}, where the maximization is over all possible distributions of X.

The second term doesn’t depend on this distribution, and the first term is maximized when 0 and 1 are equally probable at the input.



Capacity for Binary Symmetric Channel



For low noise channel, significant reduction in uncertainty about the input after observing the output. Explain why C = 1 for p = 0.

For high noise channel, little reduction. Explain why C = 0 for p = 0.5.

What happens to p = 1?

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Channel capacity tells us how fast and how accurately we can communicate …

Magic of Asymptotically Error-Free if R < C

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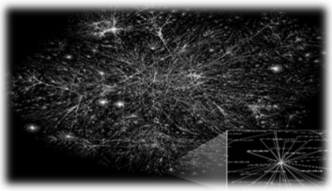
Shannon showed that one can theoretically transmit information (i.e., message bits) at any rate $R < C$ per use of the channel, with arbitrarily low error.

He also showed the converse, that transmission at any average rate $R \geq C$ incurs an error probability that is low-bounded by some positive number.

The secret: Encode blocks of k message bits into n-bit codewords, so $R = k/n$, with k and n very large.

Encoding blocks of k message bits into n-bit codewords to protect against channel errors is an example of channel coding.



Introduction to Networks

Professor: Ziyu Shao

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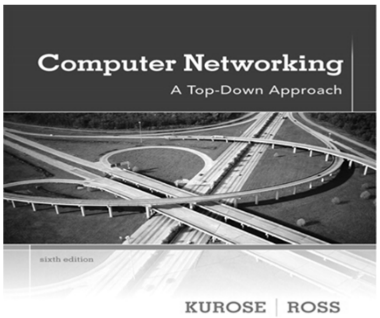
Reference

Acknowledgement:
Main contents of Parts 2,3,4 adopt materials from the official slides to accompany the following book, which is also the reference book



Computer Networking:
A Top Down Approach

6^{th} edition
Jim Kurose, Keith Ross
Addison-Wesley
March 2012

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Outline

1 Complex Networks
2 Foundation of Internet
3 Performance Metrics of Networks
4 Protocols & Layers
5 History

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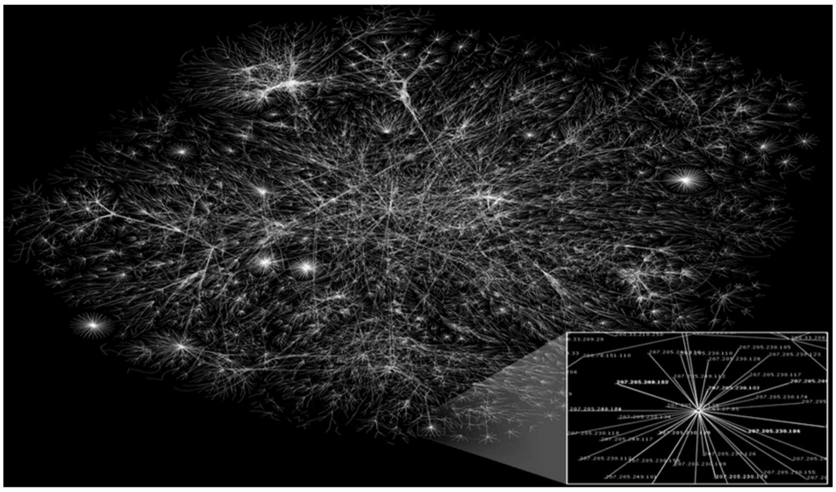
World Wide Web(WWW)



Source: http://en.wikipedia.org/wiki/File:Internet_map_1024.jpg

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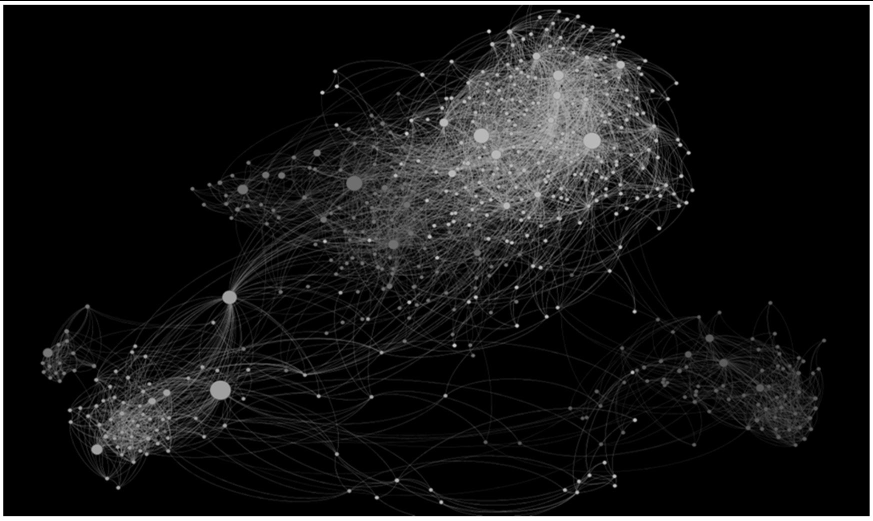
Facebook Friendship Network



Source: http://kimoquaintance.com/2011/08/22/what-can-we-learn-about-somalis-from-their-facebook-networks/

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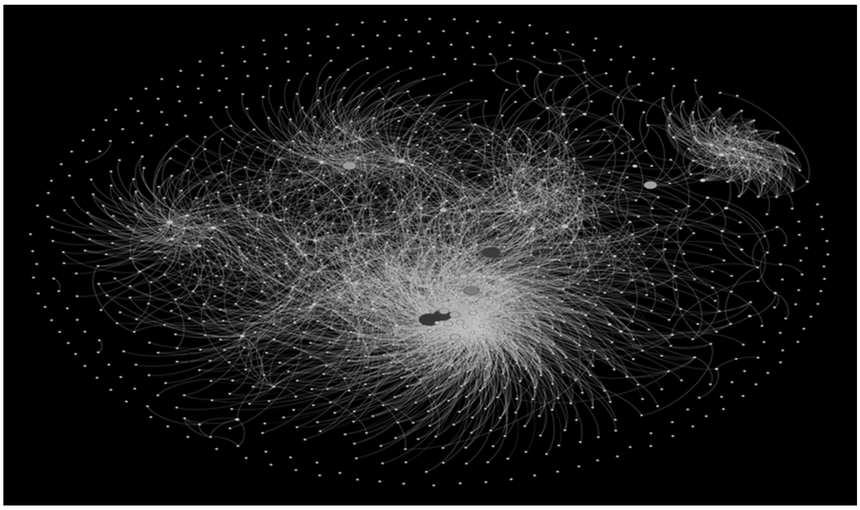
Facebook Friendship Network



Source: http://kimoquaintance.com/2011/08/22/what-can-we-learn-about-somalis-from-their-facebook-networks/

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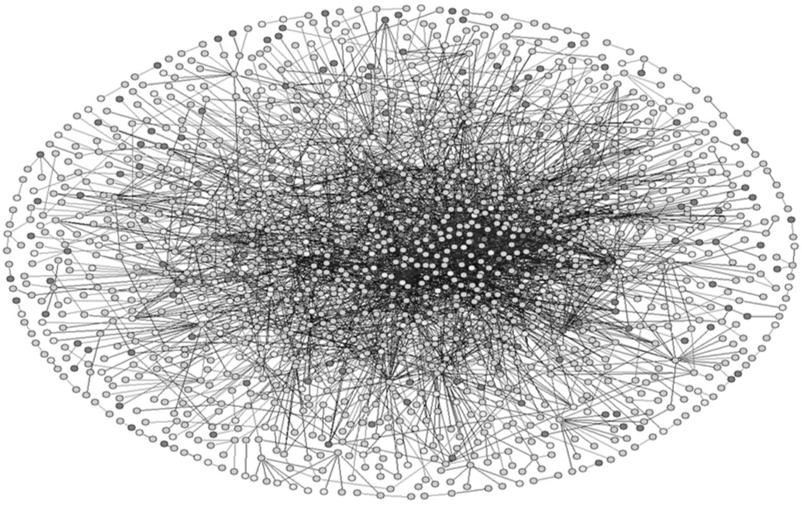
Protein Interaction Networks



Source: https://www.mdc-berlin.de/16074534/en/news/archive/2008/20080910-erwin_schr_dinger_prize_2008_goes_to_resea

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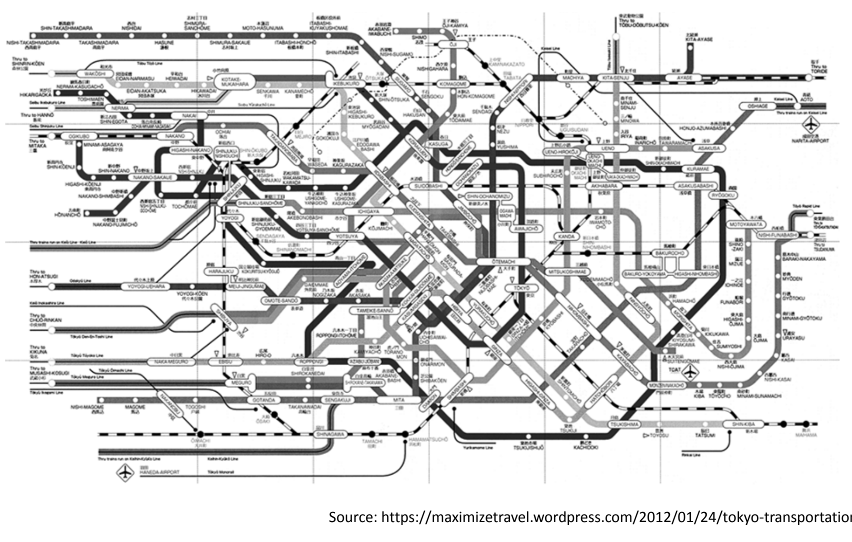
Tokyo Transportation Network



Source: https://maximizetravel.wordpress.com/2012/01/24/tokyo-transportation

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Outline

1 Complex Networks
2 Foundation of Internet
3 Performance Metrics of Networks
4 Protocols & Layers
5 History

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Internet Topology

Source: http://www.jacobsschool.ucsd.edu/news/news_releases/release.sfe?id=685

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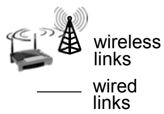
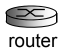
Elements of Internet

Millions of connected computing devices (*hosts*)



Communication Links (*optical fiber, copper, radio, satellite*)



Packet Switches (*routers & switches*)

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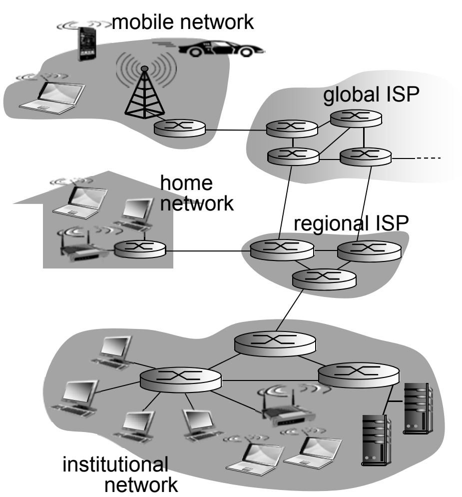
Structure of Internet



Network Edge:

end systems with hosts & access networks

Access Network:

connect end systems to edge routers

Network Core:
interconnected routers
network of networks

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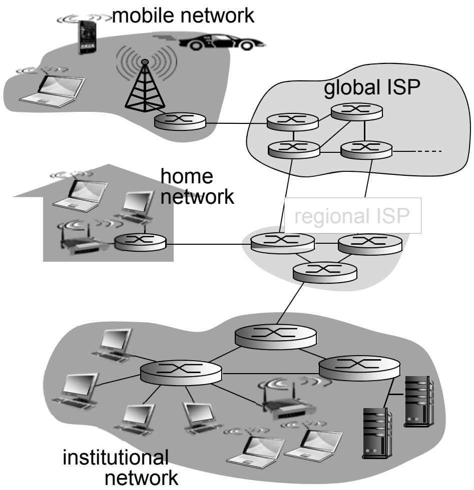
Access Network



The Last Mile Problem:
bandwidth (bits per second) of access network

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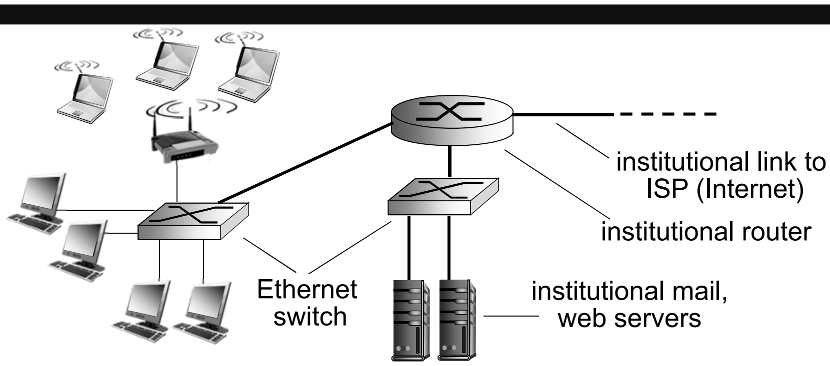
Enterprise Access Network (Ethernet)



Typically used in companies, universities, etc
10 Mbps, 100Mbps, 1Gbps, 10Gbps transmission rates
End systems typically connect into Ethernet switch

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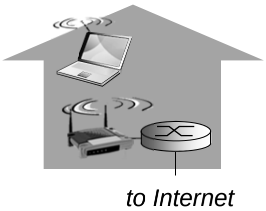
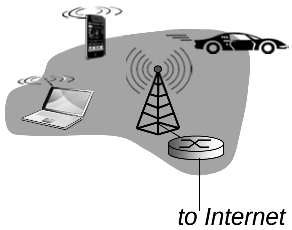
Wireless Access Network

- Shared *wireless* access network connects end system to router
  - via base station aka "access point"

wireless LANs:
- within building (100 ft)
- 802.11b/g (WiFi): 11, 54 Mbps transmission rate



wide-area wireless access
- provided by telco (cellular) operator, 10's km
- between 1 and 10 Mbps
- 3G, 4G: LTE



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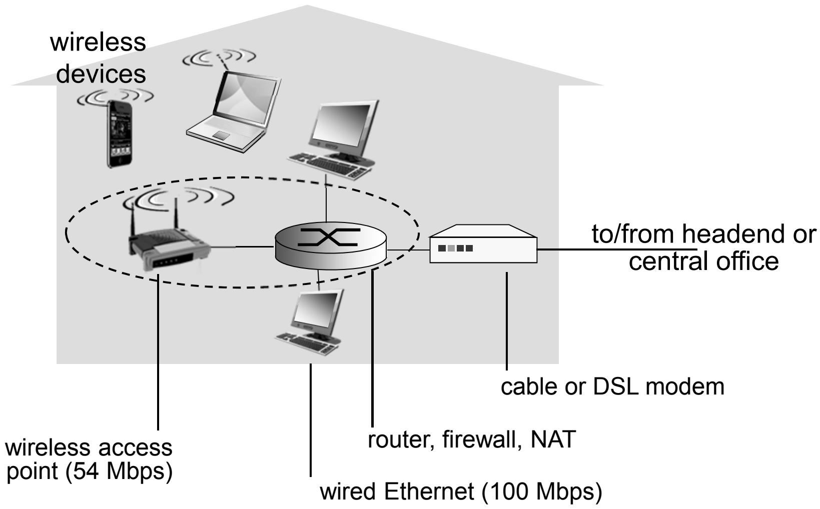
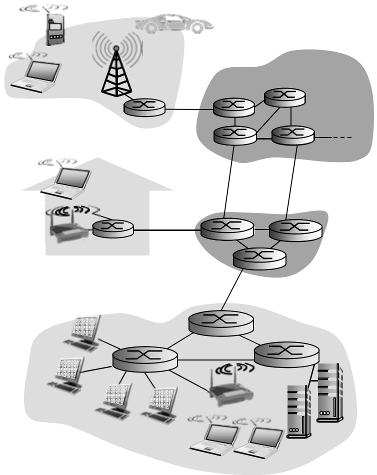
Home Access Network



Network Core



Mesh of interconnected routers

Packet Switching:
- *Hosts break messages into packets*
- *Forward packets from one router to the next, across links on path from source to destination*
- *Each packet transmitted at full link capacity*

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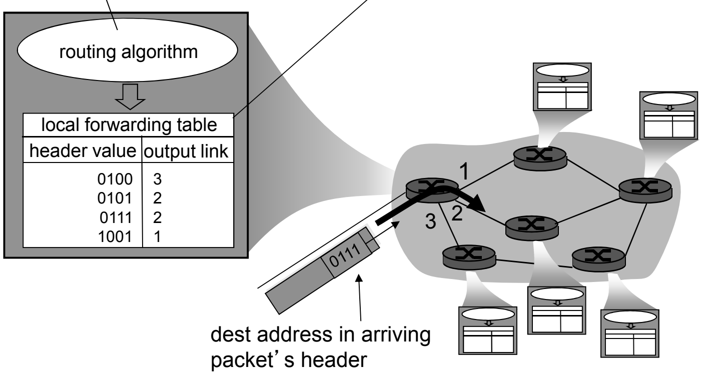
Two Key Functions of Router

routing: determines source-destination route taken by packets
- *routing algorithms*

forwarding: move packets from router's input to appropriate router output



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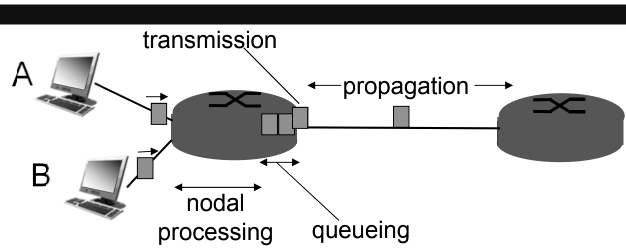
Metric 1: Packet Delay

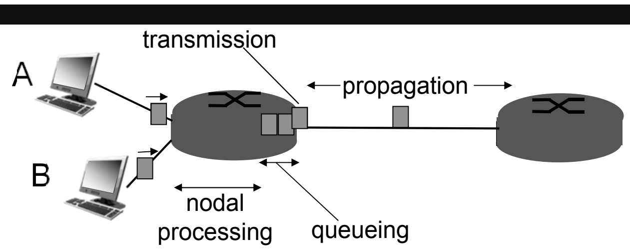


$d_{nodal} = d_{proc} + d_{queue} + d_{trans} + d_{prop}$

d_{proc}: nodal processing
- check bit errors
- determine output link
- typically < msec

d_{queue}: queueing delay
- time waiting at output link for transmission
- depends on congestion level of router

Metric 1: Packet Delay



$d_{nodal} = d_{proc} + d_{queue} + d_{trans} + d_{prop}$

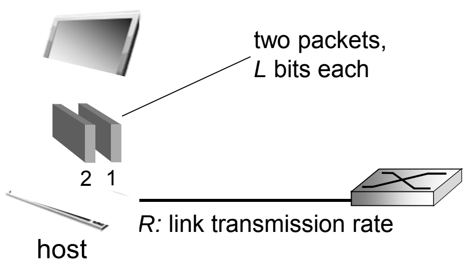
d_{trans}: transmission delay:
- *L*: packet length (bits)
- *R*: link *bandwidth (bps)*
- $d_{trans} = L/R$

d_{trans} and d_{prop} *very* different

d_{prop}: propagation delay:
- *d*: length of physical link
- *s*: propagation speed in medium ($\sim 3\times10^8$ m/sec)
- $d_{prop} = d/s$

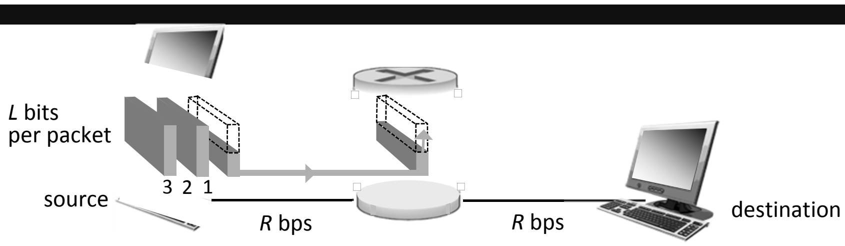
Transmission Delay: Scenario 1

host sending function:
- takes application message
- breaks into smaller chunks, known as *packets*, of length *L* bits
- transmits packet into access network at *transmission rate R*
  - link transmission rate, aka link *capacity, aka link bandwidth*



packet transmission delay = time needed to transmit *L*-bit packet into link = $\frac{L \text{ (bits)}}{R \text{ (bits/sec)}}$

Transmission Delay : Scenario 2



- takes *L*/*R* seconds to transmit (push out) *L*-bit packet into link at *R* bps
- *store and forward:* entire packet must arrive at router before it can be transmitted on next link

one-hop numerical example:

- *L* = 7.5 Mbits
- *R* = 1.5 Mbps
- one-hop transmission delay = 5 sec

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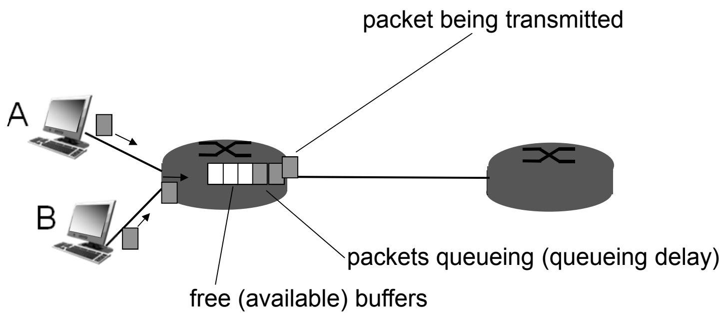
Queueing Delay

packets *queue* in router buffers

- packet arrival rate to link exceeds output link capacity
- packets queue, wait for turn



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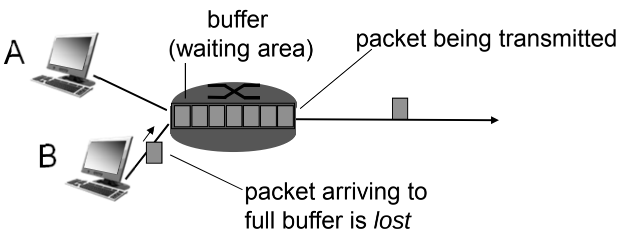
Metric 2: Packet Loss

- queue (aka buffer) preceding link in buffer has finite capacity
- packet arriving to full queue dropped (aka lost)
- lost packet may be retransmitted by previous node, by source of end system, or not at all



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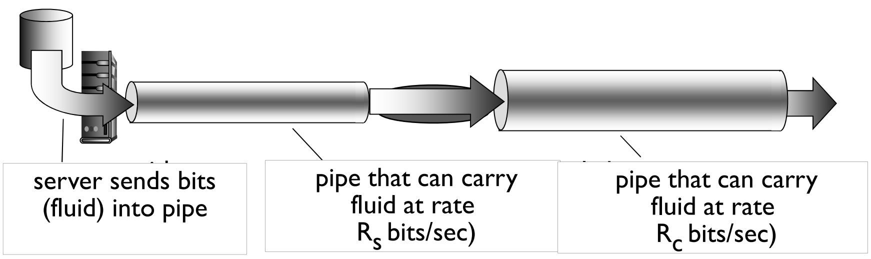
Metric 3: Throughput

- *throughput:* rate (bits/time unit) at which bits transferred between sender/receiver
  - *instantaneous:* rate at given point in time
  - *average:* rate over longer period of time



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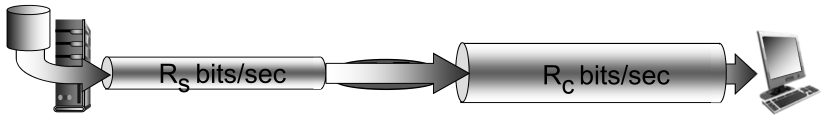
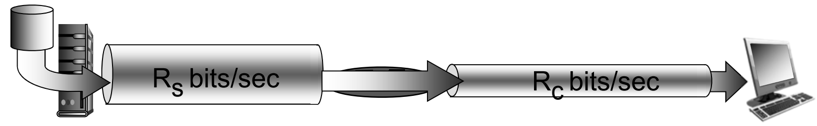
Throughput (more)

- $R_s < R_c$ What is average end-end throughput?



❖ $R_s > R_c$ What is average end-end throughput?



bottleneck link

link on end-end path that constrains end-end throughput

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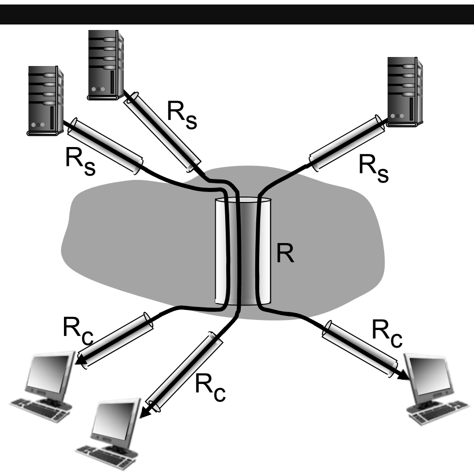
Throughput: Internet Scenario

- per-connection end-end throughput: $\min(R_c, R_s, R/10)$
- in practice: R_c or R_s is often bottleneck



10 connections (fairly) share backbone bottleneck link R bits/sec

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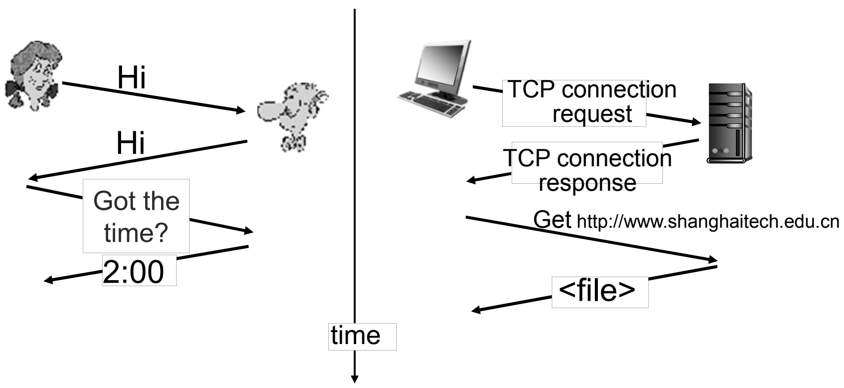
Protocols: Laws of Networks

a human protocol vs. a computer network protocol:



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Protocols: Laws of Networks

Protocols define format, order of messages sent and received among network entities, and actions taken on message transmission & receipt

All communication activity in Internet governed by protocols

Examples: TCP, UDP, IP, BGP, HTTP, 802.11

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The Big Question

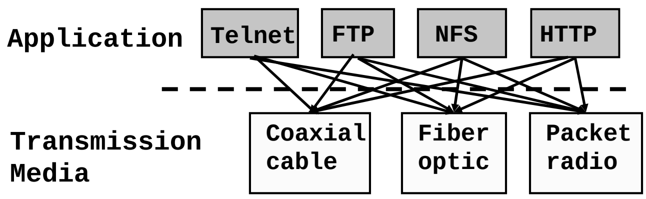
Networks are complex, with many "pieces":

- hosts
- routers
- links of various media
- applications
- protocols
- hardware, software

Question:

is there any hope of *organizing* structure of network?

The Problem



- Do we re-implement every application for every technology?
- Obviously not, but how does the Internet architecture avoid this?

Architecture

- Architecture is not the implementation itself
- Architecture is how to “organize” implementations
  - what interfaces are supported
  - where functionality is implemented
- Architecture is the modular design of the network

Layering

- Layering is a particular form of modularization
- The system is broken into a vertical hierarchy of logically distinct entities (layers)
- The service provided by one layer is based solely on the service provided by layer below
- Rigid structure: easy reuse, performance may suffers

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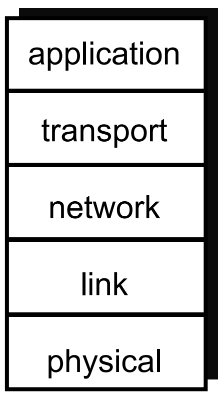
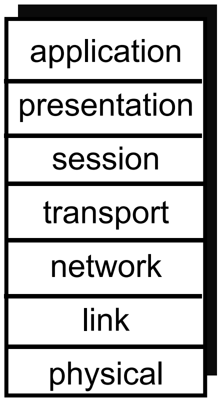
Layering Model for Internet

- *application:* supporting network applications
  - FTP, SMTP, HTTP
- *transport:* process-process data transfer
  - TCP, UDP
- *network:* routing of datagrams from source to destination
  - IP, routing protocols
- *link:* data transfer between neighboring network elements
  - Ethernet, 802.11(WiFi)
- *physical:* bits "on the wire"



OSI Reference Model for Layers

- *presentation:* allow applications to interpret meaning of data, e.g., encryption, compression, machine-specific conventions
- *session:* synchronization, checkpointing, recovery of data exchange
- Internet stack “missing” these layers!
  - these services, *if needed,* must be implemented in application
  - needed?



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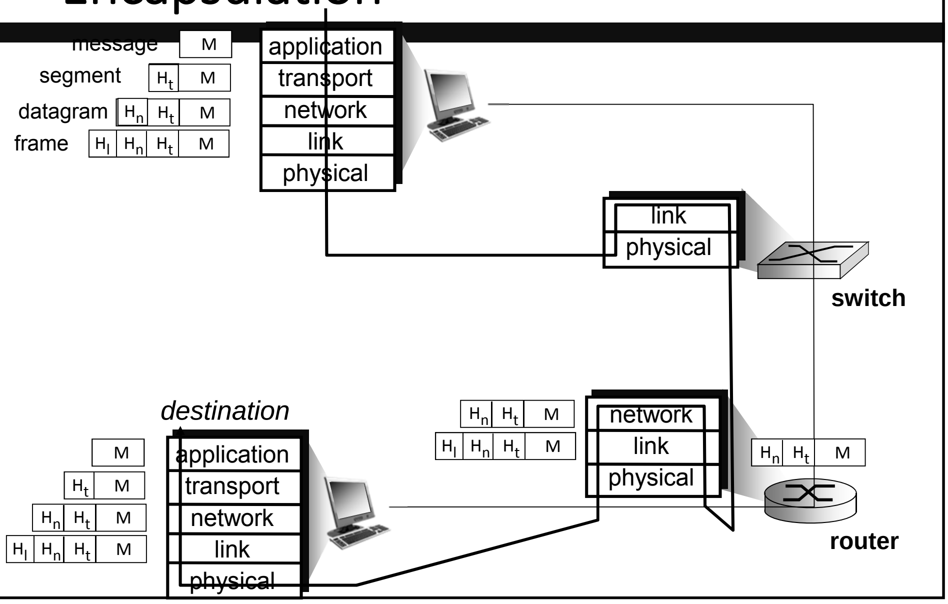
Encapsulation



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Layering Solves Problem

- Application layer doesn't know about anything below the presentation (or transport) layer, etc.
- Information about network is hidden from higher layers
- This ensures that we only need to implement an application once!

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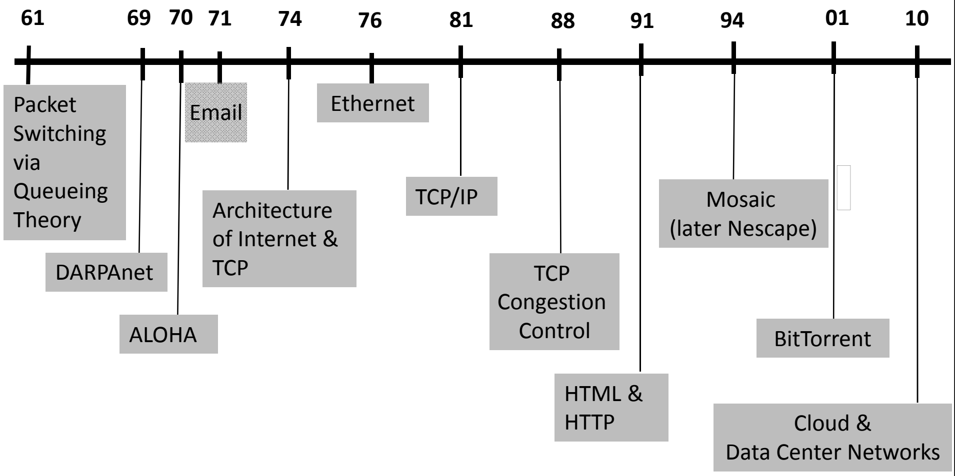
History of Moments



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Father of the Internet

Vincent Cerf

Robert Kahn

Interesting video: *https://www.youtube.com/watch?v=xA6Ccg4sdXc*

2014 public lecture by Vincent Cerf & Robert Kahn for the 40th anniversary of Internet.

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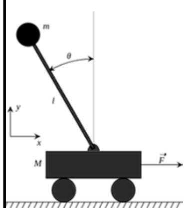
Homework: Simulation

1. Consider a sequence of packets passing through one link.

- Packets arrive one by one, and the time intervals between arriving packets satisfying a probability distribution with a mean equaling to 6 units.
- The bandwidth of the link is random, satisfying a probability distribution with a mean equaling to 0.2 packet per units.
- The size of the link buffer is 3(full buffer with three packets).
- Zero nodal processing delay & propagation delay.

You need to use python to simulate the behavior of such system with various probability distributions. Please record the average queueing delay, average delay, average throughput and loss rate. You are also required to make observations and provide intuitive explanations.



Feedback Control

by Boris Houska

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Overview

Why do we need control systems ?

What is a feedback controller ?

How do modern feedback controllers work ?

Can you stand on one leg?

Can you stand on one leg?

Try it …

Now close your eyes…

What did you experience?

Why is it difficult to keep your balance with closed eyes?

Basic Question of Control

Sensors ("Eyes")
- Cameras
- Radar
- GPS
- Inertial sensors (IMU)
- Pressures
- Temperatures
- ...

Basic Question of Control

Sensors ("Eyes")
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Actuators ("Muscles")
- Motors
- Flaps
- Valves
- Propellers
- Heaters
- Pumps
- ...

Basic Question of Control

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Actuators ("Muscles")
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- Propellers
- Heaters
- Pumps
- ...

How to connect?

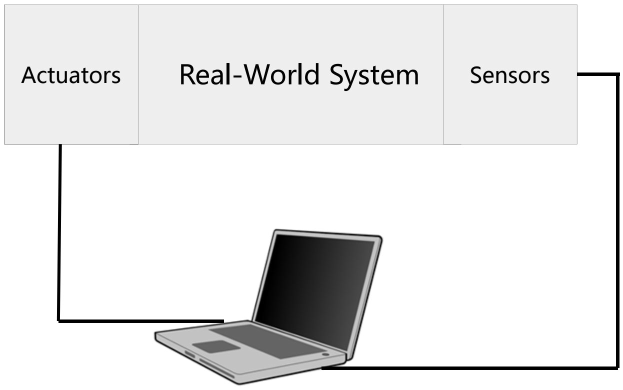
Feedback Control

Step 1: Wait for the measurement from the sensor

Feedback Control

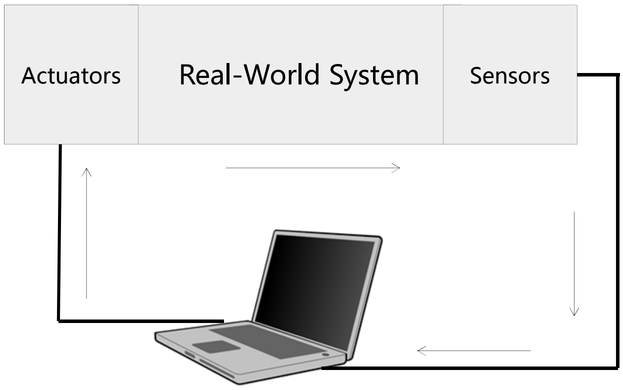
Step 2: Filter the incoming data and compute a control reaction

Feedback Control



Step 3: Send out the control signal to the actuators

Feedback Control



Run Step 1 – 3 in a loop!

Zillions of Applications

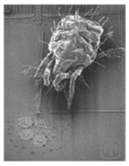
Airplanes start and land using auto-pilots

Most chemical production process are optimized and automated

Automatic heating control in building saves huge amounts of energy

… there are many many more !!!




(source: wikipedia)

Quiz

What are the actuators?

Quiz

What are the sensors?

Quiz

What are the actuators?

Quiz

What are the sensors?

Quiz

What are the actuators?

Quiz

What are the sensors?

Feedback Control

Run Step 1 – 3 in a loop!

Models of Systems

- Most systems that we study and investigate are linear systems
- Non-linear systems are controlled typically by first linearizing them
- What is a linear system? $x_1(t)$ -> $y_1(t)$, $x_2(t)$ -> $y_2(t)$
- $a_1 x_1(t) + a_2 x_2(t)$ -> output $y(t) = a_1 y_1(t) + a_2 y_2(t)$
- What is a linear time-invariant system?

  input $x(t)$ -> output $y(t)$

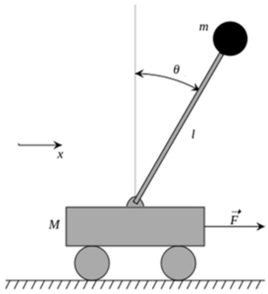
  delayed input $x(t-T_d)$ -> delayed output $y(t-T_d)$
- How to model a linear system?

  impulse response $h(t)$ or transfer function $H(f)$

Goal of this lecture

Learn how to design a controller with a few lines of Python code

Motivating Example: Inverted Pendulum



Can we bring the pendulum to its inverted position?

Sensor: Camera measuring positions and velocities

Actuator: motor adjusting the force F

Potential and Kinetic Energy

Potential energy:

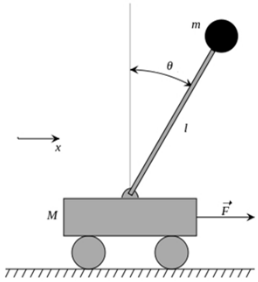
$$V = mgh$$

Kinetic energy:

$$T = \frac{1}{2}mv^2$$

Energy of the Inverted Pendulum

$$Q_j = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j}$$



Potential energy:

$$V = mgl\cos(\theta)$$

Kinetic energy:

$$T = \frac{1}{2}M\dot{x}^2 + \frac{m}{2}\left(\dot{x}^2 + 2l\dot{x}\dot{\theta}\cos(\theta) + l^2\dot{\theta}^2\right)$$

Notation: dot above variable means derivative w.r.t. time

Equations of Motion

Lagrange function:

$$L = T - V$$

Lagrange formalism:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} - \frac{\mathrm{d}L}{\mathrm{d}x} = (M+m)\ddot{x} + ml\cos(\theta)\ddot{\theta} - ml\sin(\theta)\dot{\theta}^2 = F$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}} - \frac{\mathrm{d}L}{\mathrm{d}\theta} = ml^2\ddot{\theta} + ml\cos(\theta)\ddot{x} + mgl\sin(\theta) = 0$$

Equations of Motion

Lagrange function:

$$L = T - V$$

Lagrange formalism:

Inertia

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} - \frac{\mathrm{d}L}{\mathrm{d}x} = (M+m)\ddot{x} + ml\cos(\theta)\ddot{\theta} - ml\sin(\theta)\dot{\theta}^2 = F$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}} - \frac{\mathrm{d}L}{\mathrm{d}\theta} = ml^2\ddot{\theta} + ml\cos(\theta)\ddot{x} + mgl\sin(\theta) = 0$$

Equations of Motion

Lagrange function:

$$L = T - V$$

Lagrange formalism:

Inertia centrifugal force

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} - \frac{\mathrm{d}L}{\mathrm{d}x} = (M+m)\ddot{x} + ml\cos(\theta)\ddot{\theta} - ml\sin(\theta)\dot{\theta}^2 = F$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}} - \frac{\mathrm{d}L}{\mathrm{d}\theta} = ml^2\ddot{\theta} + ml\cos(\theta)\ddot{x} + mgl\sin(\theta) = 0$$

Equations of Motion

Lagrange function:

$$L = T - V$$

Lagrange formalism:

Inertia — centrifugal force

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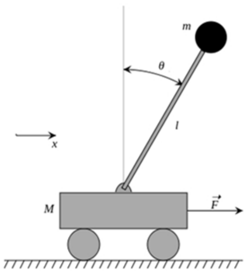
$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}} - \frac{\mathrm{d}L}{\mathrm{d}\theta} = ml^2\ddot{\theta} + ml\cos(\theta)\ddot{x} + mgl\sin(\theta) = 0$$

gravity

Simplified Equations of Motion

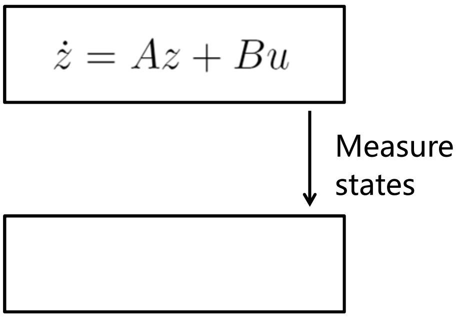


For small excitation angles:

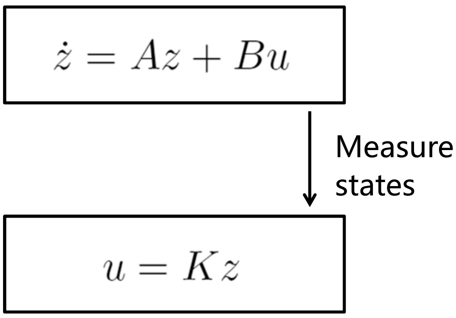
$$\sin(\theta) \approx \theta \quad \cos(\theta) \approx 1 \quad \dot{\theta}^2 \approx 0$$

$$\begin{aligned}\ddot{x} &= \frac{F}{M} - \frac{mg}{M}\theta \\ \ddot{\theta} &= -\frac{F}{Ml} + \frac{M+m}{M}\frac{g}{l}\theta\end{aligned}$$

Linear Control System



Linear Control System



Linear feedback law

Linear Control System

$$\dot{z} = Az + Bu$$

Send back controls ↑ ↓ Measure states

$$u = Kz$$

Linear feedback law

Inverted Pendulum in Standard Form

States:

$$z = \begin{pmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{pmatrix}$$

- x: Position of trolley
- θ: Angle of the pendulum
- $\dot{x}$: Velocity of the trolley
- $\dot{\theta}$: Angular velocity of the pendulum

System matrices:

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{mg}{M} & 0 & 0 \\ 0 & \frac{M+m}{M}\frac{g}{l} & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{M} \\ -\frac{1}{Ml} \end{pmatrix}$$

Matrices (Arrays) and Linear Algebra

$$\dot{z} = Az + Bu$$

A, B=? Simple way to represent coefficients

z=? Vector of important information (states)

$\dot{z}$ = ? Vector of state changes

Az= how current states impact future state changes?

Bu= how control signals impact future state changes

Matrix Representation of Linear Equations

$$y_1 = -3x_1 + x_2 + 1.5x_3 + u_1$$

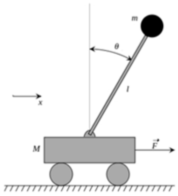
$$y_2 = 2x_1 - 4x_2 + u_1 - u_2$$

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 1 & 1.5 \\ 2 & -4 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$\mathbf{y} = A\mathbf{x} + B\mathbf{u}$$

Linear Control System



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Linear feedback law

Linear Control System

$$\dot{z} = Az + Bu$$

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Linear feedback law

Closed Loop Dynamics: $\dot{z} = (A + BK)\, z$

Closed Loop Dynamics in the State Space

Closed-loop trajectory satisfies

$$\dot{z} = (A + BK)\, z$$

Explicit solution

$$z(t) = e^{(A+BK)t} z(0)$$

Simulation of Closed-Loop Dynamics in Python

Math-Syntax:

$$z(t) = e^{(A+BK)t} z(0)$$

Python Syntax:

```
z = la.expm((A+B*K)*t)*z0;
```

Simulation of Closed-Loop Dynamics in Python

Math-Syntax:

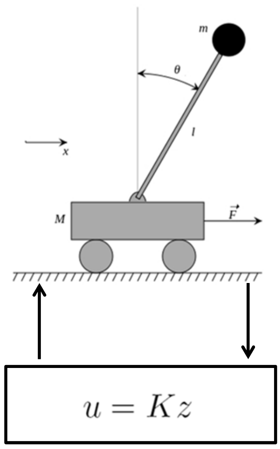
$$z(t) = e^{(A+BK)t} z(0)$$

Python Syntax:

```
z = la.expm((A+B*K)*t)*z0;
```

Simulates linear closed-loop systems with 1 line of code!

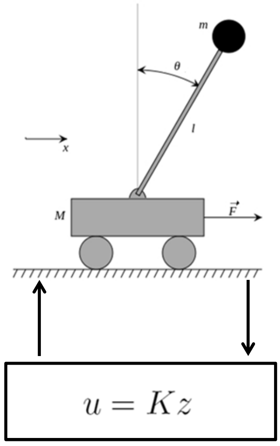
What happens for K = 0 ?



$$\begin{aligned} \ddot{x} &= \frac{F}{M} - \frac{mg}{M}\theta \\ \ddot{\theta} &= -\frac{F}{Ml} + \frac{M+m}{M}\frac{g}{l}\theta \end{aligned}$$

Linear feedback law

What happens for K = 0 ?



Linear feedback law

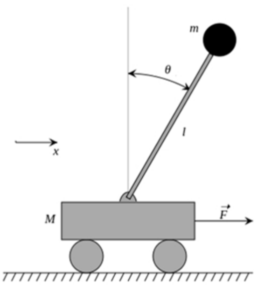
$$
\begin{aligned}
\ddot{x} &= \frac{F}{M} - \frac{mg}{M}\theta \\
\ddot{\theta} &= -\frac{F}{Ml} + \frac{M+m}{M}\frac{g}{l}\theta
\end{aligned}
$$

For K = 0 we apply no force

u = F = 0

unstable!

How to choose K ?



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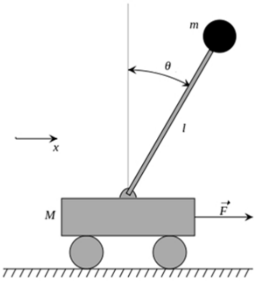
\ddot{\theta} &= -\frac{F}{Ml} + \frac{M+m}{M}\frac{g}{l}\theta
\end{aligned}
$$

Intuition:

If angle positive, choose F positive

If angle negative, choose F negative

How to choose K ?



$$\begin{aligned} \ddot{x} &= \frac{F}{M} - \frac{mg}{M}\theta \\ \ddot{\theta} &= -\frac{F}{Ml} + \frac{M+m}{M}\frac{g}{l}\theta \end{aligned}$$

Intuition:

Let's try the choice $K = (\,0,\, b,\, 0,\, 0\,)$ with $b > (m+M)g$

Associated control law: $F = Kz = b\theta$

Let's implement this!

```
def simulate( x0, K ):    # inputs: initial values, control gain

       import numpy        as np
       import scipy.linalg as la

       M = 0.4;               # mass of the trolley (in kg)
       m = 0.1;               # mass of the pendulum (in kg)
       l   = 0.2;               # length of the pendulum (in m)
       g  = 9.81;             # gravitational constant (in m/s^2)
```

Let's implement this!

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```

Guidelines:

- Give intuitive names to variables and functions

- Don't forget to comment your code

Let's implement this!

```
def simulate( x0, K ):    # inputs: initial values, control gain

  # MODEL PARAMETERS
  # ---------------------------------------------------------
     [...]

  # SETUP OF THE SYSTEM MATRICES
  # ---------------------------------------------------------
    A = np.matrix(np.zeros((4,4)));    # initialize matrix A
    B = np.matrix(np.zeros((4,1)));    # initialize matrix B

    A[0,2] = 1.0  ;# derivative of trolley position equals velocity
    A[1,3] = 1.0  ;# derivative of angle equals angular velocity
    A[2,1] = -m*g/M ; # accelaration of the trolley
    A[3,1] = (m+M)*g/(M*l);    # angular acceleration

    B[2,0] = 1.0/M     ; # influence of the force on acceleration
    B[3,0] = -1.0/(M*l); # influence of the force on the angular
                           # acceleration
```

Let's implement this!

```
def simulate( x0, K ):   # inputs: initial values, control gain

  # MODEL PARAMETERS
  # ---------------------------------------------------------
     [...]

  # SETUP OF THE SYSTEM MATRICES
  # ---------------------------------------------------------
     [...]

  ## SIMULATION OF THE CLOSED-LOOP SYSTEM
  ## ------------------------------------------------
       N = 500 ;  # plot resolution
       T  = 10.0; # time horizon (in seconds)
       t   = np.linspace(0,T,N+1);        # setup time points
       x  = np.matrix(np.zeros((N+1,4))); # time series of states
       x[0] = x0;                         # store the initial value

       X = la.expm( (A+B*K)*(T/N) );      # state transition matrix

       for i in range(N):
           x[i+1] = x[i]*np.transpose(X);   # closed-loop simulation
```

Let's implement this!

```
def simulate( x0, K ):    # inputs: initial values, control gain

  # MODEL PARAMETERS
  # ---------------------------------------------------------
     [...]

  # SETUP OF THE SYSTEM MATRICES
  # ---------------------------------------------------------
     [...]

  # SIMULATION OF THE CLOSED-LOOP SYSTEM
  # ---------------------------------------------------------
     [...]

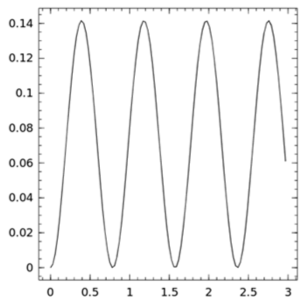
  # RETURN THE RESULT OF THE SIMULATION
  # ---------------------------------------------------------
        return (t,x);
```

20 lines of self contained code to simulate a closed-loop system for an inverted pendulum

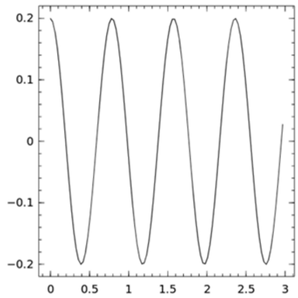
Closed-Loop Simulation Example

$$K = (\,0,\, b,\, 0,\, 0\,) \qquad b > (m+M)g$$

Trolley position [m] versus time [s]



Angle [rad] versus time [s]



Pendulum does not fall down, but oscillates

How to choose K ?

$$\begin{aligned}\ddot{x} &= \frac{F}{M} - \frac{mg}{M}\theta \\ \ddot{\theta} &= -\frac{F}{Ml} + \frac{M+m}{M}\frac{g}{l}\theta\end{aligned}$$

Intuition:

We want to "reduce" the oscillation

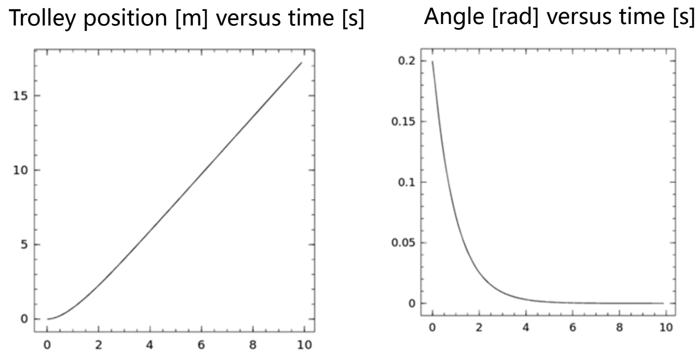
Strategy:

If angular velocity positive, increase F

If angular velocity negative, decrease F

Closed-Loop Simulation Example

$$K = (0,\, b,\, 0,\, d) \qquad b > (m+M)g \qquad d > 0$$



Pendulum stabilized at inverted position, but trolley drifts away

How to choose K ?

$$\begin{aligned} \ddot{x} &= \frac{F}{M} - \frac{mg}{M}\theta \\ \ddot{\theta} &= -\frac{F}{Ml} + \frac{M+m}{M}\frac{g}{l}\theta \end{aligned}$$

Goal of our lab exercise:

We do not only want to stabilize the pendulum, but also the trolley.

How to choose K ?

$$
\begin{aligned}
\ddot{x} &= \frac{F}{M} - \frac{mg}{M}\theta \\
\ddot{\theta} &= -\frac{F}{Ml} + \frac{M+m}{M}\frac{g}{l}\theta
\end{aligned}
$$

Goal of our lab exercise:

We do not only want to stabilize the pendulum, but also the trolley.

Strategy:

$$K = (\, a,\, b,\, c,\, d\,)$$

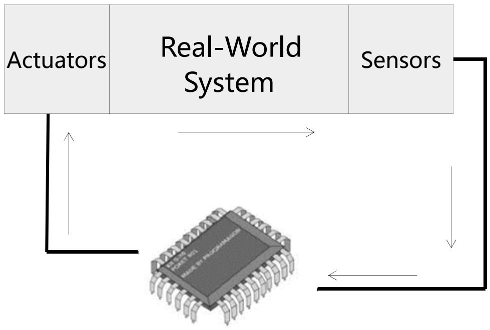
Tune all four coefficients!

Modern Feedback Control

- Nowadays, we do not tune controllers “by hand”
- Instead: use state-of-the-art optimization algorithms

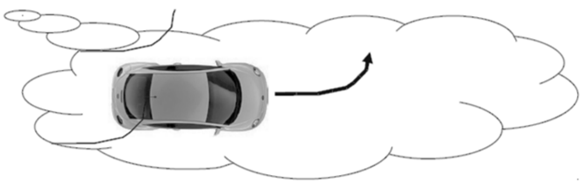
Modern Feedback Control

- Nowadays, we do not tune controllers "by hand"
- Instead: use state-of-the-art optimization algorithms
- Small optimization problems can be solved within micro-seconds and on embedded hardware



Embedded Optimization

Idea: always look a bit into the future.



Brain predicts and optimizes:
e.g. slow down **before** curve

Use repeated computation of optimal controls for feedback control!

Why Optimization + Control ?

Control objective:

Minimize time

Why Optimization + Control ?

Control objective:

Minimize fuel consumption/ safe energy

(subject to other tasks)

Why Optimization + Control ?

Control objective:

Maximize energy

Take Home Points

- Feedback control connects sensors and actuators

We tuned and simulated a closed loop system with 20 lines of code

Take Home Points

- With minor modifications we could use these 20 lines of code to tune and simulate an airplane autopilot

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Even simple controllers operate faster and more accurately than any human pilot

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- With minor modifications we could use these 20 lines of code to tune and simulate an airplane autopilot

Even simple controllers operate faster and more accurately than any human pilot

Just a few lines of code, no joke!

Take Home Points

Modern control methods use optimization algorithms